Search, Price Adjustment and Inflation Dynamics

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preliminary (missing appendix)

Abstract

Several authors have noted positive relationships between the average rate of inflation and both the response of nominal prices to monetary and cost shocks and the variance of inflation. In this paper we study the responses of both nominal and real prices to random fluctuations in costs and money growth using a monetary economy with search frictions and no constraints on sellers' ability to change prices. The economy exhibits a form of nominal price stickiness in that the price level may react incompletely or sluggishly to changes in either the stock of money or the level of nominal cost. The degree of incomplete price adjustment varies with the average rate of inflation. At low levels of inflation, prices are relatively unresponsive to both cost and money growth shocks. As the inflation rate rises, prices become more responsive to both types of shocks. The model is consistent with empirical findings suggesting that the degree of price adjustment in response to both cost and money growth shocks is increasing in the average rate of inflation and that the variance of inflation increases with its average level.

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1. Introduction

It has been argued that the responsiveness of nominal prices to various shocks is related to the average rate of inflation. For example, Devereux and Yetman (2002) present evidence that the pass-through of nominal exchange rate movements (which may be interpreted as cost shocks) to consumer prices is declining in the average rate of inflation for a sample of 107 countries during the post-Bretton Woods era. Also, Taylor (2000) argues that the response of nominal prices to increases in costs has declined with the rate of inflation over time for the U.S. and other developed countries.

This relationship, as well as the broader issue of the source of nominal rigidity in the economy is generally ignored by the large literature focusing on the *effects* of price stickiness. In this literature, price changes are typically subject to explicit costs and/or frequency limitations, and the effects of shocks in a neighborhood of a constant (often zero) inflation steady-state are considered. Many of these papers focus on cyclical monetary policy and how the central bank should respond to shocks given both the degree of nominal rigidity and underlying economic trends (including inflation).

In this paper, we develop a monetary economy with search frictions and fully flexible prices in which both nominal and real prices may adjust incompletely to random fluctuations in costs and the money growth rate. In our economy, this endogenous "stickiness" of prices may decline with average inflation, a prediction consistent with the observations of both Devereux and Yetman (2002) and Taylor (2000). Our approach differs from that taken in most of the "sticky price" literature in that we impose no cost of price adjustment or restrictions on the ability of agents to change prices each period. As a result, incomplete adjustment of prices is associated with the profit maximizing strategies of price-posting sellers. The optimal pricing response to shocks varies with the state of the economy, generating a relationship between trend inflation and both price adjustment and the dynamics of inflation. Also, in our model the search friction which generates market power and a relationship between price adjustment and inflation also generates the demand for fiat money in equilibrium.

Our model embeds the price-posting game of Burdett and Judd (1983) in a general equilibrium environment along the lines of the random matching monetary models of Shi (1999) and Head and Shi (2003). In a similar but non-stochastic environment, Head and Kumar (2004) study the welfare costs of trend inflation under certainty. In their model, the Burdett-Judd pricing framework generates price dispersion in equilibrium, with the extent of dispersion depending on the average rate of inflation. In this paper, that model is extended to include stochastic elements and our focus is on the response of nominal prices to random shocks. Here, both the average degree of price dispersion and its response to shocks are key factors determining price adjustment in equilibrium.

In the economy, shocks to both costs and the money growth rate are passed-through differentially to consumer prices by sellers pricing in different regions of the price distribution. For a fixed degree of search intensity, an increase in costs or a persistent increase in the money growth generates increased price dispersion. Increased prices dispersion, however, raises the gains to search inducing a larger fraction of buyers to observe more than one price. This lowers sellers' market power overall and limits the extent to which prices can rise in response to such shocks. The overall adjustment of prices is thus determined by the combination of two opposing effects. An increase (decrease) in costs or the money growth rate raises (lowers) prices for a fixed degree of search intensity. The endogenous response of search intensity, however, weakens (strengthens) sellers' market power thus putting downward (upward) pressure on prices.

The relative strengths of these conflicting effects depends on the average rate of inflation. At a low average rate of inflation, a relatively large fraction of buyers observes only a single price. A increase of either production costs or money growth generates a large increase in price dispersion, and thus induces a large increase in search intensity. The resulting reduction in sellers' market power limits the adjustment of prices in response to these shocks. As the rate of trend inflation rises, *ceteris paribus*, the average share of buyers observing more than one price falls, a given shock has a smaller effect on price dispersion and thus generates a smaller response of search intensity. As a result, the pass-through of both cost and monetary growth shocks increases with the average inflation rate. Moreover, at sufficiently high inflation, average prices become closely tied to marginal cost and inflation effectively moves one-for-one with changes in costs. Thus, our results on the relationship between the responses of both real and nominal prices to cost movements and the average inflation rate are consistent with the observations of both Devereux and Yetman (2002) and Taylor (2000).

We also consider the dynamics of inflation in our economy. We show that the variance of inflation induced by cost shocks of a given magnitude rises (along with the degree of price adjustment) with the trend rate of inflation. We also show that the autocorrelation of the inflation rate in response to money growth shocks varies with the average rate of inflation. At moderate *average* inflation, the rate of inflation may respond sluggishly to changes in the money growth rate. In our economy, the dynamics of inflation are affected principally by movements in expected future inflation. Sluggish movements in expected inflation may generate very persistent responses of inflation to changes in the money growth rate.

The relationship between average inflation and the extent of price adjustment in response to shocks is studied in many papers on "state-contingent" pricing models, including Dotsey, King and Wolman (1999) and Devereux and Siu (2003). Several of our results are similar to those of this literature, in spite of the fact that we impose no exogenous nominal rigidity. For example, our model predicts asymmetric responses of prices to positive and negative cost shocks, as does that of Devereux and Siu (2003). In both our model and theirs, increases in cost may lead to larger price responses than reductions in cost of the same magnitude. Also, state-contingent pricing models with menu costs (*e.g.* the theoretical model of Devereux and Yetman (2002)), predict the price level to be more responsive to shocks at higher inflation, as a larger share of firms will find it profitable to change prices in a given period the higher the rate of inflation. Craig and Rocheteau (2004) consider the implications of menu costs for the welfare costs of inflation in a model similar to ours in the sense that a search friction makes fiat money essential in equilibrium. They do not consider the adjustment of prices to shocks.

The remainder of the paper is organized as follows: Section 2 describes the environment. In section 3, we define a symmetric (Markov) monetary equilibrium for this environment and outline our numerical procedure for computing such equilibria. The effects of random shocks to costs and money growth are considered in a series of computational experiments in section 4. In Section 5 we consider the dynamics of inflation in some in parametric examples in which the expected future inflation may or may not respond sluggishly to shocks to the money growth rate. Section 6 summarizes, describes some implications of the results for future work, and concludes.

2. The Economy

2.1. The environment

Time is discrete. There are $H \geq 3$ different types of both households and non-storable consumption goods, and there are large numbers (*i.e.* unit measures) of households of each type. A type h household produces only good h and derives utility only from consumption of good $h+1$, modulo H. Each household is comprised of large numbers (unit measures) of two different types of members; "buyers" and "sellers". Individual household members do not have independent preferences and do not undertake independent actions. Rather, they share equally in *household* utility and act only on instructions *from the household.*

Members of a representative type h household who are *sellers* can produce good h in period t at marginal cost $\phi_t > 0$ utils per unit. Production costs are stochastic; ϕ_t evolves via a discrete

Markov chain with

$$\text{Prob}\left\{\phi_{t+1} = \phi' | \phi_t = \phi\right\} \equiv \pi^{\phi}(\phi', \phi) \qquad \forall t, t+1; \quad \phi', \phi \in \mathcal{P}, \tag{2.1}$$

where $\mathcal{P}$ is a finite set of possible production cost parameters. Let y_t denote the total quantity of good h produced by all the sellers from this household in period t. Then the household's total period disutility from production is equal to $\phi_t y_t$.

Members of this household who are *buyers* observe random numbers of price quotes and may purchase good $h+1$ at the lowest price that they observe individually. Let q_{kt} denote the measures of the household's buyers which observe $k \in \{0, \ldots, K\}$ price quotes. The household will choose these measures, but it *does not* choose the exact number of price quotes observed by any specific individual buyer. Rather, it chooses the probabilities with which buyers observe different numbers of quotes. Since the household contains a unit measure of buyers in total, the probability of an individual buyer observing k prices is equivalent to the measure of a household's buyers who observe k prices.[1] For each price quote observed, the household pays an information or search cost of μ utils. Thus, the household's total disutility of search in period t is equal to $\mu \sum_{k=0}^{K} k q_{kt}$.

A representative household acts so as to maximize the expected discounted sum of its period utility over an infinite horizon:

$$U = \text{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \phi_t y_t - \mu \sum_{k=0}^{K} k q_{kt} \right] \right]. \tag{2.2}$$

The household's period utility equals that which it receives from consumption of goods purchased by its buyers minus the production disutility incurred by its sellers and its search costs. Consumption utility is given by $u(c_t)$, where c_t is the total purchases of good $h+1$ by the household's buyers. We assume that $u(\cdot)$ is strictly increasing and concave, with $\lim_{c \to 0} u'(c)c = \infty$. For convenience, in most of our analysis we will let $u(\cdot)$ have the constant relative risk aversion (*CRRA*) form:

$$u(c) = \frac{c_t^{1-\alpha}}{1-\alpha}, \tag{2.3}$$

with $\alpha > 1$.

Since a type h household produces good h and consumes good $h+1$, a double coincidence of wants between members of any two households is impossible. Moreover, it is assumed that

[1] The maximum number of price quotes observed, K, is unimportant, as we will show later. We may think of K being chosen by the household, or of the household as setting $q_k = 0$ for all $k \geq K$.

households of a given type are indistinguishable and that members of individual households cannot be relocated in the future following an exchange. Since consumption goods are non-storable, direct exchanges of goods cannot be mutually beneficial. Rather, exchange is facilitated by the existence of perfectly durable and intrinsically worthless *fiat money.* A type h household may acquire fiat money by having its producers sell output to buyers of type $h-1$ households. This money may then be exchanged for consumption good $h+1$ by the household's own buyers in a future period.

In the initial period ($t = 0$) households of all types are endowed with M_0 units of fiat money. The *per household* stock of this money is denoted M_t, for each t. At the beginning of each period $t \geq 1$ households receive a lump-sum transfer, $(\gamma_t - 1)M_{t-1}$, of new units of fiat money from a monetary authority with no purpose other than to change the stock of money over time. We assume that the gross growth rate of the money stock,

$$\gamma_{t+1} = \frac{M_{t+1}}{M_t}, \tag{2.4}$$

evolves stochastically via a discrete Markov chain:

$$\text{Prob}\{\gamma_{t+1} = \gamma' | \gamma_t = \gamma\} \equiv \pi^\gamma(\gamma', \gamma) \qquad \forall t, t+1; \quad \gamma', \gamma \in \mathcal{G}, \tag{2.5}$$

where $\mathcal{G}$, like $\mathcal{P}$, is a finite set.

Finally, it is useful to define the vector, $\sigma_t = (\phi_t, \gamma_t)$, of exogenous stochastic parameters. Using (2.1) and (2.5) we define a Markov process for σ:

$$\text{Prob}\{\sigma_{t+1} = \sigma' | \sigma_t = \sigma\} \equiv \Pi(\sigma', \sigma) \qquad \forall t, t+1; \quad \sigma', \sigma \in \mathcal{S} \equiv \mathcal{P} \times \mathcal{G}. \tag{2.6}$$

In each period, the *state* of the economy is given by σ_t and the per household stock of money, M_t.

2.2. The current period trading session

In describing the optimization problem of a representative household (of any type), it is useful to begin with the current period trading session. At the beginning of period t a representative household observes the state of the economy, (M_t, σ_t) and has post-transfer *household* money holdings m_t.[2] The household chooses the probabilities with which an individual buyer observes different numbers of price quotes, $\mathcal{Q}_t \equiv \{q_{0t}, \ldots, q_{Kt}\}$, and issues trading instructions to both its buyers and sellers to maximize utility. Buyers and sellers then split up for a *trading session.* We

[2] Where possible, capital letters (*e.g. C, Q, M*) will be used to distinguish per household quantities from their counterparts for an individual household (*c, q, m*) etc.. In the exposition, we will suppress the economy state vector as it remains fixed throughout the trading session.

assume that it is not until this trading session begins that the exact number of quotes observed by *individual* buyers is known. As a result, households have no incentive to treat their members asymmetrically; they distribute money holdings equally to all buyers and issue the same instructions to all buyers and to all sellers.[3]

In the trading session, sellers post prices and buyers decide whether or not to purchase at the posted price, each acting in accordance with household instructions. As trading begins *after* $\mathcal{Q}_t$ is chosen, for now we treat the measures of buyers observing particular numbers of price quotes as fixed (as they are throughout the current period trading session) and return to their determination when we consider households' dynamic optimization later. Exchanges of goods for fiat money take place in bilateral matches between buyers and sellers of different households. Following trading, buyers and sellers reconvene and the household consumes the goods purchased by its buyers. The sellers' revenue (in fiat money) and any remaining money unspent by the buyers are pooled and carried into the next period, when they are augmented with transfer $(\gamma_{t+1}-1)M_t$ to become m_{t+1}.

