Price Adjustment, Dispersion and Inflation

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preliminary and incomplete

Abstract

The observation that consumer prices are "sticky" in the sense that the nominal price level typically responds sluggishly or less than fully to short-run changes in economic fundamentals has attracted interest among economists, particularly with regard to its implications for aggregate fluctuations and the effectiveness of monetary policy. In much of the literature, price stickiness is taken as given, with price adjustment impeded by some exogenously imposed nominal rigidity. In this paper we study the responses of both nominal and real prices to random fluctuations in costs, preferences, and money creation using a monetary economy with search frictions and no constraints on sellers' ability to change prices. In equilibrium, our economy exhibits nominal price stickiness, the degree of which varies with the average rate of inflation. At low levels of inflation, prices are unresponsive to shocks. As the inflation rate rises, prices become more responsive. Our model is consistent with empirical findings suggesting that both the variance of inflation and the degree to which cost shocks are passed-through to prices are increasing in the average rate of inflation. In contrast to models of state-contingent pricing in the presence of menu costs, our model predicts that price dispersion always increases with inflation, rather than vanishing when inflation is sufficiently high; a finding consistent with the evidence on the dispersion of prices during hyperinflations.

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1. Introduction

Nominal price "stickiness", or the tendency of consumer prices to respond sluggishly or incompletely to short-run fluctuations in costs or demand has attracted a great deal of attention from economists. This observation has generated a large literature focused on the implications of price stickiness for the ability of monetary authorities to influence the level of economic activity, at least in the short-run. In much of this literature, nominal price adjustment is impeded by the presence of exogenously imposed rigidities. These rigidities are modeled in a variety of ways; including assuming an explicit cost of changing prices or that the frequency with which prices may be changed is limited or stochastic.

In this paper, we use a monetary economy with search frictions to examine the responses of nominal and real prices to random shifts in costs, preferences, and the money growth rate. In our environment, prices are fully flexible; sellers are neither prevented from changing prices at certain times nor do they face a cost of changing their price. Our economy nevertheless exhibits incomplete adjustment of prices ("price stickiness") in equilibrium. Moreover, the extent to which prices are sticky depends on both the average level of inflation and on the nature of shocks. When inflation is low, both nominal and real prices are relatively unresponsive to cost shocks but respond strongly to preference shocks. In contrast, as inflation rises, prices respond more more strongly to cost shocks and less so to preference shocks. At very high rates of inflation, prices are effectively fully flexible, in the sense that cost movements are "passed-through" to prices one for one.

Our work is motivated in part by the observation that the responsiveness of prices to shocks to fundamentals appears to have changed over time. Several authors have presented evidence that price stickiness is inversely related to the average rate of inflation in particular instances. For example, Taylor (2000) argues that the response of nominal prices to increases in costs has declined with the rate of inflation over time for the U.S. and other developed countries. Devereux and Yetman (2002) present evidence that the pass-through of nominal exchange rate movements (which affect costs) to consumer prices is declining in the average rate of inflation for a sample of 107 countries during the post-Bretton Woods era. The predictions of our model for the effects of cost shocks are consistent with both of these sets of findings. Also, it has long been recognized that periods of high average inflation tend to be periods of high inflation volatility. Our model is consistent with this observation when inflation fluctuations are driven by cost and/or monetary shocks.

Our work is also motivated by the desire to model price stickiness endogenously to better understand the relationships among economic factors which determine the degree of nominal rigidity in the economy. While the literature examining the *effects* of exogenously imposed nominal rigidities is extensive, there have been relatively few attempts to model the sources of incomplete price adjustment in stochastic monetary economies. A notable paper that does study these issues is Eden (1994). In our model, as in Eden's, the extent to which nominal prices adjust to shocks is endogenous and responds to the state of the economy. In contrast, however, our analysis focuses on the effects of the average inflation rate for the pricing behavior of firms, whereas in Eden's model anticipated money growth has no effect on the distribution of real prices. Our focus also contrasts with that undertaken in most studies of exogenous price stickiness (see, for example Gali (1999)) as these typically abstract from the effects of average inflation. Indeed, it is common in these models to examine the short-run effects of shocks around a zero inflation steady-state.

Our model embeds the price-posting game of Burdett and Judd (1983) into a general equilibrium environment along the lines of the random matching monetary models of Shi (1999) and Head and Shi (2003). In an earlier paper, Head and Kumar (2003) study the long-run properties of a similar environment under certainty. In this paper, we extend their model to a stochastic environment, and focus on the response of nominal prices to random shocks. In this environment, the Burdett-Judd pricing framework generates price dispersion in equilibrium, with the extent of dispersion depending on the average rate of inflation. In our model, the degree of price dispersion, and its response to shocks are key factors determining price adjustment in equilibrium.

In our economy, shocks to costs, preferences, and the money growth rate are passed-through differentially to consumer prices by sellers pricing in different regions of the price distribution. Changes in the distribution affect the fraction of buyers observing more than one price changing the overall degree of sellers' market power and the altering adjustment of prices. At low rates of inflation, a relatively large fraction of buyers observes only a single price. A cost or money growth shock generates greater dispersion, and causes a large increase in this share. The resulting reduction in sellers' market power limits the adjustment of prices in response to these shocks. In contrast, preference shocks compress the distribution, increasing the share buyers observing a single price. The resulting increase in sellers' market power induces them to change prices drastically in response to preference shocks.

As the rate of trend inflation rises, *ceteris paribus*, the average share of buyers observing more than one price falls, and the response of this share (positive or negative) to shocks of any kind weakens. Pass-through of cost and monetary shocks increases, while the response of prices to preference shocks diminishes. Average prices become more closely tied to marginal cost. As a result, at sufficiently high trend inflation, inflation moves one-for-one with changes in costs.

A tight relationship between the price level and shocks to fundamentals also emerges in "state-contingent" pricing models along the lines of Dotsey, King and Wolman (1999) and Devereux and Yetman (2002). In these models, for a fixed cost of changing prices, a larger share of firms find it profitable to change their prices in a given period the higher the rate of inflation. At some point, the rate of inflation is sufficient to induce *all* firms to change their prices each period, eliminating price stickiness. These models also, however, predict that price dispersion will vanish at high levels of inflation. In contrast, in our economy price dispersion increases monotonically with inflation, a finding consistent with empirical evidence that price dispersion increases with inflation and is typically very high during hyperinflations (*e.g.* Cassella and Feinstein (1990), Lach and Tsiddon (1992), Fershtman, Fishman, and Simhon (2003)).

This version of the paper is preliminary and incomplete. Section 2 describes the environment. In section 3, we define and Markov monetary equilibrium for this environment and outline our numerical procedure for computing such equilibria. The effects of random shocks to costs, preferences, and money growth are considered in a series of computational experiments in section 4. Section 5 endogenizes money growth, assuming that the monetary authority chooses it in response to shocks to costs and preferences either optimally or following an *ad hoc* sub-optimal policy rule. Section 6 draws conclusions and describes some implications of the results for future work.

2. The Economy

2.1. The environment

The economy is a stochastic version of that studied by Head and Kumar (2003). Time is discrete. There are large numbers (*i.e.* unit measures) of $H \geq 3$ different types of both households and non-storable consumption goods. A type h household is able to produce only good h and derives utility only from consumption of good $h+1$, modulo H. Each household is comprised of large numbers (unit measures) of two different types of members; "buyers" and "sellers". Individual buyers and sellers do not have independent preferences and do not undertake independent actions. Rather, they share equally in *household* utility and act only on instructions *from the household*.