With $\mathcal{Q}_t$ fixed, the mechanism by which buyers and sellers are matched is similar to the "noisy sequential search" process of Burdett and Judd (1983). Households know the distribution of prices offered by sellers, but individual buyers may purchase only at a price they are quoted by a specific prospective seller in a particular period.[4] Let the distribution of prices posted by sellers of the appropriate type at time t be described by the cumulative distribution function (*c.d.f.*) $F_t(p_t)$ on support $\mathcal{F}_t$. Given $F_t(p_t)$, the *c.d.f.* of the distribution of the *lowest* price quote received by a buyer at time t is given by

$$J_t(p_t) = \sum_{k=0}^{K} q_k \left[1 - [1 - F_t(p_t)]^k\right] \qquad \forall p_t \in \mathcal{F}_t. \tag{2.7}$$

Individual buyers are constrained to spend no more than the money distributed to them at the beginning of the session by the household. If buyer i purchases he/she does so at the lowest price observed, spending $x_{it}(p_t)$ conditional on the price paid. Thus buyers face the expenditure constraint

$$x_{it}(p_t) \leq m_t \qquad \forall i, p_t. \tag{2.8}$$

[3] The optimality of equal treatment of symmetric members by the household may be established as in Petersen and Shi (2004). For brevity, we state it here as an assumption.

[4] We assume that buyers cannot return to sellers from whom they have purchased in the past and instead draw new price quotes from the distribution each period. This assumption enables price dispersion to persist in a stationary equilibrium of our model. Empirical evidence in Lach (2002) suggests that price dispersion is indeed persistent and that individual sellers change their prices frequently, limiting the ability of buyers to identify low price sellers for repeat purchases.

Buyers, being identical, act symmetrically if they receive the same lowest price quote (*i.e.* $x_{it} = x_t$ for all households). Because the household contains a continuum of symmetric buyers, it faces no uncertainty with regard to its *overall* trading opportunities in the trading session of the current period. Realized household consumption purchases in this period are then

$$c_t = (1 - q_{0t}) \int_{\mathcal{F}_t} \frac{x_t(p_t)}{p_t} dJ_t(p_t), \tag{2.9}$$

where q_{0t} is the probability with which a buyer observes no price quote, or alternatively, the measure of such buyers.

An individual seller produces to meet the demand of the buyers who observe his/her price and wish to purchase. Expected sales in the current period trading session for a seller who posts p_t are given by

$$y(p_t) = \frac{X_t(p_t)}{p_t} \sum_{k=0}^{K} Q_{kt} k \left[1 - F_t(p_t)\right]^{k-1}. \tag{2.10}$$

Here $X_t(p_t)$ is the spending rule of a type $h-1$ buyer, $F_t(p_t)$ is the distribution of prices posted by the seller's competitors, and Q_{kt} is the average measure of buyers observing k prices.

In (2.10), $X_t(p_t)/p_t$ represents the *quantity per sale* and the summation term is the *expected number of sales*. The expected number of sales equals the number of observations of the seller's price multiplied by the probability that in each of these instances it is the lowest price observed. The number of observations is the ratio of the measures of buyers to sellers (in this case one) times the expected number of price observations for a randomly selected buyer, $\sum_k Q_k k$. Given distribution $F_t(p_t)$, the probability that the other $k-1$ prices observed by a buyer exceed the seller's price is $[1 - F_t(p_t)]^{k-1}$.

Let $\hat{F}_t(p_t)$ be the distribution of prices posted by a representative household's sellers and denote its support $\hat{\mathcal{F}}_t$. Since this household contains a continuum of sellers, it faces no uncertainty with regard to its *total* sales in the current trading session. These are given by

$$y_t = \int_{\hat{\mathcal{F}}_t} y(p_t) d\hat{F}_t(p_t). \tag{2.11}$$

Using (2.9)—(2.11), we have

$$m_{t+1} = m_t - \int_{\mathcal{F}_t} x_t(p_t) dJ_t(p_t) + \int_{\hat{\mathcal{F}}_t} p_t y(p_t) d\hat{F}_t(p_t) + (\gamma_{t+1} - 1) M_t. \tag{2.12}$$

A representative household's money holdings going into next period's goods trading session are m_t minus the amount spent by its buyers this period; plus its sellers' receipts of money; plus the transfer received at the beginning of the next period.

We now characterize the households' choice of instructions, $x_t(p_t)$ and $\hat{F}_t(p_t)$, to its buyers and sellers respectively. Consider first the household's price-posting strategy (*i.e.* the instructions it gives to its sellers). The expected return to the household from having a seller post price p_t is

$$r(p_t) = \left[\omega_t X_t(p_t) - \phi_t \frac{X_t(p_t)}{p_t}\right] \sum_{k=0}^{K} Q_{kt} k \left[1 - F_t(p_t)\right]^{k-1}. \tag{2.13}$$

In (2.13), ω_t is the marginal value of money in the trading session of the *next* period, $F_t(p_t)$ denotes the *c.d.f.* of prices posted by sellers of other household of its type and $X_t(p_t)$ is the expenditure rule of its prospective customers, all of whom are *ex ante* identical. Note that ω_t is the value to the household of relaxing constraint (2.12) marginally.

From (2.13) it can be seen that $r(p_t)$ equals the expected *return per sale* (in brackets) times the *expected number of sales* (as in (2.10)). The former term is the value of the currency units obtained minus the disutility of production. Here it is clear that the return to posting a price lower than $p_t^* = \phi_t/\omega_t$ (the *marginal cost price*) is negative, and thus the household will instruct no seller to do so. In addition, the return for posting a price at which no buyer would buy (*i.e.* for which $X_t(p_t) = 0$) is zero.

The household maximizes returns by instructing its sellers to post only prices such that

$$p_t \in \underset{p_t}{\operatorname{argmax}}\ r(p_t) \equiv \hat{\mathcal{F}}_t \tag{2.14}$$

The household receives the same expected return from a seller who posts any price in $\hat{\mathcal{F}}_t$. We thus express the household's instructions by a *c.d.f.* $\hat{F}_t(p_t)$ on support $\hat{\mathcal{F}}_t$ and think of sellers as drawing their prices randomly from this distribution. At this stage, however, we make no claims about the characteristics of this distribution.

Consider now the expenditure rule given to the households' buyers, $x_t(p_t)$. The household's gain to having a buyer exchange $x_t(p_t)$ units of currency for consumption at p_t is given by the household's marginal utility of current consumption, $u'(c_t)$, times the quantity of consumption good purchased, $x_t(p_t)/p_t$. The household's cost of this exchange is the number of currency units given up, $x_t(p_t)$, times ω_t. Since individual buyers are small and the household may not reallocate money balances across buyers once the goods trading session has begun, it may be easily shown that the optimal spending rule instructs buyers to spend their entire money holdings if the lowest price they observe is below $u'(c_t)/\omega_t$ (the *reservation price*) and to return with money holdings unspent otherwise:

Proposition 1:

$$x_t(p_t) = \begin{cases} m_t & p_t \le \frac{u'(c_t)}{\omega_t} \\ 0 & p_t > \frac{u'(c_t)}{\omega_t}. \end{cases} \tag{2.15}$$

2.3: Dynamic optimization

To this point we have focused on the current period trading session holding fixed the probabilities of a representative household's buyers observing one and two prices and taking as given the household's marginal value of a unit of money. We now turn to the household's dynamic optimization problem. To begin with, it is useful to write household consumption as the sum of the purchases of those of its buyers who observe different numbers of prices:

$$c_t = \sum_{k=0}^{K} q_{kt} c_t^k \qquad \text{where} \qquad c_t^k = m_t \int_{\mathcal{F}_t} \frac{1}{p_t} dJ_t^k(p_t) \tag{2.16}$$

and for all $p_t \in \mathcal{F}_t$

$$J_t^k(p_t) = 1 - [1 - F_t(p_t)]^k \tag{2.17}$$

are the consumption purchases and distributions of the lowest price observed by buyers who observe exactly k prices respectively, for $k = 0, \ldots, K$. Of course, $c_t^0 = 0$ for all t. In (2.16) we have made use of the fact that buyers follow the optimal expenditure rule, (2.15). Note that the household's choice of $\mathcal{Q}$ is constrained by the requirement that it be a probability:

$$q_{kt} \ge 0, \quad k = 0, \ldots, K \qquad \text{and} \qquad \sum_{k=0}^{K} q_{kt} = 1, \qquad \forall t. \tag{2.18}$$

At time t, for a representative household (of any type), its individual money holdings, m_t, are a relevant state variable in addition to M_t and σ_t. We represent dynamic optimization problem of such a household by the following Bellman equation:

$v_t(m_t, M_t, \sigma_t) =$

$$\max_{\mathcal{Q}_t, m_{t+1}, \hat{m}_t(p_t), \hat{F}_t(p_t)} \Big\{ u(c_t) - \phi_t y_t - \mu \sum_{k=0}^{K} q_{kt} + \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \Pi(\sigma_{t+1}, \sigma_t) v_{t+1}(m_{t+1}, M_{t+1}, \sigma_{t+1}) \Big\} \tag{2.19}$$

subject to: (2.5) (2.7) (2.8) – (2.12) and (2.15) – (2.18).

The household takes as given the actions of other households, $Y_t(p_t; M_t, \sigma_t)$, $X_t(p_t; M_t, \sigma_t)$, and $\hat{\mathcal{Q}}_t(M_t, \sigma_t)$; as well as the distribution of exchange prices, $J_t(p_t; M_t, \sigma_t)$. Here M_t and σ_t are included as arguments to indicate that these actions and distributions depend on the aggregate state. The value function is written here as time varying because it depends on the distributions of *nominal* prices, which may be expected to change over time as the money stock grows given (2.4).

From the household Bellman equation, we have a first-order conditions associated with choice of m_{t+1};

$$\omega_t(m_t, M_t, \sigma_t) = \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \Pi(\sigma_{t+1}, \sigma_t) \left[\frac{\partial v_{t+1}(m_{t+1}, M_{t+1}, \sigma_{t+1})}{\partial m_{t+1}} \right], \tag{2.20}$$

and with choice of $x_t(p_t)$,

$$u'(c_t)\frac{1}{p_t} - \lambda_t(p_t; m_t, M_t, \sigma_t) - \omega_t(m_t, M_t, \sigma_t) = 0 \qquad \forall p_t, t, \tag{2.21}$$

where $\lambda_t(p_t; m_t, M_t, \sigma_t)$ is a Lagrange multiplier on the buyers' expenditure constraint, (2.8). We also have first-order and complementary slackness conditions associated with choice of q_{kt}:

$$u'(c)c_t^k \leq \mu k + \xi_t(m_t, M_t, \sigma_t) \qquad q_{kt} \geq 0 \qquad q_{kt}[u'(c)c_t^k - \mu k - \xi_t(m_t, M_t, \sigma_t)] = 0, \tag{2.22}$$

for $k = 0, \ldots, K$, where $\xi_t(m_t, M_t, \sigma_t)$ is a multiplier associated with the requirement that that q_{kt}'s sum to one. Finally, we have the envelope condition

$$\frac{\partial v_t(m_t, M_t, \sigma_t)}{\partial m_t} = \int_{\mathcal{F}_t} \lambda_t(p_t; m_t, M_t, \sigma_t) dJ_t(p_t) + \omega_t(m_t, M_t, \sigma_t) \qquad \forall t. \tag{2.23}$$

Conditions (2.20)—(2.23) together with the buyers' expenditure rule, (2.13), and the requirement that $\hat{\mathcal{F}}_t$ satisfy (2.15) characterize the household's optimal behaviour conditional on its money holdings, m_t, the aggregate state, (M_t, σ_t), and its beliefs regarding the actions of other households.

3. Equilibrium

We consider only equilibria that are symmetric and Markov. By symmetric, we mean that in equilibrium households choose common probabilities, $\hat{\mathcal{Q}}_t$, for buyers to observe different numbers of price quotes; a common distribution, $\hat{F}_t(p_t)$, from which sellers draw prices to post; and that all have the same marginal valuation of money, Ω_t; consumption, C_t; and money holdings, M_t; in each period. The equilibria we consider are Markov in that quantities; C_t, output, Y_t; the probability distribution , $\mathcal{Q}_t$; and the distributions of *real prices* (*i.e.* nominal prices divided by the per household money stock, M_t), are time invariant functions of σ, which evolves according to Markov chain (2.6).

In a symmetric equilibrium, all buyers have common reservation prices and equal money holdings so that (2.9) gives rise to a version of the quantity equation,

$$C_t = [1 - Q_{0t}]M_t \int_{\mathcal{F}_t} \frac{1}{p_t} dJ_t(p_t) \qquad \forall t. \tag{3.1}$$

If $C_t = C(\sigma)$ for all t such that $\sigma_t = \sigma$, then conditional on σ, the average nominal transaction price must be proportional to the per household money stock, M. That is, for any two time periods, t, t', such that $\sigma_t = \sigma_{t'}$:

$$\frac{M_t}{M_{t'}} = \frac{\int_{\mathcal{F}_{t'}} \frac{1}{p_{t'}} dJ_{t'}(p_{t'})}{\int_{\mathcal{F}_t} \frac{1}{p_t} dJ_t(p_t)}. \tag{3.2}$$

If conditional on σ, *all* nominal posted prices are proportional to M, then there exist N (the cardinality of $\mathcal{S}$) time-invariant distributions of *real* posted prices characterized by supports $\mathcal{F}(\sigma) \equiv \{p_t/M_t,\, p_t \in \mathcal{F}_t;$ for all t such that $\sigma_t = \sigma\}$ and conditional *c.d.f.*'s:

$$F(p \,|\, \sigma) = F_t(p_t) \qquad \forall p \in \mathcal{F}(\sigma), \quad \forall t \mid \sigma_t = \sigma. \tag{3.3}$$

If N conditional distributions satisfying (3.3) exist, then we may think of buyers as observing *real* price quotes, and define N corresponding conditional distributions of lowest real prices observed in a manner analogous to (2.7):

$$J(p \,|\, \sigma) = \sum_{k=0}^{K} Q_k(\sigma) \left[1 - [1 - F(p \,|\, \sigma)]^k\right]. \tag{3.4}$$

Similarly, if the distributions of posted and transactions prices are time-invariant conditional on σ, then households' nominal money holdings, m_t, expenditure rule for buyers, $x_t(p_t)$, and the support of sellers' posted prices, $\hat{\mathcal{F}}_t$ may be divided by the per household money stock to obtain time-invariant conditional *real* counterparts: $m(\sigma) = m_t(\sigma)/M_t(\sigma)$, $x(p \,|\, \sigma) = x_t(p_t \,|\, \sigma)/M_t(\sigma)$, and $\hat{\mathcal{F}}(\sigma) = \{p_t/M_t,\, p_t \in \hat{\mathcal{F}}_t\}$. In this Markov setting, we will drop the time subscript where possible, and use the prime ($'$) to denote the value of a variable in the next period.

We then have the following definition:

Definition: A *symmetric monetary equilibrium* (SME) is a collection of time-invariant, individual household choices, $\hat{\mathcal{Q}}(\sigma)$, $m'(\sigma)$, $x(p \,|\, \sigma)$, $\hat{F}(p \,|\, \sigma)$; common expenditure rules $X(p \,|\, \sigma)$ and probabilities $\mathcal{Q}(\sigma)$; and distributions of posted prices, $F(p \,|\, \sigma)$; conditional on $\sigma \in \mathcal{S}$, such that

1. Taking as given the distributions of posted prices, $F(p \,|\, \sigma)$, common expenditure rule, $X(p \,|\, \sigma)$, and measures of buyers observing different numbers of price quotes, $\mathcal{Q}(\sigma)$; a representative household chooses $\hat{\mathcal{Q}}_t = \hat{\mathcal{Q}}(\sigma)$, $m_{t+1} = m'(\sigma) M_{t+1}$, $x(p_t) = x(p \,|\, \sigma) M_t$, and distribution $\hat{F}_t(p_t) = \hat{F}(p \,|\, \sigma)$ for all $p \in \mathcal{F}(\sigma)$ to satisfy the household Bellman equation, (2.19).

2. Individual choices equal per household quantities: $\hat{\mathcal{Q}}(\sigma) = \mathcal{Q}(\sigma)$, $x(p \,|\, \sigma) = X(p \,|\, \sigma)$, $\hat{F}(p \,|\, \sigma) = F(p \,|\, \sigma)$ for all $p \in \mathcal{F}(\sigma)$, and individual household money holdings equal the per household money stock: $m(\sigma) = 1$.

3. Money has value in all states: For all $\sigma \in \mathcal{S}$, $F(p \,|\, \sigma) > 0$ for some $p < \infty$.

In characterizing an SME for this economy, we focus on the sequence of households' marginal valuations of money, $\{\Omega_t\}_{t=0}^{\infty}$ which determines the returns to sellers and buyers from transacting at a particular price at a particular point in time. Returning to the household optimization problem and combining (2.20), (2.21), and (2.23), we have

$$\omega_t(m_t, M_t, \sigma_t) = \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \Pi(\sigma_{t+1}, \sigma_t) \left[u_c(c_{t+1}) \frac{1}{[1 - Q_{0t+1}]} \int_{\mathcal{F}_t} \frac{1}{p_{t+1}} dJ_{t+1}(p_{t+1}) \right] \qquad \forall t. \tag{3.5}$$

In a symmetric equilibrium, substituting (3.1) into (3.5) we have

$$\Omega_t(M_t, \sigma_t) = \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \Pi(\sigma_{t+1}, \sigma_t) \left[u_c(C_{t+1}) \frac{C_{t+1}}{[1 - Q_{0t+1}]M_{t+1}} \right] \qquad \forall t. \tag{3.6}$$

Making use of (2.4) and dropping the time subscripts we define $\Omega(\sigma)$, for all $\sigma \in \mathcal{S}$,

$$\Omega(\sigma) \equiv \Omega_t M_t = \beta \sum_{\sigma' \in \mathcal{S}} \Pi(\sigma', \sigma) \left[\frac{1}{\gamma'[1 - Q_0(\sigma')]} u_c[C(\sigma')]C(\sigma') \right] \qquad \forall \sigma \in \mathcal{S}. \tag{3.7}$$

We thus associate an SME with a collection of N state-contingent values, $\Omega(\sigma_1), \ldots, \Omega(\sigma_N)$, for households' marginal value of fiat money.

Under the assumption that an SME exists, it is possible to establish several characteristics that it must necessarily possess. We begin in this way and later establish existence by computing equilibria of calibrated parametric versions of the economy. In establishing these characteristics, we rely heavily on earlier results from Head and Kumar (2004), who studied stationary equilibria of a similar economy with no aggregate uncertainty.

For all $\sigma \in \mathcal{S}$, let $\Omega(\sigma)$, $C(\sigma)$, $\mathcal{Q}(\sigma)$, $F(p \,|\, \sigma)$, and $J(p \,|\, \sigma)$ be components of an SME as defined above. In addition, let $\gamma > \beta$ for all $\gamma \in \mathcal{G}$.[5] Our first result restricts the measures of buyers observing different numbers of prices in an SME.

Proposition 2: If an SME exists, then in all states, positive measures of buyers observe one and two prices only. That is, in any SME for all $\sigma \in \mathcal{S}$, $\mathcal{Q}(\sigma)$ satisfies $Q_0 = 0$, $Q_1 > 0$, $Q_2 = 1 - Q_1$, and $Q_k = 0$ for all $k > 2$.

Proposition 2 implies that we may associate an SME with the probability of a buyer observing a *single* price in each state, which we will denote $Q(\sigma)$. In equilibrium, this will equal the measure of buyers observing one price with the remaining buyers (measure $1 - Q(\sigma)$, observing two prices.

[5] It is possible to show that there can be no SME in which the probability with which a buyer observes a single price is equal to either 0 or 1 in any state. Similarly, there can be no SME if $\gamma \leq \beta$ in any state. See appendix.

We next have a proposition based on Theorem 4 of Burdett and Judd (1983) and Proposition 1 of Head and Kumar (2004) which imposes some structure on the form of the conditional distributions of posted prices in any SME:

Proposition 3: *Suppose that $\gamma > \beta$ for all $\gamma \in \mathcal{G}$ and there exists an SME with $Q(\sigma) \in (0,1)$ for all $\sigma \in \mathcal{S}$. Then, given $\Omega(\sigma)$ the conditional distribution of real posted prices, $F(p\,|\,\sigma)$ is unique, dispersed, continuous, and has connected support satisfying: $\mathcal{F}(\sigma) = [p_\ell(\sigma), p_u(\sigma)]$ where,*

$$p_\ell(\sigma) > p^*(\sigma) = \frac{\phi}{\Omega(\sigma)} \qquad \text{and} \qquad p_u(\sigma) = \frac{u_c[C(\sigma)]}{\Omega(\sigma)}. \tag{3.8}$$

Proposition 3 establishes that there is a unique *candidate* distribution of real posted prices in each state for any SME in which a positive measure of buyers observe a single price, conditional on a representative household's marginal valuation of money, $\Omega(\sigma)$. For this candidate distribution, $p \in \mathcal{F}(\sigma)$ requires that $p \in \underset{p}{\operatorname{argmax}}\, r(p)$ where writing (2.14) using real quantities and making using or Proposition 2 we have

$$r(p) = X(p\,|\,\sigma)\left[\Omega(\sigma) - \frac{\phi}{p}\right]\left[Q(\sigma) + 2\big[1 - Q(\sigma)\big]\big[1 - F(p)\big]\right]. \tag{3.9}$$

Combining (3.8) with (3.9) and noting that $F(p_u(\sigma)\,|\,\sigma) = 0$ and $F(p_\ell(\sigma)\,|\,\sigma) = 1$ for all σ, it is possible to derive the following expressions:

$$p_\ell(\sigma) = \frac{\phi}{\Omega(\sigma)}\left[1 - \left(1 - \frac{\phi}{u_c[C(\sigma)]}\right)\frac{Q(\sigma)}{2 - Q(\sigma)}\right]^{-1} \tag{3.10}$$

and

$$F(p\,|\,\sigma) = \frac{\left[\Omega(\sigma) - \frac{\phi}{p}\right][2 - Q(\sigma)] - \left[1 - \frac{\phi}{u_c[C(\sigma)]}\right]\Omega(\sigma)Q(\sigma)}{\left[\Omega(\sigma) - \frac{\phi}{p}\right]2[1 - Q(\sigma)]} \tag{3.11}$$

for all $\sigma \in \mathcal{S}$.

From (3.11), it is convenient to derive the following expressions for the conditional densities of posted and transactions prices:

$$f(p\,|\,\sigma) = \frac{\phi}{p^2}\left[\frac{Q(\sigma) + 2[1 - Q(\sigma)][1 - F(p\,|\,\sigma)]}{[\Omega(\sigma) - \phi/p]2[1 - Q(\sigma)]}\right] \tag{3.12}$$

and

$$j(p\,|\,\sigma) = \big[Q(\sigma) + 2[1 - Q(\sigma)][1 - F(p\,|\,\sigma)]\big]f(p\,|\,\sigma). \tag{3.13}$$

Expressions (3.8) and (3.10)—(3.12) describe the conditional distributions of posted and transactions prices in an SME as functions of $Q(\sigma)$ and $C(\sigma)$. Taking $J(p\,|\,\sigma)$ (and implicitly, Q and

C) as given, an individual household's consumption depends on the probability with which its *own* buyers observe a single price. Given Proposition 2, the optimal choice of this probability, $q(\sigma)$ may be easily seen to satisfy

$$q(\sigma) = \underset{q}{\operatorname{argmax}}\ u\left[\left(qc^1(\sigma) + (1-q)c^2(\sigma)\right)\right] - \mu(2-q) \qquad \sigma \in \mathcal{S}, \tag{3.14}$$

where $c^k(\sigma)x$, $k = 1, 2$ are as defined at (2.16) and (2.17). The optimal measure of buyers to have observe one price, $q(\sigma)$, may then be derived:

$$q(\sigma) = \begin{cases} 0 & \text{if } \mu < \mu_L \equiv u_c\left(c^2(\sigma)\right)\left[c^2(\sigma) - c^1(\sigma)\right] \\ 1 & \text{if } \mu > \mu_H \equiv u_c\left(c^1(\sigma)\right)\left[c^2(\sigma) - c^1(\sigma)\right] \\ \frac{{u_c}^{-1}\left(\frac{\mu}{c^2(\sigma)-c^1(\sigma)}\right) - c^2(\sigma)}{c^1(\sigma)-c^2(\sigma)} & \text{if } \mu_L \leq \mu \leq \mu_H. \end{cases} \tag{3.15}$$

If the search cost is *below* μ_L, then the household will choose to have *all* of its buyers observe more than one price (*i.e.* $q(\sigma) = 0$). Similarly, if $\mu > \mu_H$, the household will choose to have *no* buyer observe a second quote (*i.e.* $q(\sigma) = 1$). From Proposition 2, it can be seen that such a solution in any state is inconsistent with the existence of an SME. Note that the bounds, μ_L and μ_H (and so $q(\sigma)$ itself), depend on $c^1(\sigma)$ and $c^2(\sigma)$ and are functions of Q. Letting the optimal choice be written $q(Q, \sigma)$) existence of an SME hinges on finding a vector of fixed points such that for all $\sigma \in \mathcal{S}$, $q(Q, \sigma) = Q(\sigma)$ *and* $\mu_L(\sigma) \leq \mu \leq \mu_H(\sigma)$.

Proposition 4: *For a fixed σ and for any $Q \in (0, 1)$, there exists a $\mu(\sigma)$ such that $q(Q, \sigma) = Q(\sigma)$.*

Proposition 4 establishes the existence of fixed points for each particular σ, each associated with an individual search cost $\mu(\sigma)$. In the environment, however, we have specified search costs independently of the state. In general, there may not exist a single μ such that there is a fixed point of (3.15) for all $\sigma \in \mathcal{S}$. In this case, trade would break down in those states for which μ lay outside the interval $[\mu_L(\sigma), \mu_H(\sigma)]$, and there would exist no SME by our defintion. Note, however, that if the variation in σ is sufficiently small, then the intervals, $[\mu_L(\sigma), \mu_H(\sigma)]$ will have a non-empty intersection, and there will indeed exist a single μ in this intersection for which fixed points of (3.15) exist in all states.[6]

[6] For a version of this economy with no aggregate uncertainty (effectively a single σ), Head and Kumar (2004) establish formally the existence of an equilibrium of the type considered here. A simple argument relying on the continuity of c^1 and c^2 in the parameters ϕ and γ can be used similarly to establish the existence of an SME here, for a sufficiently restricted state space, $\mathcal{S}$.

Restrictions on the state space $\mathcal{S}$ for which we can guarantee that fixed points of (3.15) exist in all N states for a common μ, are complicated and are also not general in that they depend crucially on the parameters and functional forms adopted. For this reason, we do not derive them explicitly. Rather, given a parmeterization or our economy, we compute search cost parameters for which SME's exist. As a practical matter, we find that the interesection of the intervals $[\mu_L(\sigma), \mu_H(\sigma)]$ is non-empty even for substantial variation across both ϕ and γ.

The process by which we compute equilibria is described in general terms here. For a detailed description of our computational algorithm, see the appendix. We begin by choosing specific values of the economy's parameters.[7] This requires us to set the discount factor, β; the search cost, μ; sets of values for both the cost parameter, $\mathcal{P}$, and money creation rate, $\mathcal{G}$; as well as the transition probabilities, $\Pi(\sigma'|\sigma)$ for $\sigma',\ \sigma\ \in \mathcal{S}$.