Members of a representative type h household who are *sellers* can produce good h in period t at marginal cost $\phi_t > 0$ utils per unit. Production costs are stochastic; ϕ_t evolves via a discrete Markov chain with

$$\text{Prob}\{\phi_{t+1} = \phi_j | \phi_t = \phi_i\} \equiv \pi^{\phi}_{ji} \qquad \forall t, t+1; \quad \phi_j, \phi_i \in \mathcal{P}, \tag{2.1}$$

where $\mathcal{P}$ is a finite set of possible production cost parameters. Let x_t denote the total quantity of good h produced by all the sellers from this household in period t. Then the household's total disutility from production in this period is equal to $\phi_t x_t$.

Members of this household who are *buyers* observe random numbers of price quotes and may purchase good $h+1$ at the lowest price that they observe individually. The household chooses the *expected* number of price quotes observed by an individual buyer. Or, alternatively, because each household contains a continuum of symmetric buyers, it chooses the measures of these buyers that observe different numbers of prices. For each observed price quote, the household pays an information or search cost of μ utils. We assume that buyers observe either one or two price quotes only[1]. In this case the household's total disutility of information gathering or search in period t is equal to $\mu \sum_{k=1}^{2} k q_{kt}$, where q_{kt} is the measure of buyers observing k price quotes at time t.

In period t, a representative type h household receives utility $u_t(c_t)$ from consumption of c_t units of good $h+1$, where c_t is equal to the total purchases of its buyers in that period. For convenience, we assume that for all t, $u_t(c)$ takes the constant relative risk aversion (CRRA) form:

$$u_t(c) = \frac{c^{1-\alpha_t} - 1}{1 - \alpha_t} \tag{2.2}$$

where α_t is a random preference parameter which evolves via a discrete Markov chain:

$$\text{Prob}\left\{\alpha_{t+1} = \alpha_j | \alpha_t = \alpha_i\right\} \equiv \pi_{ji}^{\alpha} \qquad \forall t, t+1; \quad \alpha_j, \alpha_i \in \mathcal{A}, \tag{2.3}$$

with $\mathcal{A}$, like $\mathcal{P}$ a finite set. Throughout the paper we require that $\alpha > 1$ for all $\alpha \in \mathcal{A}$. In this case we have $\lim_{c \to 0} u_{tc}(c)c = \infty$, where $u_{tc}(c)$ denotes the derivative of u_t with respect to c.

A representative household's *total* period t utility is equal to that which it receives from consumption of goods purchased by its buyers minus the production disutility incurred by its sellers and its information gathering or search costs. The household acts so as to maximize the expected discounted sum of its period utility over an infinite horizon:

$$U = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \phi_t x_t - \mu \sum_{k=1}^{2} k q_{kt} \right] \right]. \tag{2.4}$$

[1] The assumption that buyers observe only one or two price quotes is not as restrictive as it may seem. Monetary equilibria of the type on which this paper focuses exist *only* if strictly positive measures of buyers observe one *and* two prices only. Moreover, it can be shown that under certain conditions households will choose to have buyers observe only one or two prices, thus guaranteeing existence. These claims are proved by Head and Kumar (2003) and the arguments are summarized in appendix C.

Since a type h household produces good h and consumes good $h+1$, a double coincidence of wants between members of any two households is impossible. Moreover, it is assumed that households of a given type are indistinguishable and that members of individual households cannot be relocated in the future following an exchange. Since consumption goods are non-storable, direct exchanges of goods cannot be mutually beneficial. Rather, exchange is facilitated by the existence of perfectly durable and intrinsically worthless *fiat money*. A type h household may acquire fiat money by having its producers sell output to buyers of type $h-1$ households. This money may then be exchanged for consumption good $h+1$ by the household's own buyers in a future period.

In the initial period ($t=0$) households of all types are endowed with M_0 units of fiat money. The *per household* stock of this money is denoted M_t, for each t, At the beginning of each period $t \geq 1$, households receive a lump-sum transfer, $(\gamma_t - 1)M_{t-1}$, of new units of fiat money from a monetary authority with no purpose other than to change the stock of money over time. We assume that the gross growth rate of the money stock,

$$\gamma_{t+1} = \frac{M_{t+1}}{M_t}, \tag{2.5}$$

evolves stochastically via a discrete Markov chain:

$$\text{Prob}\{\gamma_{t+1} = \gamma_j | \gamma_t = \gamma_i\} \equiv \pi_{ji}^{\gamma} \qquad \forall t, t+1; \quad \gamma_j, \gamma_i \in \mathcal{G}, \tag{2.6}$$

where again $\mathcal{G}$ is a finite set.

Finally, it is useful to define the vector, $\sigma = (\alpha, \phi, \gamma)$, of exogenous stochastic parameters. Using (2.1), (2.2), and (2.6) we define a Markov process for σ:

$$\text{Prob}\{\sigma_{t+1} = \sigma_j | \sigma_t = \sigma_i\} \equiv \Pi_{ji} \qquad \forall t, t+1; \quad \sigma_j, \sigma_i \in \mathcal{S}, \tag{2.7}$$

where $\mathcal{S} \equiv \mathcal{A} \times \mathcal{P} \times \mathcal{G}$. In each period, the *state* of the economy is given by σ_t and the per household stock of money, M_t.

2.2. The current period trading session

In describing the optimization problem of a representative household (of any type), it is useful to begin with the current period trading session. At the beginning of period t a representative household observes the state of the economy, (M_t, σ_t) and has post-transfer *household* money

[2] Where possible, capital letters (*e.g.* C, Q, M) will be used to distinguish per household quantities from their counterparts for an individual household (c, q, m) etc..

holdings m_t.[2] The household chooses the probabilities with which an individual buyer observes one (as opposed to two) price quotes, q_{1t}, and issues trading instructions to both its buyers and sellers in order to maximize utility. Buyers and sellers then split up for a *trading session.* We assume that it is not until this trading session begins that the exact number of quotes observed by *individual* buyers is known. As a result, households have no incentive to treat their members asymmetrically; they distribute money holdings equally to all buyers and issue the same instructions to all buyers and to all sellers[3].

In the trading session, sellers post prices and buyers decide whether or not to purchase at the posted price, each acting in accordance with household instructions. Exchanges of goods for fiat money take place in bi-lateral matches between buyers and sellers of different households. Following trading, buyers and sellers reconvene and the household consumes the goods purchased by its buyers. The sellers' revenue (in fiat money) and any remaining money unspent by the buyers are pooled and carried into the next period, when they are augmented with transfer $(\gamma_{t+1} - 1)M_t$ to become m_{t+1}.

With q_{1t} fixed, the mechanism by which buyers and sellers are matched is similar to the "noisy sequential search" process of Burdett and Judd (1983). Households know the distribution of prices offered by sellers, but individual buyers may purchase only at a price they are quoted by a specific prospective seller in a particular period[4]. Let the distribution of prices posted by sellers of the appropriate type at time t be described by the cumulative distribution function (*c.d.f.*) $F_t(p_t)$ on support $\mathcal{F}_t$. Given $F_t(p_t)$, the *c.d.f.* of the distribution of the *lowest* price quote received by a buyer at time t is given by

$$J_t(p_t) = \sum_{k=1}^{2} q_k \left[1 - [1 - F_t(p_t)]^k\right] \qquad \forall p_t \in \mathcal{F}_t. \tag{2.8}$$

Individual buyers are constrained to spend no more than the money distributed to them at the beginning of the session by the household. If buyer i purchases he/she does so at the lowest price

[3] In the exposition, we will for now we suppress the economy state vector, (M_t, σ_t), as it remains fixed throughout the trading session. Similarly, we postpone analysis of the choice of q_{1t} until later and for now treat q_k, $k = 1, 2$ as fixed, as they are once the trading session has begun.