Having fixed parameters, we then choose initial values for consumption, $C_0(\sigma)$, and the probability of a buyer observing a single price, $Q_0(\sigma)$, for each $\sigma \in \mathcal{S}$. Using these values, we construct $\Omega_0(\sigma)$, and the distributions of posted and transactions prices using (3.8) and (3.10)—(3.13). We label these distributions $F_0(p\,|\,\sigma)$ and $J_0(p\,|\,\sigma)$, respectively. Using these, we construct $c_0^1[Q_0(\sigma)]$ and $c_0^2[Q_0(\sigma)]$ and use them to compute fixed points of (3.15) for each σ, which we call $Q_1(\sigma)$. Finally, using $Q_1(\sigma)$ and $F_0(p\,|\,\sigma)$ we construct $J_1(p\,|\,\sigma)$ and $C_1(\sigma)$. This procedure is repeated (T times) until for all σ, $C_T(\sigma) - C_{T-1}(\sigma)$ and $Q_T(\sigma) - Q_{T-1}(\sigma)$ are sufficiently small.

Overall, we find that this algorithm works well in that for a wide range of parameter values it successfully computes an SME very quickly. While we do not formally rule out non-uniqueness, experimentation with different starting values in no case produced multiple equilibria for a fixed set of parameters.

4. Price Responses to Shocks in Equilibrium

We now consider the effects of random fluctuations in costs and money creation (each in isolation) in numerically computed SME's. We focus on the responses of the level and dispersion of prices to these shocks; and on the magnitude and persistence of the fluctuations in inflation that result from them.

We describe the *level* of prices by the average transaction price.[8] The average *real* price at

[7] We report the parameter values chosen in our baseline calibration in the section 4.

[8] Throughout this section we focus on *transactions* rather than *posted* prices. We do this because changes in the former more accurately signal the quantitative effects of shocks on output, consumption, and welfare. Qualitatively, both transactions and posted prices respond similarly to both cost and money growth shocks.

time t (in state σ) is:

$$\bar{p}(\sigma) = \int_{\mathcal{F}(\sigma)} p \, dJ(p \,|\, \sigma), \tag{4.1}$$

and the average nominal price (or the "price level") at time t is

$$\bar{P}_t = M_t \bar{p}(\sigma) = \int_{\mathcal{F}_t} p_t \, dJ_t(p_t). \tag{4.2}$$

The nominal price level in an SME is not stationary, and thus it is written as a function of the time period, t, rather than the current state, σ. We define the inflation rate as the net growth rate of the nominal price level:

$$I_t = \% \Delta P_t \equiv \frac{\bar{P}_t - \bar{P}_{t-1}}{\bar{P}_{t-1}} \times 100. \tag{4.3}$$

Note that the inflation rate, like the price level, is a function of time rather than the current state.

We consider two measures of the the *dispersion* of real prices. One measure is the range of its support, *i.e.* the ratio of the upper support to the lower support: $p_u(\sigma)/p_\ell(\sigma)$. For some purposes we will also use the coefficient of variation (the ratio of the standard deviation of the distribution to its mean).

In our environment, production costs are measured in units of utility. It is useful, however, to express costs either in units of goods (real costs) or currency (nominal costs). We define *real marginal cost* in state σ:

$$mc(\sigma) = \frac{\phi}{\Omega(\sigma)}. \tag{4.4}$$

Nominal marginal cost is given by

$$MC_t = mc(\sigma_t) M_t. \tag{4.5}$$

Note that both nominal and real marginal costs are affected not only by the production disutility ϕ, but also by anything that changes Ω. As such, changes in the money creation rate, γ, induce movements in both real and nominal marginal cost.

4.1: Benchmark Parameterization

We begin with a benchmark parameterization of the economy. We set the discount factor, β, equal to .99, a value commonly used in dynamic general equilibrium models calibrated to quarterly observations. The length of the period chosen is significant here as β controls the cost of carrying unspent money into the next period. Our choice of $\beta = .99$ is comparable to the base case of the cash-in-advance model Cooley and Hansen (1989). We maintain the assumption of *CRRA* utility throughout and set $\alpha = 1.5$, a value consistent with the requirement that $\lim_{C \to 0} u'(C)C = \infty$, and within the range typically examined in calibrated macroeconomic models.

As described in detail by Head and Kumar (2004), in our economy an increase in trend inflation above its lower bound (the Friedman rule: $\gamma = \beta$) raises price dispersion, inducing increased search and eroding market power; an effect which puts downward pressure on the average real price. Of course an increase in trend inflation also raises the inflation tax, *increasing* the average price. In this economy the former effect dominates at low inflation, so that increased trend inflation raises welfare in a non-stochastic SME. This effect, however, diminishes as trend inflation increases, and at some point further increases in inflation do not increase search intensity sufficiently to offset the increased inflation tax and welfare falls. Thus, there exists a trend inflation rate exceeding the Friedman rule which maximizes household welfare in a non-stochastic SME. We choose the search cost parameter, μ, so this inflation rate is equal to 3.1% ($\gamma = 1.0076$) as it is the average inflation rate for the U.S. during the Greenspan era (1987-present).

Our chosen combination of μ and γ implies an average real markup over marginal costs of 1.05, a number that we consider reasonable given the wide range of markups estimated by several studies of U.S. manufacturing (*e.g.* Morrison (1990), Basu and Fernald (1997), Chirinko and Fazzari (1994)). Finally, we set the average level of the production disutility parameter, $\phi = .1$. Given values for the other parameters, ϕ controls only the level of output in a stationary equilibrium.

We specify Markov chains for the stochastic parameters so that in each case the percentage standard deviation and autocorrelation of aggregate output in an SME with fluctuations induced by random variation in that parameter alone are equal to 1.60 and .83 respectively, values equal to their counterparts in quarterly U.S. GDP, detrended with the Hodrick-Prescott filter for the period 1959-2002. Many Markov chains fit this criterion; we choose the following symmetric processes xfor illustrative purposes only. For all t,

$$\begin{aligned} \phi_t &\in \mathcal{P} = \{.096, .1, .104\} \\ \gamma_t &\in \mathcal{G} = \{1.0017, 1.0076, 1.014\} \end{aligned} \tag{4.6}$$

with

$$\pi^{\phi} = \pi^{\gamma} = \begin{bmatrix} .88 & .06 & .06 \\ .06 & .88 & .06 \\ .06 & .06 & .88 \end{bmatrix}. \tag{4.7}$$

4.2: The pass-through of cost shocks to prices

We first consider the effects of random fluctuations in costs. For all t, $\phi_t \in \mathcal{P}$ as specified in (4.6) with $\Pi = \pi^{\phi}$ given by (4.7). To begin with we fix the rate of money creation at its benchmark level, $\gamma = 1.0076$. Thus, we consider the SME of an economy with three states, each associated with a different level of production disutility which we will refer to has the low, medium, and high cost states. While the state is determined by the realization of ϕ alone, for notational purposes we continue to use σ to indicate the current state.

Figure 1 depicts the densities of real transactions prices in each of the three states. The figure also includes the average transaction price and the measure of buyers observing a single price in each state. These densities together with the transition matrix (4.7) effectively describe the SME. In the figure it can be seen that as real costs fall and rise, the densities of transactions prices shift to the left and the right, respectively. That is, higher costs are associated with higher real transactions prices. Changes in ϕ affect $\Omega(\sigma)$ (see (4.5)) so that a 4% reduction in ϕ from .1 to .096 reduces real cost of producing one unit of output by 3.72%, while a 4% increase from .1 to .104 raises this cost by 3.26%. To facilitate comparisons of the magnitude of the change in the average real transaction price to the cost shift that precipitates it, we introduce the following measure:

$$rpt = \frac{\%\Delta\bar{p}}{|\%\Delta mc|}. \tag{4.8}$$

Here rpt is the ratio of the percentage change in the average real transaction price to the percentage change in real marginal cost, a measure of the "pass-through" of cost movements to real prices. In this example, the average real transaction price, $\bar{p}(\sigma)$, falls by 1.79% when costs fall implying $rpt = -.48$, and rises by 2.23% when costs rise implying $rpt = .68$.

The numbers above suggest that the pass-through of cost changes to real prices is incomplete. The degree of pass-through depends on both the changes in the distribution of posted prices in response to cost movements *and* changes in households' search intensity. Consider an increase in cost and focus first on the response of sellers' posted prices. When ϕ rises, households raise the prices of near the top of the distribution by more than those in the middle or near the bottom. High price sellers sell predominately to buyers who have no alternative—they observe only one price quote. A given increase in such a seller's price thus causes the household to relatively small loss of sales to competitors. In contrast, those sellers pricing in the lower range of the support of the price distribution make a larger share of their sales to buyers who have an alternative—they observe two price quotes. The household limits these sellers' price increases to avoid a large loss of sales to competitors. Effectively, high price sellers pass through a large share of the cost increase to buyers, while low price sellers pass through less.

This argument suggests an increase in price dispersion as cost changes are passed through differentially by sellers in different regions of the price distribution. An increase in price dispersion, however, increases the value of observing a second price quote and thus induces households' to increase their search intensity (*i.e.* to lower q). A reduction in equilibrium Q weakens market power and lowers real transactions prices overall in two ways. First it lowers the mark-up, pushing all prices closer to the marginal cost price, $p^*(\sigma)$. Second, it widens the gap between the distributions of posted and transaction prices.

The overall change in the distribution of transaction prices in response to cost shocks is decomposed into two effects in Figure 2. In the figure, the dashed lines depict the distributions of transactions prices in the high and low cost states for an economy in which search intensity is fixed at the equilibrium level for the medium cost state ($Q = .699$ in this case). The solid densities are the same as depicted in Figure 1, and represent the full general equilibrium effect of stochastic changes in cost on transaction prices. In the picture it is clear that the response of search intensity is crucial in generating incomplete pass-through. For example, in the absence of an increase in search intensity, the percentage increase in price in response to a shift in costs from medium to high would be 3.57%, for an *rpt* of 1.10 rather than .68.

The effect of a cost increase on price dispersion is ambiguous. On the one hand, with fixed search intensity, because of differential pass-through a cost increase raises the dispersion of both posted and transaction prices substantially. On the other, the increase in equilibrium search intensity that this induces both mitigates the widening of the support and causes the mass of the distribution to shift toward its lower support as the average mark-up falls. For the example depicted in Figures 1 and 2, as costs increase the support of the distribution of transactions prices widens in the sense that p_u/p_ℓ increases. At the same time, however, the coefficient of variation of the distribution falls.

Finally, note that the effects of a reduction in cost are qualitatively symmetric to those of an increase, but not quantitatively so. When costs fall, prices at the upper end of the distribution are reduced by a relatively large amount as households cut these sellers' prices in order to gain a large increase in sales. Prices at the lower end of the distribution are reduced by less as they are associated with low mark-ups already and the gains to garnering more sales by cutting the price are small. The effect of differential pass-through in this case is to *compress* the price distribution, *reducing* the returns to search. Households thus reduce their search intensity, raising equilibrium Q and mitigating the fall in the average price. Again the effect on price dispersion in equilibrium is ambiguous. Quantitatively, the overall pass-through of a reduction in cost is smaller than that of an increase, as evidenced by and *rpt* of -.48 rather than .68.

We now consider the relationship between the degree of cost pass-through and average inflation, which in these experiments is equal to trend rate of money creation. Figures 3 and 4 respectively depict densities of real transaction prices for cases in which average inflation is two and four percent *per annum.* These figures have the same horizontal scale as Figures 1 and 2 and so it is clear that the price distributions change by less in response to a given change in ϕ when the average rate of money creation is low than when it is high. Moreover, the degree of pass-through of *real* cost changes to real prices is increasing in the average inflation rate. For example a shift in costs from medium to high results in an *rpt* of .32 when inflation is 2% and an *rpt* of .76 when inflation is 4%.

The relationship between inflation and the degree to which cost changes are passed through to average real prices depends on the effects of the shock on price dispersion and the household search decision. Higher inflation is associated with greater search intensity and lower market power on average. To see this, note that the fractions of buyers observing a single price in the low, medium, and high cost states are .893, .847, and .776 respectively when inflation is two percent, as opposed to .693, .590, and .496 respectively when inflaiton is four percent. Moreover, the "differential pass-through" of cost shocks described above is increasing in the share of buyers observing a single price at the time of the cost shift. With low inflation a larger share of sellers increase their prices substantially in response to a cost shock, resulting in a larger increase in price dispersion for fixed Q. This in turn leads to a larger increase in search intensity in response to a shock at low inflation, and thus lower pass-through. [9]

We now consider the pass-through of *nominal* cost fluctuations to nominal price changes, at different levels of trend inflation. In our economy, while both the price level and nominal costs trend at rate $(\gamma - 1) \times 100$, they are also affected by cost shocks which change the distribution of real prices. Since nominal costs and prices have the same trend, we measure the pass-through of cost changes only. Our pass-through measure is computed as a an average of the pass-through that occurs in each state transition, weighted by the frequency of each transition in an SME. Inflation between two periods in which the state (*i.e.* costs) change from state i to state j is given by

$$\Delta P_{ij} = \frac{P_j - P_i}{P_j} = \gamma \left(\frac{p_j}{p_i} \right) - 1. \tag{4.8}$$

We correct for trend by subtracting the average inflation rate:

$$\tilde{\Delta} P_{ij} \equiv \Delta P_{ij} - \gamma. \tag{4.9}$$

[9] Figures 1 through 4 taken together illustrate that increases in average inflation may be associated with lower real prices and higher consumption overall if they result in sufficient increases in average search intensity. This is indeed the case throughout the range considered here (two to four percent inflation). Note, however, that while consumption does increase with inflation over this range, search costs (which are proportional to the measure of buyers observing two prices) do as well. As stated earlier, household welfare is maximized at trend inflation of 3.1%. As inflation increases beyond four percent consumption not only will welfare be decreased, but at some point consumption will also begin to fall

Similarly, we consider only changes in nominal marginal cost occuring due changes in ϕ:

$$\tilde{\Delta} MC_{ij} \equiv \gamma \left(\frac{MC_j}{MC_i} - 1 \right), \tag{4.10}$$

and measure nominal pass-through between any two periods across which the cost state changes by the ratio, $\tilde{\Delta} P_{ij}/\tilde{\Delta} MC_{ij}$. Finally, our measure of average nominal pass-through, *npt*, is constructed by weighting these ratios by the frequency of the particular switches in the SME, conditional on a switch taking place:

$$npt = \sum_{i=1}^{N} \left(\frac{\bar{\pi}_i^{\phi}}{\sum_{h \neq i} \pi_{ih}^{\phi}} \right) \sum_{j \neq i} \pi_{ij}^{\phi} \frac{\tilde{\Delta} \bar{\pi}_{ij}^{\phi}}{\tilde{\Delta} MC_{ij}}, \tag{4.11}$$

where $\bar{\pi}_i^{\phi}$ is the unconditional probability of state ϕ_i occuring.