[4] We assume that buyers cannot return to sellers from whom they have purchased in the past, and instead draw new price quotes from the distribution each period. This assumption enables price dispersion to persist in a stationary equilibrium of our model. Empirical evidence in Lach (2002) suggests that price dispersion is indeed persistent and that individual sellers change their prices frequently, limiting the ability of buyers to identify low price sellers for repeat purchases.

observed, spending $\hat{m}_{it}(p_t)$ conditional on the price paid. Thus buyers face the exchange constraint

$$\hat{m}_{it}(p_t) \leq m_t \qquad \forall i, p_t. \tag{2.9}$$

Buyers, being identical, act symmetrically if they receive the same lowest price quote. Because the household contains a continuum of symmetric buyers, it faces no uncertainty with regard to its *overall* trading opportunities in the trading session of the current period. Realized household consumption purchases in this period are then

$$c_t = \int_{\mathcal{F}_t} \frac{\hat{m}_t(p_t)}{p_t} dJ_t(p_t). \tag{2.10}$$

An individual seller produces to meet the demand of the buyers who observe his/her price and wish to purchase. Expected sales in the current period trading session for a seller who posts p_t are given by

$$x(p_t) = \frac{\hat{M}_t(p_t)}{p_t} \sum_{k=1}^{2} Q_{kt} k \left[1 - F_t(p_t)\right]^{k-1}. \tag{2.11}$$

Here $\hat{M}_t(p_t)$ is the spending rule of a type $h-1$ buyer, $F_t(p_t)$ is the distribution of prices posted by the seller's competitors, and Q_{kt} is the average measure of buyers observing $k = 1, 2$ prices.

In (2.11), $\hat{M}_t(p_t)/p_t$ represents the *quantity per sale* and the summation term is the *expected number of sales*. The expected number of sales equals the number of observations of the seller's price multiplied by the probability that in each of these instances it is the lowest price observed. The number of observations is the ratio of the measures of buyers to sellers (in this case one) times the expected number of price observations for a randomly selected buyer, $\sum_k Q_k k$. Given distribution $F_t(p_t)$, the probability that the other $k-1$ prices observed by a buyer exceed the seller's price is $[1 - F_t(p_t)]^{k-1}$, $k = 1, 2$.

Let $\hat{F}_t(p_t)$ be the distribution of prices posted by a representative household's sellers and denote its support $\hat{\mathcal{F}}_t$. Since this household contains a continuum of sellers, it faces no uncertainty with regard to its *total* sales in the current trading session. These are given by

$$x_t = \int_{\hat{\mathcal{F}}_t} x(p_t) d\hat{F}_t(p_t). \tag{2.12}$$

Using (2.10)—(2.12), we have

$$m_{t+1} = m_t - \int_{\mathcal{F}_t} \hat{m}_t(p_t) dJ_t(p_t) + \int_{\hat{\mathcal{F}}_t} p_t x(p_t) d\hat{F}_t(p_t) + (\gamma_{t+1} - 1) M_t. \tag{2.13}$$

A representative household's money holdings going into next period's goods trading session are m_t minus the amount spent by its buyers this period; plus its sellers' receipts of money; plus the transfer received at the beginning of the next period.

We now characterize the households' choice of instructions, $\hat{m}_t(p_t)$ and $\hat{F}_t(p_t)$, to its buyers and sellers respectively. Consider first the spending rule, $\hat{m}_t(p_t)$. The household's gain to having a buyer exchange $\hat{m}_t(p_t)$ units of currency for consumption at p_t is given by the household's marginal utility of current consumption, $u_c(c_t)$, times the quantity of consumption good purchased, $\hat{m}_t(p_t)/p_t$. The household's cost of this exchange is the number of currency units given up, $\hat{m}_t(p_t)$, times the marginal value of money in the trading session of the *next* period which we will denote ω_t. Note that ω_t is the value of relaxing constraint (2.13) marginally. Since individual buyers are small and the household may not reallocate money balances across buyers once the goods trading session has begun, the optimal spending rule instructs buyers to spend their entire money holdings if the lowest price they observe is below $u_{tc}(c_t)/\omega_t$ (the reservation price) and to return with money holdings unspent otherwise:

Proposition 1:

$$\hat{m}_t(p_t) = \begin{cases} m_t & p_t \leq \frac{u_{tc}(c_t)}{\omega_t} \\ 0 & p_t > \frac{u_{tc}(c_t)}{\omega_t}. \end{cases} \tag{2.14}$$

With regard to the household's price posting policy, the expected return from having a seller post a particular price at time t depends on the distribution of prices posted by sellers of other households of its type, $F_t(p_t)$, and the strategies of its prospective buyers, $\hat{M}_t(p_t)$. Let $\bar{p}_t$ denote the household's belief regarding the reservation price of its potential customers (all of whom are *ex ante* identical). The household will instruct no seller to post $p_t > \bar{p}_t$, as doing so generates no sales and an expected return to the household of zero.

The expected return to the household from having a seller post a price no greater than $\bar{p}_t$ is

$$r(p_t) = \left[\omega_t \hat{M}_t(p_t) - \phi_t \frac{\hat{M}_t(p_t)}{p_t}\right] \sum_{k=1}^{2} Q_{kt} k \left[1 - F_t(p_t)\right]^{k-1}; \tag{2.15}$$

In (2.15) $r(p_t)$ equals the expected *return per sale* (in brackets) times the *expected number of sales* (as in (2.11)). The former term is the value of the currency units obtained minus the disutility of production. Here it is clear that the return to posting a price lower than $p_t^* = \phi_t/\omega_t$ (the *marginal cost price*) is negative, and thus the household will instruct no seller to do so.

The household maximizes returns by instructing its sellers to post only prices such that

$$p_t \in \underset{p_t^* \leq p_t \leq \bar{p}_t}{\operatorname{argmax}} \; r(p_t) \equiv \hat{\mathcal{F}}_t \tag{2.16}$$

The household receives the same return from a seller who posts any price in $\hat{\mathcal{F}}_t$. We thus express the household's instructions by a *c.d.f.* $\hat{F}_t(p_t)$ on support $\hat{\mathcal{F}}_t$ and think of sellers as drawing their prices randomly from this distribution.

2.3: Dynamic optimization

To this point we have focused on the current period trading session holding fixed the probabilities of a representative household's buyers observing one and two prices and taking as given the household's marginal value of a unit of money. We now turn to the household's dynamic optimization problem. To begin with, it is useful to write household consumption as the sum of the purchases of those of its buyers who observe one and two prices:

$$c_t = q_{1t}c_t^1 + q_{2t}c_t^2 \qquad \text{where} \qquad c_t^k = m_t \int_{\mathcal{F}_t} \frac{1}{p_t} dJ_t^k(p_t) \tag{2.17}$$

and for all $p_t \in F_t(p_t)$

$$J_t^1(p_t) = F_t(p_t) \qquad \text{and} \qquad J_t^2(p_t) = 2F_t(p_t) - [F_t(p_t)]^2 \tag{2.18}$$

are the distributions of the lowest price observed by buyers who observe exactly one and two prices, respectively. In (2.17) we have made use of the fact that buyers follow the spending rule, (2.14). Note that the household's choice of q_{1t} is constrained by the requirement that it be a probability:

$$q_{kt} \geq 0, \quad k = 1,2 \qquad \text{and} \qquad q_{2t} = 1 - q_{1t}, \qquad \forall t. \tag{2.19}$$

At time t, for a representative household (of any type), its individual money holdings, m_t, are a relevant state variable in addition to M_t and σ_t. For $\sigma_t = \sigma_i$, we represent dynamic optimization problem of such a household by the following Bellman equation:

$v_t(m_t, M_t, \sigma_i) =$

$$\max_{q_t, m_{t+1}, \hat{m}_t(p_t), \hat{F}_t(p_t)} \left\{ u(c_t) - \phi_t x_t - (2 - q_t)\mu + \beta \sum_{\sigma_j \in \mathcal{S}} \Pi_{ji} v_{t+1}(m_{t+1}, M_{t+1}, \sigma_j) \right\} \tag{2.20}$$

subject to:

(2.5) (2.7) (2.9) – (2.13) (2.16) and (2.19),

where q is the probability that a buyer receives *one* price quote. The household takes as given the actions of other households, $X_t(p_t; M_t, \sigma_t)$, $\hat{M}_t(p_t; M_t, \sigma_t)$, and $Q_t(M_t, \sigma_t)$; as well as the distribution of exchange prices, $J_t(p_t; M_t, \sigma_t)$. Here M_t and σ_t are included as arguments to indicate that these actions and prices depend on the aggregate state.

From the household Bellman equation, we have

$$\omega_t(m_t, M_t, \sigma_i) = \beta \sum_{\sigma_j \in \mathcal{S}} \Pi_{ji} v_{t+1_m}(m_{t+1}, M_{t+1}, \sigma_j). \tag{2.21}$$

We also have first-order conditions associated with the choice of $\hat{m}_t(p_t)$:

$$u_{tc}(c_t)\frac{1}{p_t} - \lambda_t(p_t; m_t, M_t, \sigma_i) - \omega_t(m_t, M_t, \sigma_i) = 0 \qquad \forall p_t, t, \tag{2.22}$$

where $\lambda_t(p_t; m_t, M_t, \sigma_i)$ is a Lagrange multiplier on the buyers' exchange constraint, (2.9). The optimal probability with which a buyer observes one price, q_t^*, may be shown to satisfy

$$q_t^* \in \underset{q_t \in [0,1]}{\operatorname{argmax}}\ u\left[q_t c_t^1 + (1-q_t)c_t^2\right] - (2-q_t)\mu. \tag{2.23}$$

Finally, we have the envelope condition

$$v_{tm}(m_t, M_t, \sigma_i) = \int_{\mathcal{F}_t} \lambda_t(p_t; m_t, M_t, \sigma_i) dJ_t(p_t) + \omega_t(m_t, M_t, \sigma_i), \qquad \forall t. \tag{2.24}$$

Conditions (2.21)—(2.24) together with the buyers' acceptance rule, (2.14), and the requirement that $\hat{\mathcal{F}}_t$ satisfy (2.16) characterize the household's optimal behaviour conditional on its money holdings, m_t, the aggregate state, (M_t, σ_i), and its beliefs regarding the actions of other households.

3. Equilibrium

We consider only equilibria that are symmetric and Markov. By symmetric, we mean that in equilibrium households choose a common probability, Q_t, for a buyer to observe a single price and a common distribution, $\hat{F}_t(p_t)$, from which sellers draw prices to post and that all have the same marginal valuation of money, Ω_t; consumption, C_t; and money holdings, M_t; in each period. The equilibria we consider are Markov in that *quantities*; C_t, output, X_t; the probability Q_t; and the distributions of *real prices* (*i.e.* nominal prices divided by the per household money stock, M_t), are required to be time invariant functions of σ, which evolves according to Markov chain (2.7).

In a symmetric equilibrium, all buyers have common reservation prices and equal money holdings so that (2.10) gives rise to a version of the quantity equation,

$$C_t = M_t \int_{\mathcal{F}_t} \frac{1}{p_t} dJ_t(p_t) \qquad \forall t. \tag{3.1}$$

If $C_t = C(\sigma)$ for all t such that $\sigma_t = \sigma$, then conditional on σ, the average nominal transaction price must be proportional to the per household money stock, M. That is, for any two time periods, t, t', such that $\sigma_t = \sigma_{t'}$:

$$\frac{M_t}{M_{t'}} = \frac{\int_{\mathcal{F}_{t'}} \frac{1}{p_{t'}} dJ_{t'}(p_{t'})}{\int_{\mathcal{F}_t} \frac{1}{p_t} dJ_t(p_t)}. \tag{3.2}$$

If conditional on σ, *all* nominal posted prices are proportional to M, then there exist N (the cardinality of $\mathcal{S}$) time-invariant distributions of *real* posted prices characterized by supports $\mathcal{F}(\sigma) \equiv \{p_t/M_t,\, p_t \in \mathcal{F}_t;$ for all t such that $\sigma_t = \sigma\}$ and conditional *c.d.f.*'s:

$$F(p\,|\,\sigma) = F_t(p_t) \qquad \forall p \in \mathcal{F}(\sigma), \quad \forall t \mid \sigma_t = \sigma. \tag{3.3}$$

If N conditional distributions satisfying (3.3) exist, then we may think of buyers as observing *real* price quotes, and define N corresponding conditional distributions of lowest real prices observed in a manner analogous to (2.8):

$$J(p\,|\,\sigma) = Q(\sigma)F(p\,|\,\sigma) + [1 - Q(\sigma)]\left[2F(p\,|\,\sigma) - F(p\,|\,\sigma)^2\right]. \tag{3.4}$$

Similarly, if the distributions of posted and transactions prices are time-invariant conditional on σ, then households' nominal money holdings, m_t, spending rule for buyers, $\hat{m}_t(p_t)$, and the support of sellers' posted prices, $\hat{\mathcal{F}}_t$ may be divided by the per household money stock to obtain time-invariant conditional *real* counterparts: $m(\sigma) = m_t(\sigma)/M_t(\sigma)$, $\hat{m}(p\,|\,\sigma) = \hat{m}_t(p_t\,|\,\sigma)/M_t(\sigma)$, and $\hat{\mathcal{F}}(\sigma) = \{p_t/M_t,\, p_t \in \hat{\mathcal{F}}_t\}$.

We then have the following definition:

Definition: A symmetric *Markov monetary equilibrium* (MME) is a collection of time-invariant, individual household choices, $q(\sigma)$, $m'(\sigma)$, $\hat{m}(p\,|\,\sigma)$, $\hat{F}(p\,|\,\sigma)$; spending rules $\hat{M}(p\,|\,\sigma)$ and distributions of posted prices, $F(p\,|\,\sigma)$; probabilities, $Q(\sigma)$; and consumption levels, $C(\sigma)$, conditional on $\sigma \in \mathcal{S}$, such that

1. In all periods such that $\sigma_t = \sigma$, taking as given the distribution of posted prices, $F(p\,|\,\sigma)$, spending rule, $\hat{M}(p\,|\,\sigma)$, and probability $Q(\sigma)$, a representative household chooses $q_t = q(\sigma)$, $m_{t+1} = m'(\sigma)$, $\hat{m}_t(p_t\,|\,\sigma) = \hat{m}(p\,|\,\sigma)$, and distribution $\hat{F}_t(p_t\,|\,\sigma) = \hat{F}(p\,|\,\sigma)$ for all $p \in \mathcal{F}(\sigma)$ to maximize (2.20) subject to (2.5), (2.7), (2.9)—(2.13), (2.16), and (2.19).

2. Individual choices equal per household quantities: $q(\sigma) = Q(\sigma)$; and $\hat{m}(p\,|\,\sigma) = \hat{M}(p\,|\,\sigma)$ and $\hat{F}(p\,|\,\sigma) = F(p\,|\,\sigma)$ for all $p \in \mathcal{F}(\sigma)$.

3. Individual household consumption and money holdings equal their per household counterparts: $c(\sigma) = C(\sigma)$ and $m'(\sigma) = 1$, respectively.