Figure 5 depicts nominal pass-through and the average level of Q in equilibrium for levels of trend inflation ranging from two percent to fourteen percent. Over this range our estimates of pass-through increase monotonically but at a decreasing rate from .242 to .975 as the rate of trend inflation increases. The relationship depicted here is in accordance with the empirical findings of Devereux and Yetman (2002) who found pass-through of nominal exchange rate movements to consumer prices to be increasing in average inflation at a decreasing rate in a panel of 107 countries over the post-Bretton Woods period. It is also in accordance with the arguments of Taylor (2000) who cites evidence that the responsiveness of prices to cost increases has declined with average inflation for several developed countries during the late 1980's and 1990's.

Intuition for the increase in nominal cost pass-through as the trend inflation rate rises may again be traced to the average share of buyers observing a single price. As described above, this fraction decreases from .841 to .161 as the rate of trend inflation increases over this range. The higher Q, the greater the change in price dispersion associated with a cost shock at that fixed Q, and thus the greater the equilibrium change in price dispersion and the greater the response of search intensity. Since it is the response of search intensity that mitigates pass-through, at low inflation (where search intensity is very responsive to shocks) pass-through is low. At sufficiently high inflation, the response of search intensity to a cost shock is minimal, and nominal pass-through is effectively complete.

Finally, consider the dynamics of inflation induced by cost fluctuations in our economy. The relationship between *npt* and trend inflation depicted in Figure 5 directly implies that the variance of inflation induced by nominal cost shocks of a given variance increases with the trend rate of inflation (again at a decreasing rate). Thus, our economy provides a reason why cost shocks of given magnitude may result in much more volatility of changes in the price level during a high inflation era than they do in an era of low inflation. Inflation induced by cost changes is not, however, persistent in our economy. The household adjusts to the change in costs within the period in which the cost shock is realized and inflation either jumps above or falls below its trend level in that period only.

4.3: Price adjustment in response to monetary shocks

We now consider the effect of shocks to the money creation rate, holding fixed the disutility of production at $\phi = .1$. Again we consider a three-state economy with σ in this case determined by γ alone. To begin with we let $\gamma \in \mathcal{G}$ as specified in (4.6) with $\Pi = \pi^{\gamma}$ given by (4.7). We then consider the effects of changes in the average rate of inflation.

The response of real prices to monetary shocks in equilibrium is the result of a combination of the same two effects that are present in the case of cost shocks. Figure 6 illustrates the effects of money growth shocks on the densities of real transacations prices in equilibrium, and decomposes the overall effect into those that would occur with and without endogenous search intensity in a manner similar to that of Figure 2. In the figure, it can be see that higher money creation is associated with *lower* average transactions prices (and thus, higher consumption) for these three states.[10] A higher rate of money creation is also associated with greater price dispersion in the sense of a higher p_u/p_ℓ and a lower fraction of buyers observing a single price.

To understand the mechanism by which persistent changes in the money affect real prices, consider an increase in the money growth rate, say from 3.1% to 5.6% (average to high). Such an increase in γ raises expected future inflation, reducing $\Omega(\sigma)$ and increasing price dispersion for a fixed fraction of buyers observing a single price. Households raise high-price sellers' prices by more than those of low price sellers because in doing so they lose only a relatively small proportion of their sales to competitors. This accounts for the *increase* in real prices (to the dashed line in the figure) and an increase in price dispersion when search intensity is held fixed.

Increased price dispersion, , however, induces households' to increase search intensity, lowering the fraction of buyers observing a single price. This reduces market power overall and limits the increase in real prices associated with a depreciation of money. In the case depicted, the increase in search intensity is dominant. For example, with fixed search intensity, an increase of the money growth rate would cause the average real transacition price to rise and price dispersion to increase. In Figure 6 it is clear that the resulting increase in search intensity is sufficient not only to offset

[10] Again, this would not remain true if money creation in the high state were sufficiently high. Beyond some point, higher expected inflation raises the average transaction price and reduces consumption.

these effects on real prices, but to so weaken market power that real prices actually fall. A reduction of the money growth rate has the opposite effect.

A reduction in the average real price in response to an increase in the rate of money creation implies that the nominal price does not rise sufficiently to offset the increase in the money supply. Consider the case of an increase in the money creation rate from medium to high (*i.e.* from 3.1% to 5.6% *per annum*) depicted in Figures 6 and 7. In this case, the money supply grows by 1.37% over the previous period, but the real price falls from .224 to .222. Using (4.1) and (4.2) the change in the average nominal price in this case is given by

$$\frac{\bar{P}_t}{\bar{P}_{t-1}} - 1 = \frac{\bar{p}_t}{\bar{p}_{t-1}} \frac{M_t}{M_{t-1}} - 1 = \left[\frac{.222}{.224}\right] \times 1.0137 - 1 = 1.0046. \tag{4.12}$$

That is, in this case an increase in the money supply of 1.37% leads to an increase in the nominal price level of just .46%. Alternatively, in a period in which the annualized rate of money creation rises from 3.1% to 5.6%, the annualized inflation rate *falls* from 3.1% to 1.87%.

At higher levels of average inflation, this incomplete adjustment of nominal prices to changes in the money growth rate disappears as changes in market power are dominated by changes in the marginal value of money, $\Omega(\sigma)$. For example, consider a case in which the average rate of money creation is 15%, and the low and high money growth states represent the same percentage increase and decrease as in the case depicted in Figures 6 and 7. In this case the average real transaction price *rises* as the money creation rate increases from low (12.3%) to medium (15%) to high (17.8%). A higher money growth rate still increases search intensity, as equilibrium Q falls from .181 to .146 to .121 as γ rises. These increases in search intensity are, however, insufficient to overcome the increased erosion of the value of money by the inflation tax. Moreover, an increase in the real price suggests that in the period in which the money growth rate rises the nominal price level must rise by *more* than the money stock. In this case, when γ moves from average to high, the money supply grows by 4.18% and the nominal price level rises by 4.51% (and the annualized inflation rate jumps from 15% to 19.28% rather than to 17.8%).

These two examples illustrate that either of the two opposing forces acting on real prices may dominate, and that which effect does depends on the average level of inflation. At relatively low average inflation (*e.g.* at 3.1% annually, as in our benchmark paramterization), the response of search intensity dominates so that the response of nominal prices is smaller than the change in the money stock. At high inflation, the inflation tax dominates and the response of nominal prices is larger than the change in the money stock.[11] x

[11] It is also possible that at a given average rate of inflation, shocks which either decrease or increase the rate

The relationship between the adjustment of nominal prices to shocks to the money growth rate does not, however, imply monotonic relationships between the average level of inflation and either its variance or autocorrelation due to monetary shocks. Rather, the variance of inflation *falls* with the trend inflation rate at very low rates of inflation, before becoming increasing in average inflation at some "moderate" rate. Beyond this point, the variance of inflation rises with the average inflation rate. Similarly, the first-order autocorrelation of inflation is smaller than that of money creation at average inflation rates that are either high or very low. For some range of average inflation rates in between, the inflation rate may display persistent or sluggish responses to stochastic changes in the money growth rate.

Non-monotonic relationships between both the variance and auto-correlation of inflation and its average level arise from the fact that at very low inflationt the response of search intensity to money growth shocks dominates. In this case, inflation intially *falls* in response to an increase in the money growth rate. The result is a high variance of inflation and negative first-order autocorrelation. As the inflation rate rises, the search intensity effect diminishes, causing inflation to *rise* incompletely in response to an increase in money growth. For trend inflation in this range, inflation responds sluggishly to monetary shocks. As inflation continues to rise, however, eventually the increase in real prices caused by an increase in the money growth rate causes inflation to *rise* dramatically, overshooting its new level. This leads to higher variance and again, a negative autocorrelation in response to shocks of a given magnitude as trend inflation increases further.

Overall, the dynamics of inflation in response to monetary shocks in our economy are ambiguous, and depend on the particular parameterization of the economy. In the next section, we consider an example of how the economy can produce sluggish or persistent movements in inflation in response to monetary shocks at moderate trend inflation rates.

5. Expectations and Inflation Dynamics

In this section, we consider the dynamics of inflation in response to monetary shocks. We retain most parameters from our benchmark calibration, but change the process for money creation. We again let γ take on three values,

$$\gamma \in \{\gamma_L, \gamma_M, \gamma_H\} = \{1.008, 1.012, 1.016\}. \tag{5.1}$$

of money creation will *raise* the average real transaction price. For example, this will occur in the example considered here if the average inflation rate is six percent and the shocks represent increases and decreases in the money creation rate of the same relative magnitude as the case depicted in Figures 6 and 7.

These values imply annual money creation rates of 3.3, 5, and 6.7 percent, respectively. To begin with, we replace the transition matrix given by (4.7) with a process such that:

$$\text{Prob}\{\gamma' = \gamma_i | \gamma = \gamma_i\} = .8 \quad \text{and} \quad \text{Prob}\{\gamma' = \gamma_{j \neq i} | \gamma = \gamma_i\} = .1, \quad i, j \in \{L, M, H\}. \tag{5.2}$$

Figure 7 depicts the inflation rate and the growth rate of the money stock for a thirty-five period episode in which the economy experiences all possible state transitions in an SME. An important aspect of the dynamics of inflation is clearly evident in the figure. The economy experiences as many inflation rates as there are state transitions, in this case nine. This is the reason why at (4.3) inflation is written as a function of time rather than the state, σ. When the money creation rate remains at its level in the previous period the inflation rate is equal to γ. Whenever it changes, however, the adjustment of real prices causes inflation to differ from the growth rate of the money stock. This implies that in response to a persistent change in the money creation rate, the inflation rate will take two periods to adjust. If the money growth rate increases or decreases for one period only and then returns to its previous level, the inflation rate will deviate from the original rate of money creation for three periods.

As described in the previous section, the *degree* of price adjustment in response to shocks to the money growth rate depends on the average level of inflation as this determines the relative strengths of two conflicting effects. At very low inflation rates, in response to an increase in γ the increase in search intensity may dominate by so much that the reduction of the average real price causes inflation to *fall* in the initial period. In contrast, increases in the money creation rate when inflation is very high may induce real prices to rise as the increase in search intensity fails to dominate the increased erosion of the value of money. In this case inflation will initially increase by *more* than the increase of the money growth rate.

Both when inflation is very low and when it is very high, any change in the money growth rate will thus result in negatively autocorrelated movements of the inflation rate, although the pattern of negative and positive changes in response to either an increase or decrease in γ will be reversed. The relationship between the average of the inflation rate and its variance will also differ in these two cases. When average inflation is very low, an increase will lower its variance for a given percentage variance of the money growth rate. In contrast, at high inflation, further increases in the average inflation rate will *increase* the variance of inflation.

For a case with “moderate” average inflation (such as that depicted in Figure 7), the adjustment of inflation to its new level is monotonic. To see this, consider an increase in the money creation rate from γ_M to γ_H.[12] In this case inflation initially rises to a rate less than γ_H as an increase in search intensity lowers real prices and thus mitigates the increase in the price level. As long as money creation continues at γ_H, real prices will subsequently remain constant, and inflation will equal the rate of money creation. This implies a further increase in inflation in the second period following the change in γ. If after one period the money creation rate were to return to γ_M, the inflation rate would fall, but to a level still exceeding the money creation rate owing to an *increase* in real prices associated with a reduction in search intensity. If money creation were to then remain at γ_M, inflation would change yet again, this time to coincide with the rate of money creation.

In such a case nominal prices are "sticky" in the following senses: First, the nominal price level fails to rise or fall by enough to maintain a constant real price. Second, the inflation rate deviates from the rate of money creation for two or three periods in response to a persistent or competely transitory change in γ, respectively.

The autocorrelation of inflation may then be either greater or less than that of the money growth rate. When inflation is either very high or very low, inflation will display less autocorrelation than γ because of its non-montonic adjustment to changes in the money growth rate. When inflation is moderate, inflation will display greater autocorrelation than the money growth rate. Similarly, when inflation is moderate in this sense, its variance will be increasing in its average rate, for a given relative variance of shocks.