4. Money has value in equilibrium: $\Omega_t > 0$, for all t.

In characterizing an MME for this economy, we begin with the sequence of households' marginal valuations of money, $\{\Omega_t\}_{t=0}^{\infty}$. In this economy, Ω_t is a key variable as it determines the returns to sellers and buyers from transacting at a particular price at a particular point in time. Returning to the household optimization problem and combining (2.21), (2.22), and (2.24), we have

$$\omega_t = \beta \mathrm{E}_t \left[u_{t+1_c}(c_{t+1}) \int_{\mathcal{F}_t} \frac{1}{p_{t+1}} dJ_{t+1}(p_{t+1}) \right] \qquad \forall t. \tag{3.5}$$

In a symmetric equilibrium, substituting (3.1) into (3.5) we have

$$\Omega_t = \beta \mathrm{E}_t \left[u_{t+1_c}(C_{t+1}) \frac{C_{t+1}}{M_{t+1}} \right] \qquad \forall t. \tag{3.6}$$

Making use of (2.5) and rearranging yields

$$\Omega_t M_t = \beta \mathrm{E}_t \left[\frac{1}{\gamma_{t+1}} u_{t+1_c}(C_{t+1}) C_{t+1} \right]. \tag{3.7}$$

Using (2.7) we define $\Omega(\sigma)$, for all $\sigma \in \mathcal{S}$, using the notation introduced in the definition of an MME:

$$\Omega(\sigma) \equiv \Omega_t M_t = \beta \sum_{\sigma' \in \mathcal{S}} \Pi(\sigma', \sigma) \left[\frac{1}{\gamma'} u'_c[C(\sigma')] C(\sigma') \right] \qquad \forall \sigma \in \mathcal{S}, \quad \forall t. \tag{3.8}$$

We thus associate an MME with a collection of N state-contingent values, $\Omega(\sigma_1), \ldots, \Omega(\sigma_N)$, for households' marginal value of fiat money.

Under the assumption that an MME exists, it is possible to establish several characteristics that it must necessarily possess. We will begin in this way and return to the issue of existence later. To this end, suppose that for all $\sigma \in \mathcal{S}$, $\Omega(\sigma)$, $C(\sigma)$, $Q(\sigma)$, $F(p\,|\,\sigma)$, and $J(p\,|\,\sigma)$ are components of an MME as defined above. In addition, let $Q(\sigma) \in (0,1)$ for all σ, and $\gamma > \beta$ for all $\gamma \in \mathcal{G}$.[5] The following proposition, based on Theorem 4 of Burdett and Judd (1983) and Proposition 1 of Head and Kumar (2003) imposes some structure on the form of the conditional distributions of posted prices in an MME:

Proposition 2: *Suppose that $\gamma > \beta$ for all $\gamma \in \mathcal{G}$ and there exists an MME with $Q(\sigma) \in (0,1)$ for all $\sigma \in \mathcal{S}$. Then, given $\Omega(\sigma)$ the conditional distribution of real posted prices, $F(p\,|\,\sigma)$ is unique, dispersed, continuous, and has connected support satisfying: $\mathcal{F}(\sigma) = [\underline{p}(\sigma), \bar{p}(\sigma)]$ where,*

$$\underline{p}(\sigma) > p^*(\sigma) = \frac{\phi}{\Omega(\sigma)} \qquad \text{and} \qquad \bar{p}(\sigma) = \frac{u_c[C(\sigma)}{\Omega(\sigma)}. \tag{3.9}$$

Proposition 2 establishes that there is a unique *candidate* distribution of real posted prices in each state for any MME in which a positive measure of buyers observe a single price, conditional on a representative household's marginal valuation of money, $\Omega(\sigma)$. For this candidate distribution, $p \in \mathcal{F}(\sigma)$ requires that $p \in \operatorname*{argmax}_p r(p)$ where writing (2.15) using real quantities we have

$$r(p) = \hat{M}(p\,|\,\sigma) \left[\Omega(\sigma) - \frac{\phi}{p} \right] \Big[Q(\sigma) + 2\big[1 - Q(\sigma)\big]\big[1 - F(p)\big] \Big]. \tag{3.10}$$

[5] As in Head and Kumar (2003), it is possible to show that there can be no MME in which the probability with which a buyer observes a single price is equal to either 0 or 1 in any state. Similarly, there can be no MME if $\gamma \leq \beta$ in any state. See appendix C.

Combining (3.9) with (3.10) and noting that $F(\bar{p}(\sigma)\,|\,\sigma) = 0$ and $F(\underline{p}(\sigma)\,|\,\sigma) = 1$ for all σ, it is possible to derive the following expressions:

$$\underline{p}(\sigma) = \frac{\phi}{\Omega(\sigma)}\left[1 - \left(1 - \frac{\phi}{u_c[C(\sigma)]}\right)\frac{Q(\sigma)}{2 - Q(\sigma)}\right]^{-1} \tag{3.11}$$

and

$$F(p\,|\,\sigma) = \frac{\left[\Omega(\sigma) - \frac{\phi}{p}\right][2 - Q(\sigma)] - \left[1 - \frac{\phi}{u_c[C(\sigma)]}\right]\Omega(\sigma)Q(\sigma)}{\left[\Omega(\sigma) - \frac{\phi}{p}\right]2[1 - Q(\sigma)]} \tag{3.12}$$

for all $\sigma \in \mathcal{S}$.

From (3.12), it is convenient to derive the following expressions for the conditional densities of posted and transactions prices:

$$f(p\,|\,\sigma) = \frac{\phi}{p^2}\left[\frac{Q(\sigma) + 2[1 - Q(\sigma)][1 - F(p\,|\,\sigma)]}{[\Omega(\sigma) - \phi/p]2[1 - Q(\sigma)]}\right] \tag{3.13}$$

and

$$j(p\,|\,\sigma) = \big[Q(\sigma) + 2[1 - Q(\sigma)][1 - F(p\,|\,\sigma)]\big]f(p\,|\,\sigma). \tag{3.14}$$

Expressions (3.9) and (3.11)—(3.13) describe the conditional distributions of posted and transactions prices in an MME as functions of $Q(\sigma)$ and $C(\sigma)$. Taking $J(p\,|\,\sigma)$ (and implicitly, Q and C) as given, an individual household's consumption depends on the probability with which its *own* buyers observe a single price as indicated by (2.17) and (2.18). Define,

$$C^k[Q(\sigma)] \equiv \int_{\mathcal{F}(\sigma)} \frac{1}{p}\, dJ_k(p\,|\,\sigma) \qquad k = 1, 2, \tag{3.15}$$

where $J_k(p\,|\,\sigma)$ is the analog of (2.18) derived from (3.12) and implicitly depends on $Q(\sigma)$. Using (3.15) and suppressing the argument $Q(\sigma)$, the solution for optimal q, (2.23), may be written

$$q^* = \begin{cases} 0, & \text{if } \mu < \mu_L \equiv u_c\left(C^2\right)\left[C^2 - C^1\right]; \\ 1, & \text{if } \mu > \mu_H \equiv u_c\left(C^1\right)\left[C^2 - C^1\right]; \\ \frac{1}{C^1 - C^2}\left[{u_c}^{-1}\left(\frac{\mu}{C^2 - C^1}\right) - C^2\right], & \text{if } \mu_L \le \mu \le \mu_H. \end{cases} \tag{3.16}$$

In (3.16), μ_L may be thought of as a lower bound on the search or information cost such that if the cost is *below* μ_L, then the household will choose to have *all* of its buyers observe more than one price ($q^* = 0$). Similarly, μ_H is an upper bound on the search or information cost. If $\mu > \mu_H$, the household will choose to have *no* buyer observe a second quote ($q^* = 1$). Note that the critical levels, μ_L and μ_H (and so q^* itself), depend on C^1 and C^2 and are functions of Q. The following proposition establishes the existence of fixed points of (3.16):

Proposition 3: *For a fixed σ and for any $Q \in (0,1)$, there exists a μ such that $q^*(Q) = Q$.*

An MME requires that $q^*[Q(\sigma)] = Q(\sigma)$ for all σ. Proposition 3, however, establishes the existence of a fixed point for a particular σ, and in general there may not exist a single μ such that there is a fixed point of (3.16) for all $\sigma \in \mathcal{S}$. Because we have specified search costs independently of the state, existence of an MME in our model requires restrictions on $\mathcal{S}$ so that for a given μ there is a fixed point of (3.16) in all N states. We do not derive explicit restrictions that will suffice. Rather, in the computational/quantitative examples that we consider below, we find that as a practical matter, if the variation across the three stochastic parameters is not too great, $Q(\sigma)$ does indeed exist for all $\sigma \in \mathcal{S}$.

Given the difficulty in specifying conditions under which fixed points of (3.16) exists for all σ for a single μ, we do not approach the existence of an MME formally.[6] Rather, we construct examples of MME's for several parameterizations of our economy and use them to consider the responses of both nominal and real prices to shifts in preferences (α), costs (ϕ), and the rate of money creation (γ). We describe the process by which we compute equilibria in general terms here. For a detailed description of our computational algorithm, see appendix B.

We begin by choosing specific values of the economy's parameters[7]. This requires us to set the discount factor, β, the search cost, μ, and sets of values for the preference parameter, $\mathcal{A}$; cost parameter, $\mathcal{P}$; money creation rate, $\mathcal{G}$, as well as the transition probabilities, Π_{ji} for $\sigma_i, \ \sigma_j \in \mathcal{S}$.

Having fixed parameters, we then choose initial values for consumption, $C_0(\sigma)$, and the probability of a buyer observing a single price, $Q_0(\sigma)$, for each $\sigma \in \mathcal{S}$. Using these values, we construct $\Omega_0(\sigma)$, and the distributions of posted and transactions prices using (3.9) and (3.11)—(3.14). We label these distributions $F_0(p \,|\, \sigma)$ and $J_0(p \,|\, \sigma)$, respectively. Using these, we construct $C_0^1[Q_0(\sigma)]$ and $C_0^2[Q_0(\sigma)]$. Next we compute fixed points of (3.16) for each σ and call these $Q_1(\sigma)$. Finally, using $Q_1(\sigma)$ and $F_0(p \,|\, \sigma)$ we construct $J_1(p \,|\, \sigma)$ and $C_1(\sigma)$. This procedure is repeated (T times) until for all σ, $C_T(\sigma) - C_{T-1}(\sigma)$ and $Q_T(\sigma) - Q_{T-1}(\sigma)$ are sufficiently small.

Overall, we find that this algorithm works well in that for a wide range of parameter values it successfully computes an MME very quickly. While we do not formally rule out non-uniqueness, experimentation with different starting values in no case produced multiple equilibria for a fixed set of parameters.

[6] For a version of this economy with no aggregate uncertainty (effectively a single σ), Head and Kumar (2003) formally establish existence of an equilibrium of the type considered here.

[7] We report the parameter values chosen in our baseline calibration in the section 4.

4. Price Responses to Shocks in Equilibrium

We study random fluctuations in costs, preferences, and money creation, each in isolation, in numerically computed MME's. We focus on the effects of these fluctuations on both the level and dispersion of prices; and on the magnitude and persistence of fluctuations in inflation.

We describe the *level* of prices by the average transaction price. For real prices, we write:

$$p_t^{ave} = \int_{\mathcal{F}(\sigma_t)} p \, dJ(p \,|\, \sigma_t). \tag{4.1}$$

The average nominal price is given by

$$P_t^{ave} = M_t p_t^{ave} = \int_{\mathcal{F}_t} p_t \, dJ_t(p_t). \tag{4.2}$$

We consider the *dispersion* of the distributions of either real or nominal prices with respect to the range of their supports, by which we mean the ratio of the upper support to the lower support of the distributions of real prices, $\bar{p}(\sigma_t)/\underline{p}(\sigma_t)$. We define the inflation rate as the net growth rate of the nominal price level:

$$I_t = \frac{P_t^{ave} - P_{t-1}^{ave}}{P_{t-1}^{ave}} \times 100. \tag{4.3}$$

In our environment, marginal production cost is expressed in units of utility. It is useful in the analysis of cost fluctuations to express costs either in units of goods (real marginal cost) or currency (nominal marginal cost). We define *nominal marginal cost* at time t:

$$MC_t = \frac{\phi_t}{\Omega_t}. \tag{4.4}$$

Real marginal cost is given by

$$mc_t = \frac{MC_t}{M_t}. \tag{4.5}$$

Since all producers face the same costs, the distribution of transactions prices induces distributions of nominal and real markups. The average nominal and real markups are defined by the ratios P_t^{ave}/MC_t and p_t^{ave}/mc_t, respectively.

4.1: Benchmark Parameterization

We characterize equilibria numerically, and for this reason we begin with a benchmark parameterization of the economy. We set the discount factor, β, equal to .96, as this is a value commonly used in dynamic general equilibrium models calibrated to annual observations. We choose *mean* values for the stochastic parameters ϕ, α, and γ in part to calibrate certain measures for our economy in a *deterministic* stationary monetary equilibrium. That is, in a equilibrium in which all of these parameters (and thus the state vector, σ) are constant.

We set $\alpha = 1.5$ on average, a value consistent with the requirement that $\lim_{C\to 0} u'(C)C = \infty$, and within the range typically examined in calibrated macroeconomic models. We set the mean rate of money creation, γ, in conjunction with the search cost parameter, μ, so that inflation is at its optimal rate for the deterministic economy, by which we mean the constant inflation rate that maximizes a representative household's utility given that there are no shocks to costs or preferences.[8] With $\mu = .008$, the optimal inflation rate in the deterministic economy is roughly 4% *per annum* (*i.e.* $\gamma = 1.04$ *on average*). Our chosen combination of μ and γ implies an average real markup of 1.10, a number within the (fairly wide) range of markups estimated by several studies of U.S. manufacturing (Morrison (1990), Basu and Fernald (1997), Chirinko and Fazzari (1994)). Finally, given the values of the other parameters of the economy, $\mu = .008$ lies well in the interior of the range $[\mu_L, \mu_H]$ calculated as in (3.16). Thus constant search costs at this level are consistent with existence of an MME for substantial fluctuations in costs, preferences, and money creation. Finally, we set the average level of the production disutility parameter, $\phi = .1$, more or less arbitrarily. Given values for the other parameters, ϕ controls only the level of output in a stationary equilibrium.

We specify Markov chains for the stochastic parameters so that in each case the percentage standard deviation and autocorrelation of aggregate output in an MME with fluctuations induced by random variation in that parameter alone are equal to 2.56 and .70 respectively, values equal to their counterparts in annual U.S. GDP, detrended with the Hodrick-Prescott filter for the period 1959-2001. Many Markov chains fit this criterion; we choose the following symmetric processes simply for illustrative purposes. For all t,

$$\phi_t \in \mathcal{P} = \{.094, .1, .106\}, \quad \alpha_t \in \mathcal{A} = \{1.48, 1.5, 1.52\}, \quad \text{or} \quad \gamma_t \in \mathcal{G} = \{1.013, 1.04, 1.067\}, \tag{4.6}$$

with

$$\pi^\phi = \pi^\alpha = \pi^\gamma = \begin{bmatrix} .8 & .1 & .1 \\ .1 & .8 & .1 \\ .1 & .1 & .8 \end{bmatrix}. \tag{4.7}$$

4.2: The pass-through of cost shocks to prices

We first consider the effects of random fluctuations in costs. For all t, $\phi_t \in \mathcal{P}$ as specified in (4.6) with $\Pi = \pi^\phi$ given by (4.7). To begin with we fix the rate of money creation at $\gamma = 1.04$ and

[8] In our economy, as in that of Head and Kumar (2003), inflation induces search and erodes market power. To a point, this effect dominates the inflation tax, resulting in an optimal inflation rate exceeding the Friedman rule ($\gamma = \beta = .96$).

throughout this sub-section we fix $\alpha = 1.5$. Thus we focus on the MME of a three-state economy. While the state is determined by the realization of ϕ_t alone, for notational purposes we continue to use σ_t to indicate the current state.