Regardless of its average rate, in this economy inflation deviates from the rate of money creation only in periods in which expected inflation changes. This is because any adjustment of real prices to a change in expected inflation takes place instantaneously—there is no propagation mechanism within the model through which the effects of money growth shocks on prices and real quantities can take place over time. The only dynamic aspect of the economy is that the expected future value of money, $\Omega(\sigma)$, determines prices and quantities in equilibrium. In each period, however, the prices and quantities are able to adjust to their appropriate level given $\Omega(\sigma)$. As noted earlier, money is neutral, even "in the short-run". A non-persistent change in the money growth rate, say from γ_M to γ_H and then back to γ_M over three periods as described above, *will* have real effects, but only because each shift in the money growth rate changes *expected* inflation.

We now modify the economy so that the expectation of future inflation may evolve slowly over

[12] We choose this case simply for illustrative purposes. From inspection of Figure 7 it is clear both that any increase in the money creation rate will have similar effects, and that any decrease will have symmetric, but opposite effects on both nominal and real prices.

time in response to money growth shocks and thus serve as a mechanism to propagate the effects of such shocks. Our approach is similar to that taken by Andolfatto and Gomme (2003). Suppose that at each point in time the monetary authority operates under one of several "regimes", and that this regime is unobserved. Households observe the history of money transfers (including the current transfer) and they know the stochastic process by which money growth evolves. That is, they know the probability of each money growth rate occuring in each regime and they know the probability of a switch from one regime to another in each period. Agents compute their expectation of future inflation rationally, knowing the process and having observed the history of money growth rates.

The only component of the environment that we have modified here is the particular form of the Markov process for the money growth rate. This modification requires neither the definition of equilibrium nor the algorithm for computing an equilibrium to be changed. The change in the information structure does, however, increase the cardinality of the state, N, as the current realization of the money growth rate is no longer sufficient to determine expected future inflation. Rather, households must keep track of the history of γ. In this case the expected future value of money will evolve slowly in response to changes in γ, as each realization augments, rather than determines completely, the information set.

To illustrate qualitatively the potential effects of a persistently evolving $\Omega(\sigma)$, we consider a simple example of a money growth process as described above.[13] Let money growth take on only the three values posited in (5.1). There are two regimes; a *low* inflation regime (R_L), in which γ may be only low or medium (*i.e.* $\gamma \in \{\gamma_L, \gamma_M\}$), and a *high* inflation regime (R_H), in which it may be only *high* or medium. Note that in this case, realization of either γ_L or γ_H *will* reveal the regime, allowing households to disregard the history except for the current realization. Only when γ_M is realized is the history of money growth rates relevent, and even in this case, only the history back to the last realization of either γ_L or γ_H matters. We simplify further by setting $\text{Prob}\{\gamma = \gamma_M\} = \pi_M$, independent of both the regime and the current realization of γ. Finally, assume that probability of a change in regimes is constant. That is, $\pi_R \equiv \text{Prob}\{R' = R\}$, $R', R \in \{R_L, R_H\}$. For our example, we set the probability of a regime shift in any period to .2 ($\pi_R = .8$), and set $\pi_M = .45$.

In this case it is not possible to depict all possible state transitions, since there are an infinite number. Persistent realizations of either γ_L or γ_H, and transitions from one to the other are,

[13] None of these simplifying assumptions are necessary, but they are helpful in making the example easy to characterize. Moreover, this simple example is sufficient to illustrate the effects of sluggish inflation expectations on the dynamics of inflation in our environment.

however, associated with inflation dynamics qualititatively similar to those depicted in Figure 7. In these cases, the realization of the money creation rate reveals the regime, and thus causes an instanteous adjustment of both $\Omega(\sigma)$ and the distribution of real prices. For this reason, we now focus on the effect of periods in which the money growth rate is "uninformative" ($i.e$ equal to γ_M). Figure 8 depicts money growth and inflation rates for a thirty-five period episode in which the economy with unobserved regime experiences several such periods in an SME.

Consider first persistent realizations γ_M following an episode when the regime was known due to an observed money growth rate of either γ_L or γ_H. In these cases, inflation adjusts slowly and converges monotonically to the unconditional mean of inflation for the economy, γ_M. The reason for the slow adjustment of inflation is that each successive realization changes causes expected inflation (and hence $\Omega(\sigma)$) to change, owing to the increasing probability of a regime shift over time. Each successive adjustment of $\Omega(\sigma)$ is of course associated with a change in real prices and hence an inflation rate deviating from the rate of money creation.[14]

Next, consider a case in which money growth has been equal to γ_M for so long that inflation has converged to this level, then jumps to γ_H for one period and subsequently returns for an extended period to γ_M. In this case, the inflation rate jumps initially as the shock indicates that the economy is in the high inflation regime with certainty. Subsequently, inflation returns slowly (over seven periods, in this case) to its unconditional mean. Thus, the economy exhibits a persistent inflation response to a one-time above average increase in the money stock. In the example considered here, γ_M occurs frequently, and thus the first-order autocorrelation of inflation (XX) is substantially higher than that of the money growth rate (YY).

In general, both the variance and autocorrelation of inflation in the economy with unobserved regime depend not only on the average inflation rate, but also on the expected frequencies of both regime shifts and uninformative realizations of the money growth rate. If γ_M occurs frequently, then the autocorrelation of inflation may exceed that of the money growth rate even if some state transitions induce negative autocorrelation in the inflation rate as will happen at very high and very low average inflation. Similarly, the variance of inflation depends on both its persistence and the magnitude of the initial nominal price response to a shock. The examples presented here are sufficient to illustrate, however, that persistent inflation fluctuations may be generated through the

[14] While expected inflation converges to the unconditional mean only asymptotically, here it is clear that by the seventh consecutive realization of γ_M, additional periods of money growth at this level have only a negligible effect on the expected future value of money.

interaction of a slowly evolving expected future value of money with incomplete price adjustment arising from the combination of sellers' price-posting strategies and buyers' search intensity.

6. Conclusions

This paper has considered a stochastic monetary economy in which both nominal and real prices may respond incompletely to stochastic fluctuations in costs and the rate of money creation in spite of the fact that there are no exogenously imposed constraints on sellers' ability to adjust prices. Both cost and money growth shocks, result in two opposing effects: For a given search intensity on the part of buyers, high price sellers respond to a shock by more than low price sellers, resulting in a change in price dispersion. This change in price dispersion, however, induces buyers to change their search intensity.

In response to cost shocks, sellers desire to increase or decrease prices always dominates, but the search intensity effect may substantially limit pass-through to both real and nominal prices. As the rate of average inflation increases, the search intensity effect weakens and prices become more responsive to cost shocks. This finding is consistent with the observation that prices and inflation have become less responsive to cost movements as the average inflation rate has fallen, and with the observation that the extent to which cost shocks in the form of nominal exchange rate movements are passed through to consumer prices is declining in the average rate of inflation across countries.

In response to a change in the money growth rate, the responses of both real and nominal prices to changes in the money growth rate also depend on the average rate of inflation. At moderate average inflation, real prices may fall in response to a money growth shock, and hence nominal prices may exhibit a form of price stickiness in the sense that they fail to increase in proportion to the stock of money. The dynamics of inflation in response to such shocks depend not only on the degree of price adjustment, but also on the speed with which the expected future value of money adjusts to stochastic changes in the rate of money creation. If this value adjusts slowly, then even purely transitory changes in the money creation rate can produce very persistent responses of inflation. This is particularly true at moderate inflation rates, where the inflation rate responds sluggishly to changes in the rate of money growth rate even if $\Omega(\sigma)$ responds instantaneously.

Our analysis suggests that strategic interaction among price posting sellers and buyers who choose search intensity may lead endogenously to a form of nominal rigidity. In principle, the environment can account for both incomplete and/or delayed responses of nominal prices to shocks, and is consistent with a wide range of possible inflation dynamics. It would certainly be possible to quantify these effects in computational experiments using calibrated versions of the economy.

Our environnment has the advantage of being relatively simple in the sense that the distribution of prices is a *function* of the (low dimension) state vector σ rather than a state variable itself. A consequence of this is that there is considerable scope for adding more "realistic" components to the model without losing tractability. As the point of this paper is to illustrate the mechanism by which the expected future value of money interacts with the price-posting game played by buyers and sellers, we do not add any such components here. Rather, we leave this for further research.

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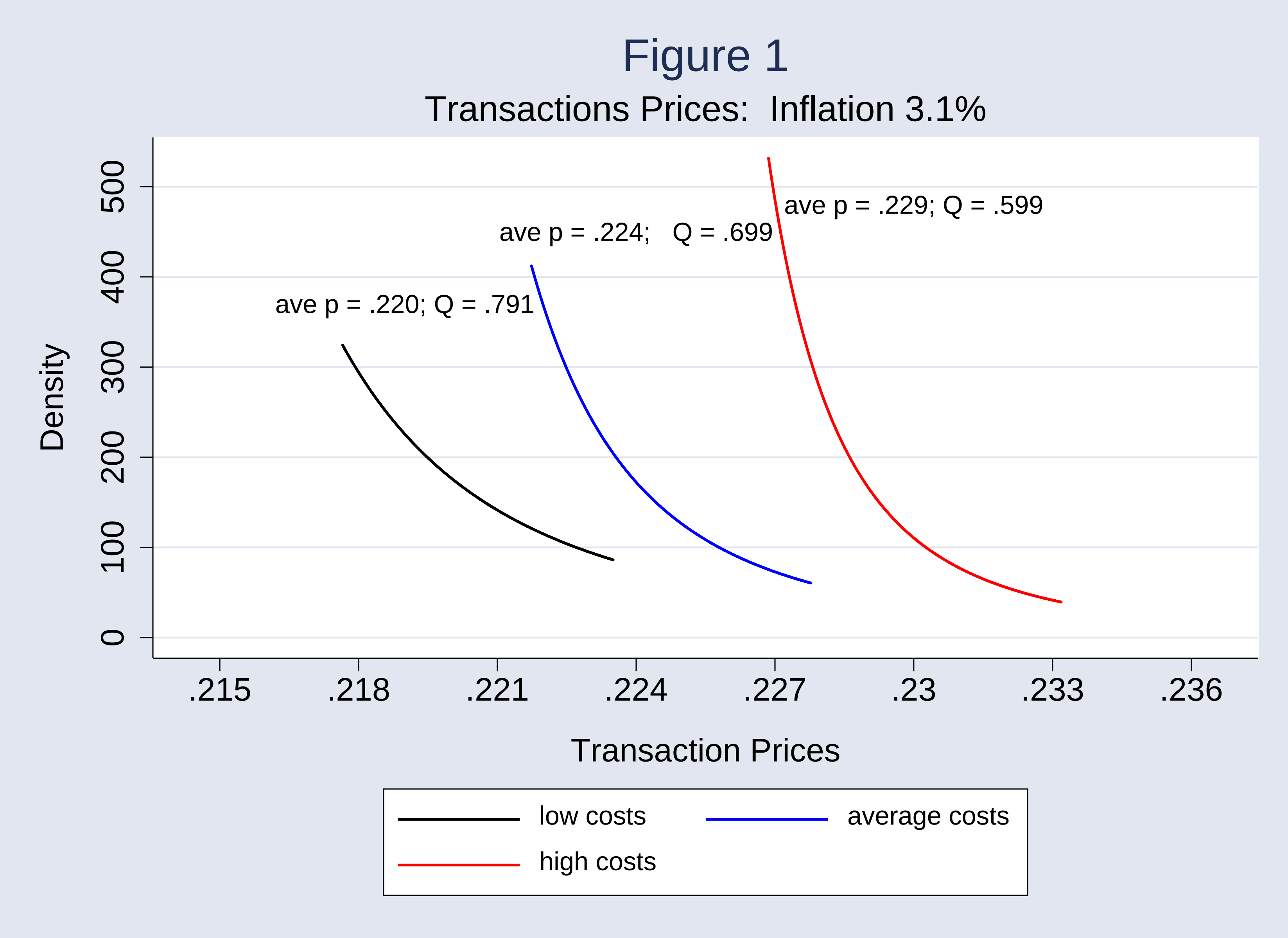


Figure 1
Transactions Prices: Inflation 3.1%
Density
0
100
200
300
400
500
.215
.218
.221
.224
.227
.23
.233
.236
Transaction Prices
ave p = .220; Q = .791
ave p = .224; Q = .699
ave p = .229; Q = .599
low costs
average costs
high costs