Figure 1 depicts depicts the densities of real transactions prices in each of the three states. Changes in ϕ_t affect $\Omega(\sigma_t)$, so that increases in ϕ from .094 to .1 and from .1 to .106 increase real costs by 5.25% and 4.81% respectively, using (4.5). In Figure 1 it is clear that as real costs increase, the densities of transactions prices shift from left to right; higher costs are associated with higher real transactions prices. The average real transaction price, $p^{ave}(\sigma)$, increases from .235 to .242 and to .250, increases of 2.98% and 3.31% as costs rise. From (3.1) it is clear that an increase in p^{ave} is always associated a reduction in equilibrium output and consumption.

Comparison of the percentage increase in the average real price to that of real costs suggests a sense in which cost "pass-through" is incomplete in our economy. When the disutility of production increases, *ceteris paribus* the return to households from posting a given price falls, and they instruct their sellers to draw from a new distribution of generally higher prices. The price distribution does not, however, simply shift rightward. In the cases depicted in Figure 1, $\bar{p}(\sigma)/\underline{p}(\sigma)$ (denoted "pu/pl" in the figure) increases (slightly) with costs. The intuition for this is important for understanding the responses of prices to shocks in our economy. Facing an increase in ϕ, households direct those sellers pricing near the top of the distribution to pass a relatively large share of the cost increase through to buyers. These sellers sell predominately to buyers who have no alternative—they observe only one price quote. A given increase in such a seller's price thus causes a relatively small loss of sales to competitors. Those sellers pricing in the lower range of the support of the price distribution, in contrast, are directed to raise their prices by a smaller amount. A much larger share of these sellers' sales are to buyers who have an alternative—they observe two price quotes. The household limits these sellers' price increases to avoid a large loss of sales to competitors.

Overall, the household's reaction to a cost increase extends the upper tail of the distribution of posted prices, and moves the mean of the distribution closer to its lower support. For a fixed fraction of buyers observing a single price, these changes in the distribution increase the return to search. As a result, households choose to increase the share of their buyers observing a second price. For the cases depicted in Figure 1, Q falls from .624 to .559 and to .500 as real costs increase. This reduces markups overall and limits the increase in price dispersion associated with differential pass-through across sellers.. In this example, the coefficient of variation of transaction prices actually falls while $\bar{p}(\sigma)/\underline{p}(\sigma)$ rises (although both movements are quantitatively small relative to the increase in $p^{ave}(\sigma)$).

We now consider the relationship between the degree of pass-through and the rate of money creation, which in this experiment with constant money growth is equal to the trend rate of inflation. Figures 2.1 and 2.2 depict densities of real transaction prices for the three cost states for $\gamma = 1.02$ (low inflation) and $\gamma = 1.06$ (high inflation), respectively. It is clear in these figures that the average transaction price changes by less in response to a given change in ϕ when the average rate of money creation is low than when it is high. Moreover, the degree of pass-through of *real* cost changes to real prices is increasing in γ. In the low inflation case, as ϕ rises from .094 to .1 and .106, real prices rise by 1.59% and 1.56% in response to cost changes of 6.10% and 5.24%, respectively. This contrasts with the high inflation case in which these changes in ϕ generate price changes of 3.90% and 3.75% in response to cost changes of 5.12% and 4.42%, respectively.

These differences in the degree to which changes in real costs are passed through to average real prices hinge on the effects of the average rate of money creation on price dispersion and the household search decision. With $\gamma = 1.02$, the fractions of buyers observing a single price in the three cost states are $Q = .827$, .787, and .740 as costs move from low to high. In contrast, with $\gamma = 1.06$ the corresponding fractions are $Q = .457$, .403, and .357, respectively. Higher trend inflation is associated with a higher fraction of buyers observing two prices and thus with lower market power overall.[9] As the rate of money creation rises, households instruct sellers to increase prices differentially to compensate for the eroded future value of currency (*i.e.* a reduction in $\Omega(\sigma)$ for all σ). As with cost increases, those sellers pricing at the upper end of the support of the price distribution are instructed to raise their prices the most, as they expect to lose the smallest share of their sales to competitors. Also as with cost increases, for a fixed Q, the resulting increase in price dispersion induces the household to increase the share of its buyers observing two price quotes. In equilibrium this lowers Q reducing sellers' market power. The mean of the distribution of real posted prices shifts toward its lower support and thus becomes more sensitive to cost changes. This accounts for an increase in the extent to which real cost movements are passed through to the average real transaction price as the rate of trend inflation rises.

We now consider the pass-through of *nominal* cost fluctuations to nominal price changes, at

[9] This effect is also responsible for the *increase* in consumption associated with increases in trend inflation from 2% to 4% and 6%. Note, however, that while consumption does increase with inflation over this range, search costs (which are proportional to the measure of buyers observing two prices) do as well. Thus, an increase in consumption may not be associated with and increase in welfare. Welfare is of course higher at trend inflation of 4% than when it is either 2% or 6%. Also, as inflation increases further, consumption will not continue to increase, but rather will begin to fall.

different levels of trend inflation. In our stochastic economy, the inflation rate as defined in (4.3) is endogenous because it is determined not only by exogenous money creation (trend inflation at rate $(\gamma - 1) \times 100$), but also by the equilibrium response of nominal prices to random fluctuations in costs. We define the pass-through of nominal cost changes to the nominal price level by the response of P_t^{ave} to fluctuations in nominal costs as given by (4.4).

To estimate the degree of pass-through by this measure in our economy we first generate a realization of the economy's equilibrium stochastic process 10,000 periods in length. We then adjust nominal price changes by subtracting the net rate of trend inflation,

$$\tilde{I}_t = I_t - 100(\gamma - 1), \tag{4.8}$$

to obtain a time series of inflation generated only by movements in costs. We compute a corresponding series of nominal cost changes as follows:

$$\widetilde{MC}_t = \left[\frac{MC_t - MC_{t-1}}{MC_{t-1}}\right] 100 = \left[\frac{\gamma mc_t}{mc_{t-1}} - 1\right] 100, \tag{4.9}$$

where MC_t and mc_t are given by (4.4) and (4.5) respectively. We then estimate (by OLS) the degree of pass-through of nominal cost movements to nominal prices using the following regression equation:

$$\tilde{I}_t = \delta \widetilde{MC}_t + \epsilon_t, \tag{4.10}$$

where ϵ_t is assumed to be an iid, mean zero, normal error.[10]

Figure 3 depicts estimated pass-through coefficients, $\hat{\delta}$, and the average level of Q in equilibrium for levels of trend inflation ranging from two percent to twelve percent. Over this range our estimates of pass-through increase monotonically but at a decreasing rate from .27 to .95 as the rate of trend inflation increases.[11]

This relationship is in accordance with the empirical findings of Devereux and Yetman (2002) who found pass-through of nominal exchange rate movements to consumer prices to be increasing in average inflation at a decreasing rate in a panel of 107 countries over the post-Bretton Woods period. It is also in accordance with the arguments of Taylor (2000) who cites evidence that the

[10] We consider (4.10) to be a reasonable correspondent to the regression equation estimated by Devereux and Yetman (2002) in their analysis of the pass-through of nominal exchange rate movements to the nominal price level.