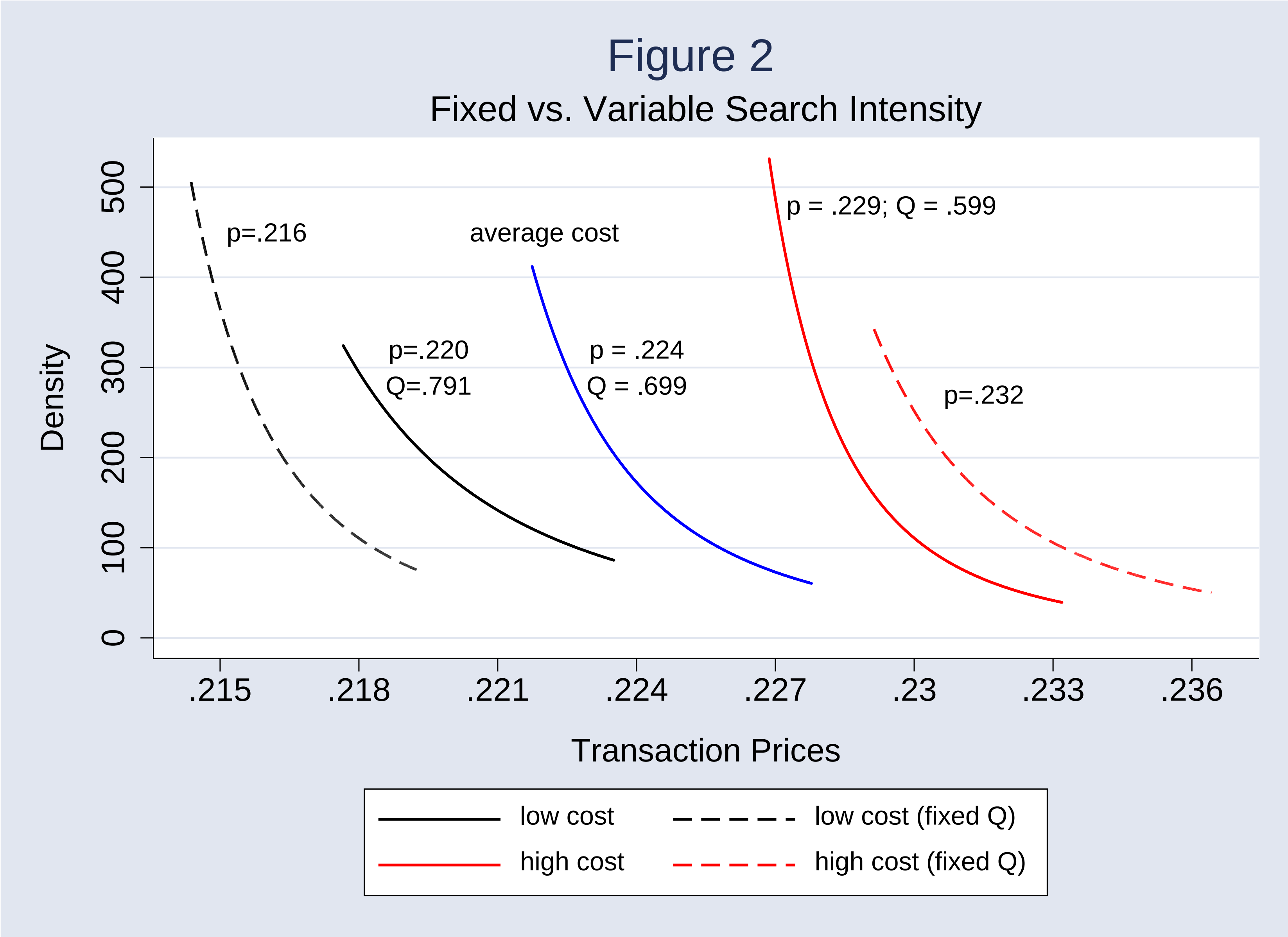
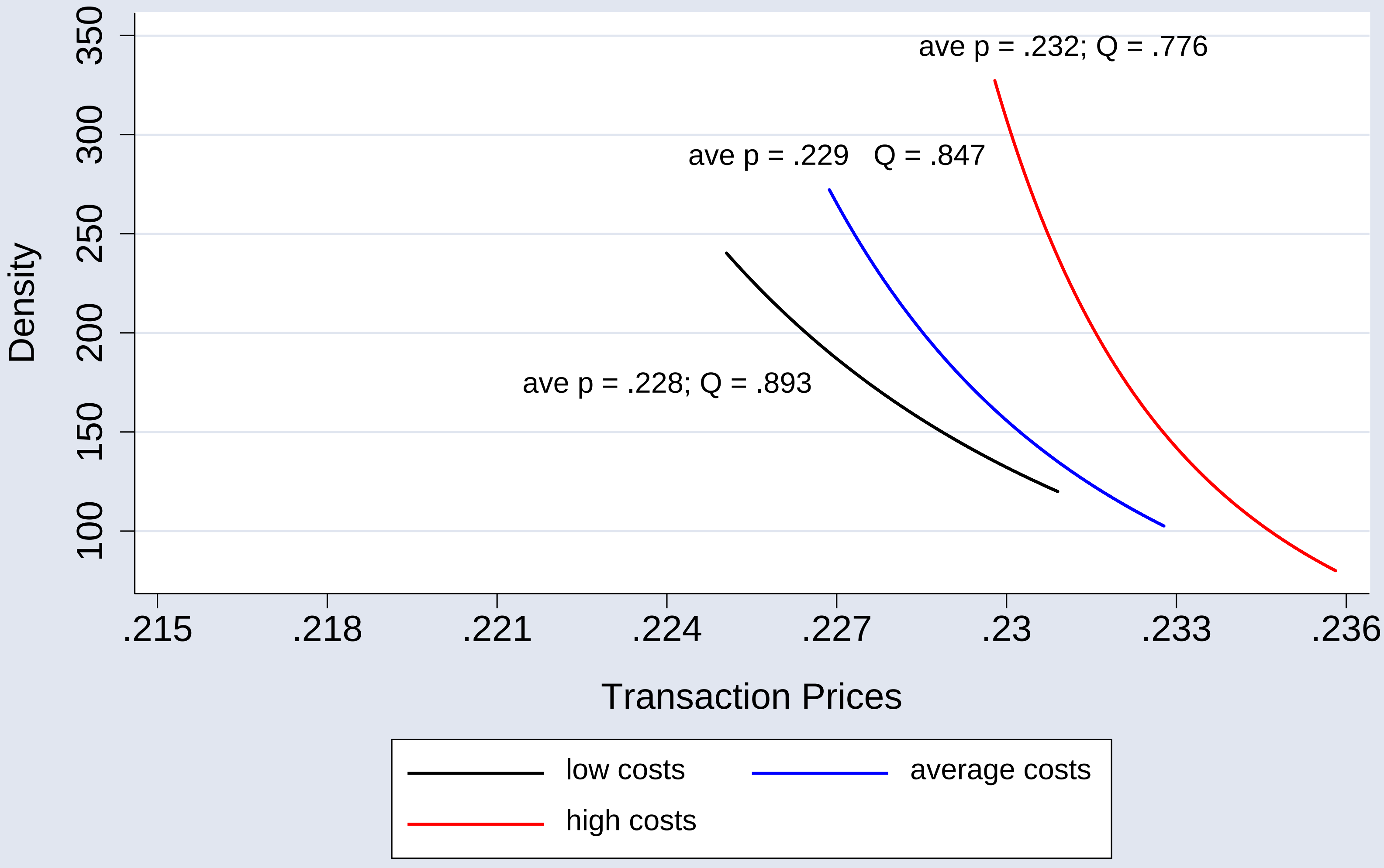


Figure 2
Fixed vs. Variable Search Intensity
Density
0
100
200
300
400
500
p=.216
average cost
p = .229; Q = .599
p=.220
Q=.791
p = .224
Q = .699
p=.232
.215
.218
.221
.224
.227
.23
.233
.236
Transaction Prices
low cost
low cost (fixed Q)
high cost
high cost (fixed Q)

Figure 3

Transactions Prices: Inflation 2%



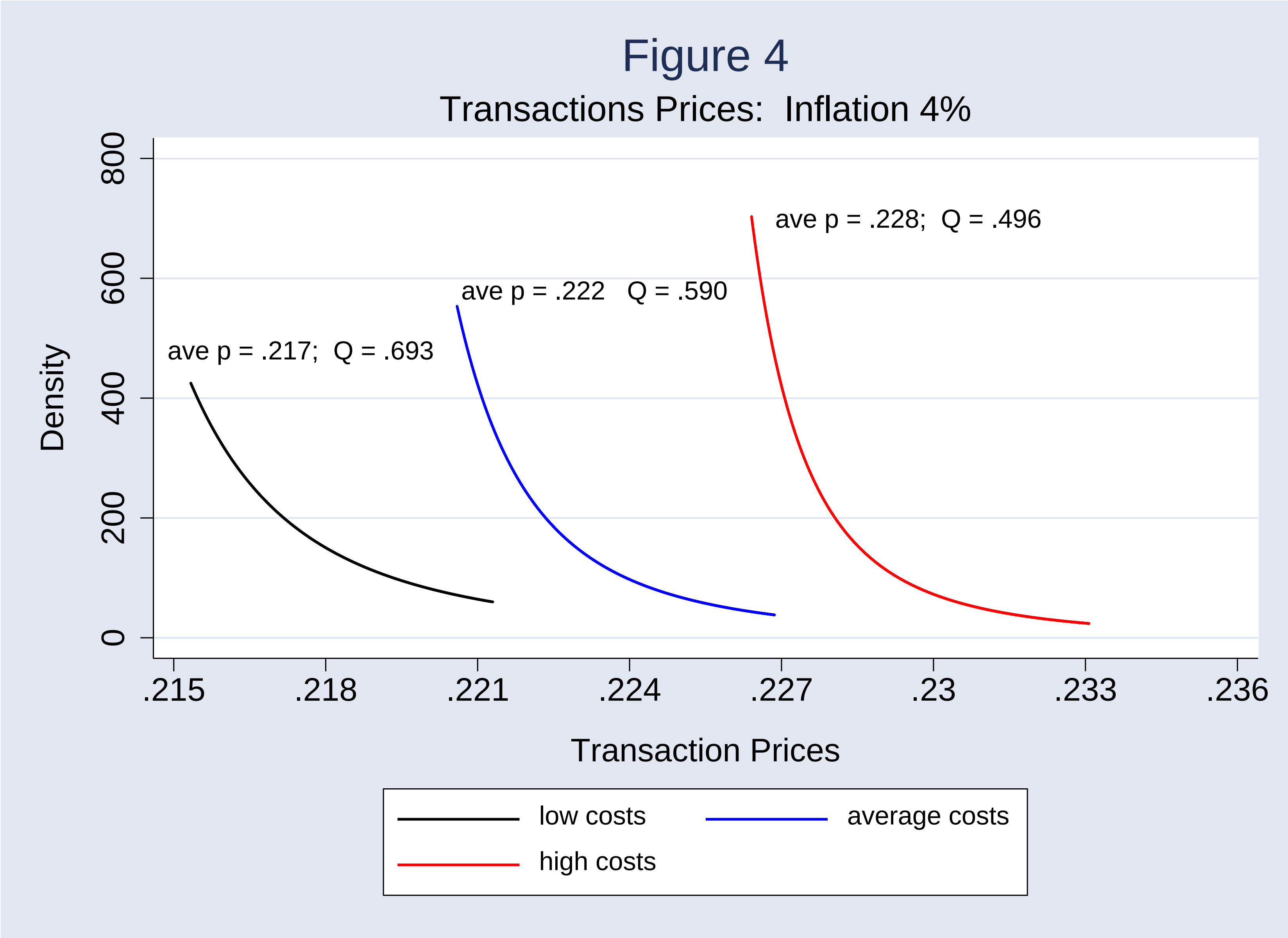


Figure 4
Transactions Prices: Inflation 4%
Density
0
200
400
600
800
.215
.218
.221
.224
.227
.23
.233
.236
Transaction Prices
ave p = .217; Q = .693
ave p = .222   Q = .590
ave p = .228; Q = .496
low costs
average costs
high costs

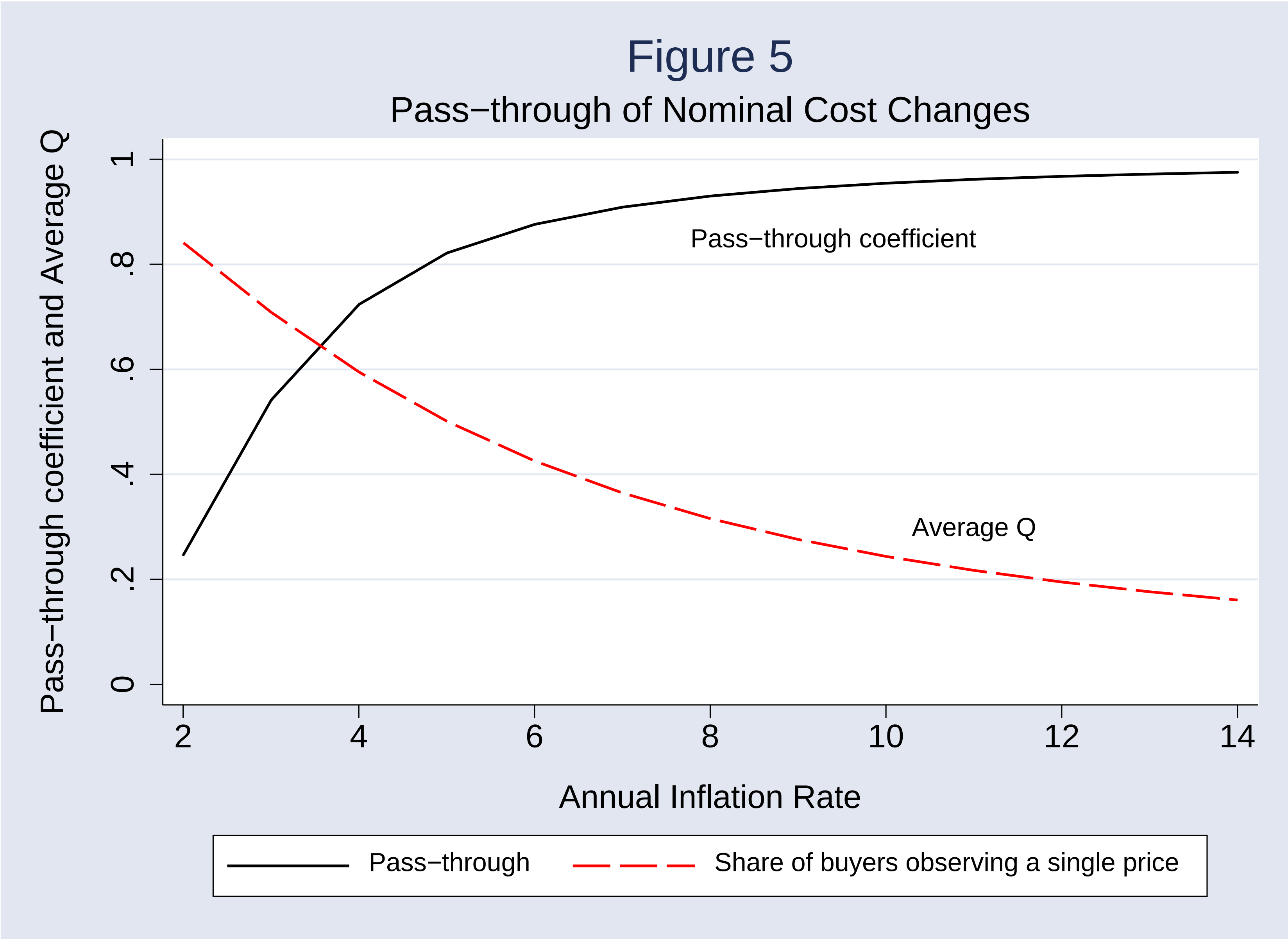


Figure 5
Pass−through of Nominal Cost Changes
Pass−through coefficient and Average Q
0
.2
.4
.6
.8
1
2
4
6
8
10
12
14
Annual Inflation Rate
Pass−through coefficient
Average Q
Pass−through
Share of buyers observing a single price

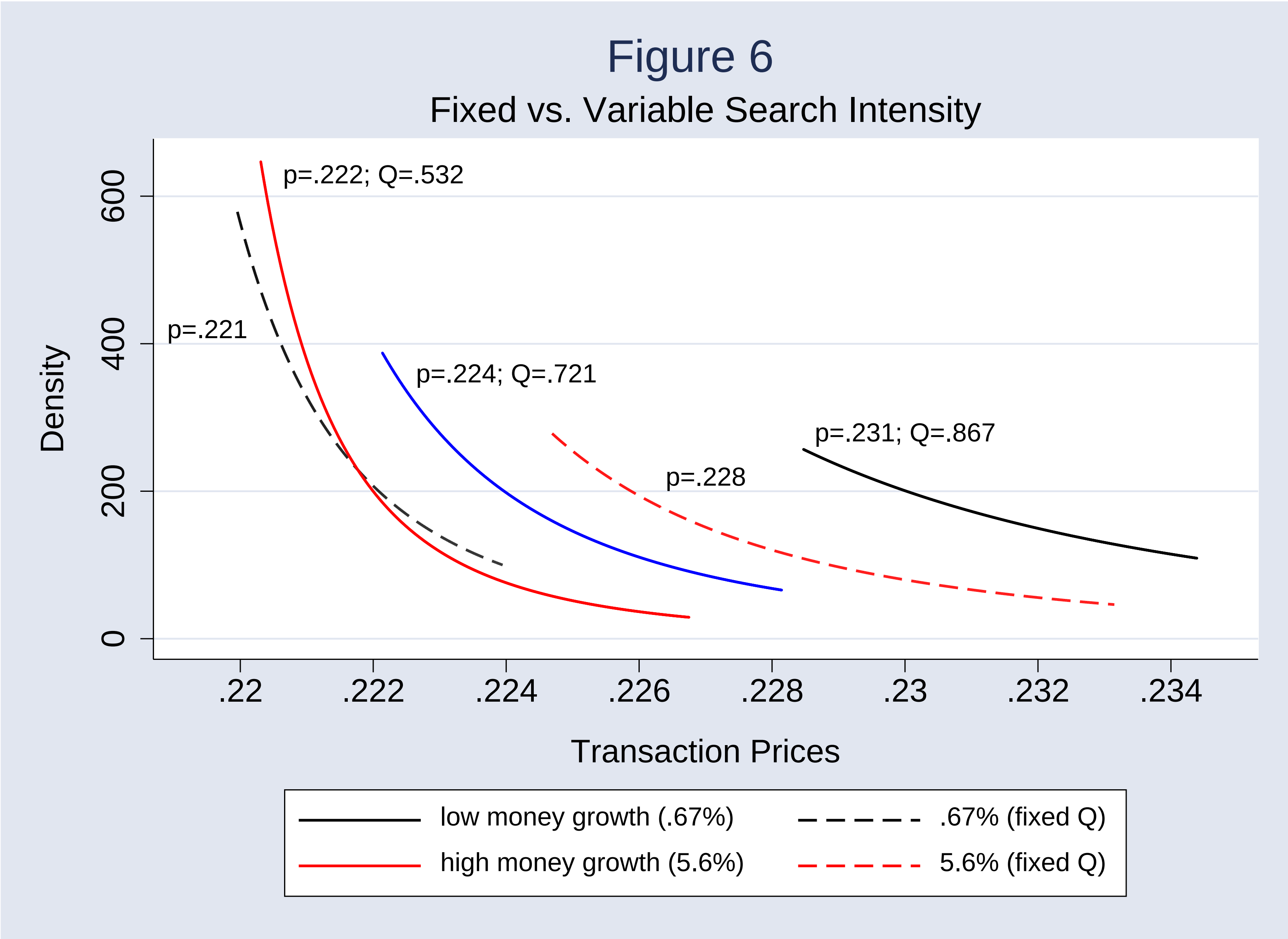


Figure 6
Fixed vs. Variable Search Intensity
p=.222; Q=.532
p=.221
p=.224; Q=.721
p=.231; Q=.867
p=.228
Density
0
200
400
600
.22
.222
.224
.226
.228
.23
.232
.234
Transaction Prices
low money growth (.67%)
.67% (fixed Q)
high money growth (5.6%)
5.6% (fixed Q)

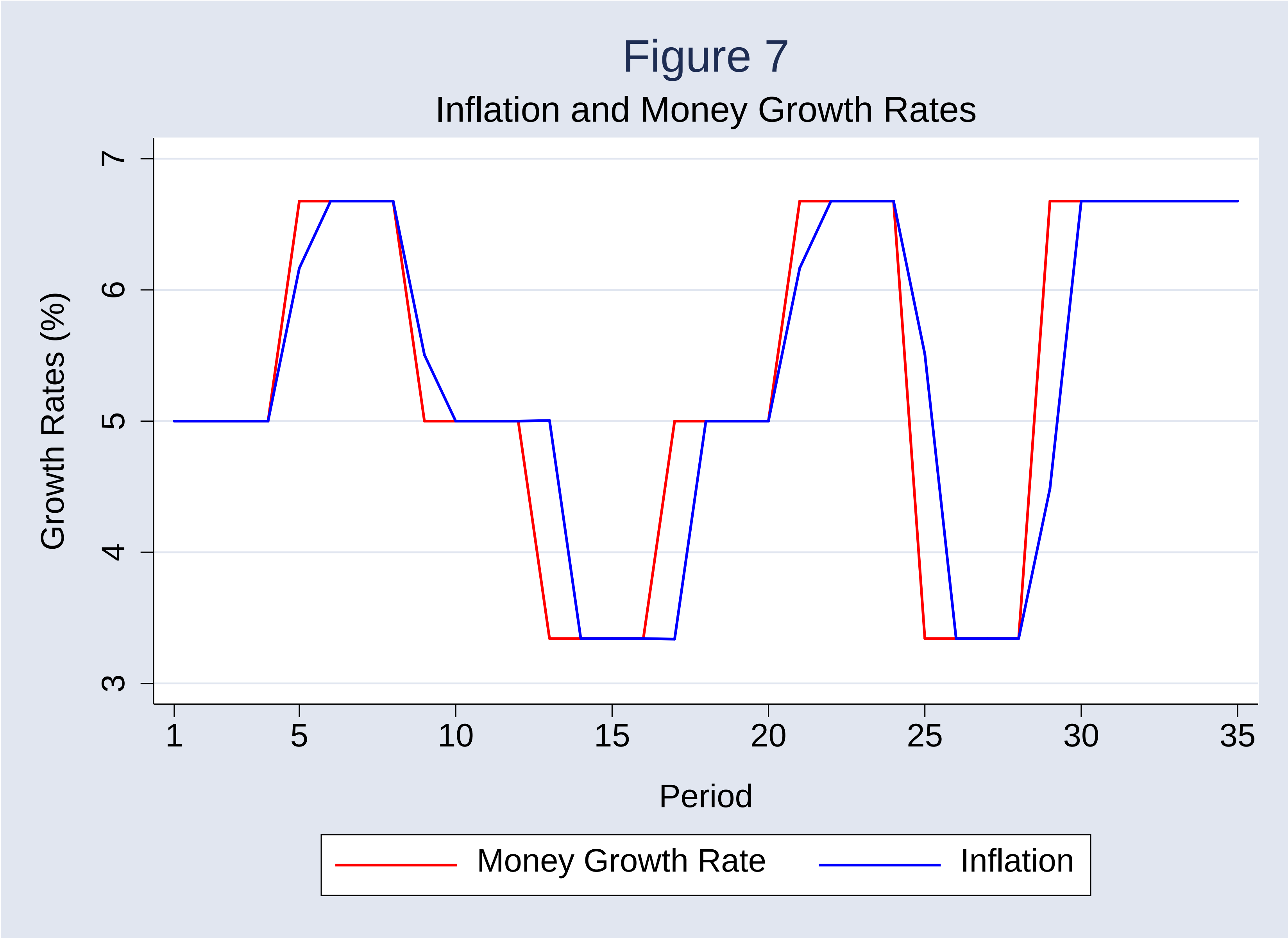


Figure 7
Inflation and Money Growth Rates
Growth Rates (%)
3
4
5
6
7
1
5
10
15
20
25
30
35
Period
Money Growth Rate
Inflation

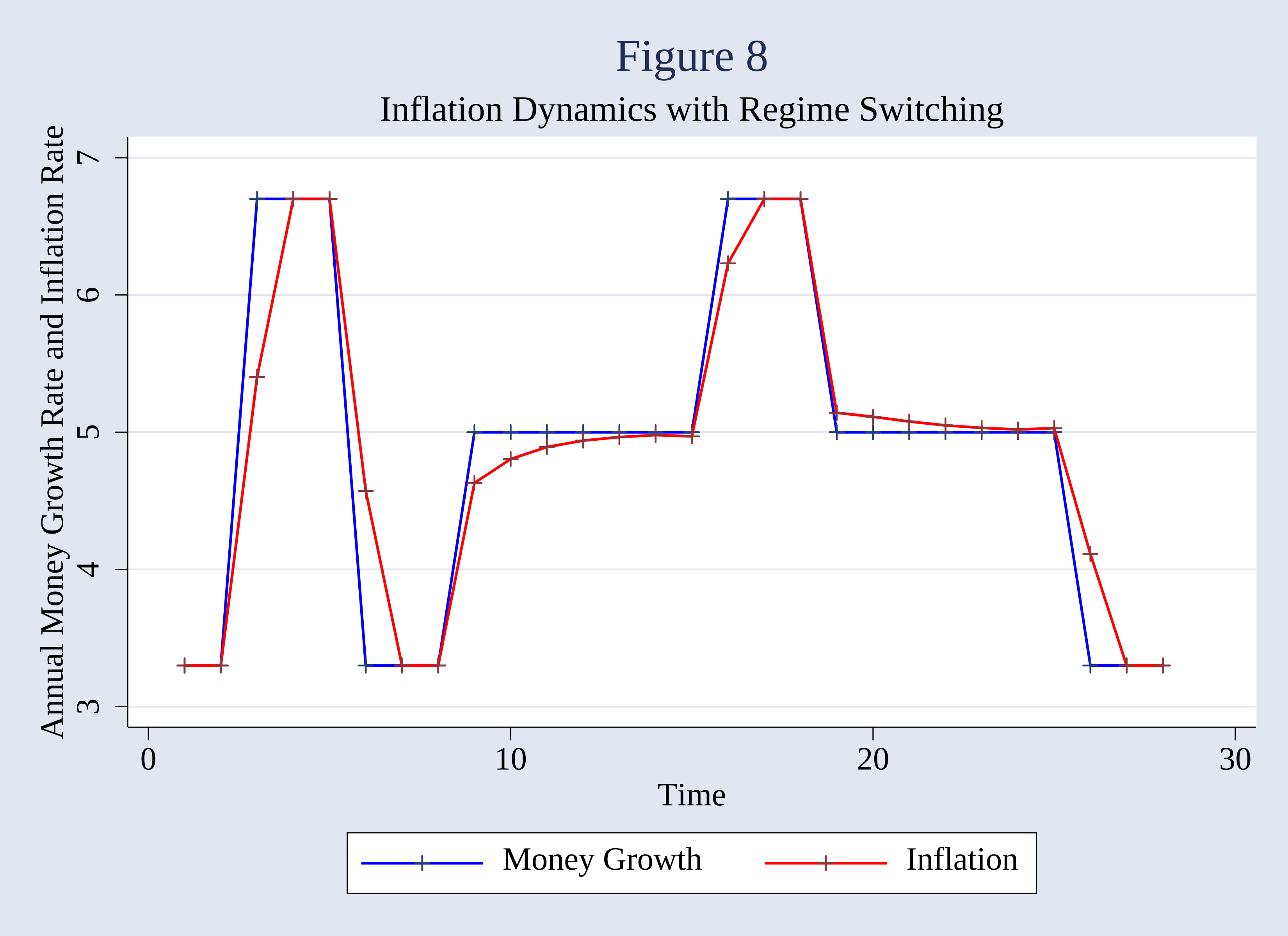


Figure 8
Inflation Dynamics with Regime Switching
Annual Money Growth Rate and Inflation Rate
3
4
5
6
7
0
10
20
30
Time
Money Growth
Inflation