[11] These estimates are very precise. All standard errors lie between .0001 and .0004.

responsiveness of prices to cost increases has declined with average inflation for several developed countries during the late 1980's and 1990's.

Intuition for the increase in nominal cost pass-through as the trend inflation rate rises may again be traced to the average share of buyers observing a single price. This fraction decreases at a decreasing rate as the rate of trend inflation increases. For a fixed Q, an increase in trend inflation increases price dispersion and raises the return to increasing the share of buyers observing a single price quote. As a result, an increase in γ lowers Q in all states and reduces sellers' market power. With lower Q, the average nominal price shifts toward the lower support of the distribution of transaction prices and is more sensitive to changes in nominal costs.

Finally, we consider the dynamics of inflation induced by cost fluctuations in our economy. Figure 4 illustrates that the variance of inflation is increasing in the average rate of money creation. as the response of nominal prices to changes in nominal costs increases, the variance of inflation rises relative to that of nominal costs. Inflation induced by cost changes is not, however, persistent in our economy. The household adjusts to the change in costs within the period in which the cost shock is realized and inflation either jumps above or falls below its trend level in that period only.

4.3: Prices and inflation dynamics with monetary shocks

We now consider the effect of shocks to the money creation rate, holding fixed the disutility of production at $\phi = .1$ and the curvature parameter in preferences at $\alpha = 1.5$. Again we consider a three-state economy with σ determined by γ alone. To begin with we let $\gamma \in \mathcal{G}$ as specified in (4.6) with $\Pi = \pi^{\gamma}$ given by (4.7). In other experiments we allow the average rate of trend inflation to differ from the optimum rate of 4% .

Figure 5 depicts the densities of real transactions prices in the three money growth states, each identified by the value of γ in that state. In the figure, it can be see that higher money creation is associated with *lower* average transactions prices (and thus, higher consumption) for these three states.[12] A higher rate of money creation is also associated with greater price dispersion in the sense of a higher $\bar{p}/\underline{p}$ and a lower fraction of buyers observing a single price.

The overall response of the economy to monetary shocks in equilibrium is a combination of two effects. Consider an increase in the money growth rate, say from 4% to 6.7%. Because the increase in money growth is persistent, it raises expected future inflation. This reduces $\Omega(\sigma)$ and increases price dispersion for a fixed fraction of buyers observing a single price. As with cost increases, high

[12] This would not necessarily be true if money creation in the high state were higher than 6.7%. Beyond some point, higher expected inflation raises the average transaction price and reduces consumption.

price sellers raise their price by more than low price sellers because in doing so they lose only a relatively small proportion of their sales to competitors. Increased price dispersion, however, causes households to raise the fraction of buyers observing a second price. As in the case of differential pass-through of cost shocks, this shifts the mass of the distribution of transactions prices towards its lower support and limits the increase in real prices associated with a depreciation of money. In the cases depicted, the reduction in market power dominates, and real prices fall in response to a persistent increase in the money growth rate.

At higher levels of average inflation, the response of prices to a reduction in $\Omega(\sigma)$ dominates. For example, consider a case in which the average rate of money creation is 15%, and the low and high money growth states represent the same percentage increase and decrease as in the case depicted in Figure 5. In this case the average real transaction price rises from .245 to .248 and .251 as the money creation rate moves from low (12%) to 15% to high (18%). In this case a higher money growth rate still reduces market power, as Q falls from .180 to .152 to .134 as γ rises. These reductions are just not sufficient to overcome the inflation tax.

We describe the effect of monetary shocks on nominal prices by considering the dynamics of the inflation rate as defined by (4.3). Figure 6 plots ratios of the standard deviation and first-order autocorrelation of inflation to the corresponding statistic for money creation as the average rate of money creation rises from 4% to 20%. In each case, the low and high money growth states represent the same percentage decrease and increase. Statistics are computed from simulations of the equilibrium stochastic process for 10,000 periods. It can be seen that the model can account for an increase in the variance of inflation as the average rate of inflation (which in our economy is always equal to the average rate of money creation) rises. At relatively low levels of inflation, the economy also generates inflation persistence.

Here persistent inflation is associated with the incomplete response of the price level to changes in the money supply in periods when the money growth rate shifts. At low levels of average inflation, inflation is less volatile than the money growth rate, and more persistent.

4.4: Preference shocks

We now consider the effects of random shifts in preferences, holding costs and the rate of money creation constant. In this case the state σ is determined by $\alpha \in \mathcal{A}$ as specified in (4.6) with $\Pi = \pi^\alpha$ again given by (4.7). We initially fix the rate of money creation at $\gamma = 1.04$; and throughout we fix the cost parameter at $\phi = .1$. Figure 7 depicts densities of transactions prices for the three states. In the figure it can be seen that an increase in α shifts the distribution of transactions prices to the right, increasing the average transaction price. An increase in α also *reduces* $\bar{p}(\sigma)/\underline{p}(\sigma)$, and *increases* the share of buyers observing a single price.

In our environment, preference shocks are shifts in households' intertemporal elasticity of substitution. An increase in α lowers the elasticity and in the presence of inflation lowers $\Omega(\sigma)$, raising prices overall. Unlike an increase in the cost parameter ϕ which also lowers Ω, a preference shift does not directly raise the the marginal cost price. Rather, from (3.11) it can be seen that an increase in α raises $\underline{p}$ relative to both p^* increasing market power overall and compressing the distribution of posted prices. This in turn reduces households' return to having a given measure of buyers observe a second price. For the cases depicted in Figure 7, Q increases from .501 to .561 and .619 as α rises from 1.48 to 1.5 and 1.52, respectively.

The effects of preference shocks contrast significantly with those of cost shocks, even though increases in α do induce increases in real costs because of their effect on $\Omega(\sigma)$. For the case depicted in Figures 7, the real disutility of production increases by 1.12% and .97% as α increases from 1.48 to 1.5 and further to 1.52. This increase in costs is *more* than passed through to real prices, which increase by 2.66% and 2.85% as α rises. Increases in costs due to direct cost shocks lower market power, generating incomplete pass-through, whereas increases in costs due to preference shifts increase market power, resulting in large increases in real prices.

The effects of preference shocks on nominal prices also contrast sharply with those of cost shocks. Figure 8 (an analog of Figure 3) depicts pass-through coefficients and average Q for rates of trend inflation between 3% and 12%, estimated again using (4.10). In this case it can be seen that pass-through is always above one, but *declines* as inflation increases. The effect of trend inflation on average Q tends to eliminate the response of Q to increases in α, resulting in a drop in pass-through. At high rates of inflation, pass-through of cost changes due to either direct cost shocks or preference shocks to the nominal price level is effectively one-for-one. At low inflation, however, the two different types of cost movements are passed-through to drastically different degrees. Inflation dynamics in response to preference shocks also differ markedly from those associated with cost movements. As trend inflation increases, the variance of inflation induced by preference shocks falls.

5. Endogenous Money

In this section, we consider versions of our economy in which the monetary authority chooses the rate of money creation in response to cost and preference shocks.

—TO BE COMPLETED

6. Conclusions

This paper has considered a stochastic general equilibrium monetary economy in which both nominal and real prices respond incompletely to stochastic fluctuations in costs, preferences, and money creation, in spite of the fact that there are no exogenously imposed constraints on sellers' ability to adjust prices. As the rate of average inflation increases, the responsiveness of prices to cost shocks increases. The findings are therefore consistent with the observation that prices and inflation have become less responsive to cost movements as the average inflation rate has fallen, and with the observation that the extent to which cost shocks in the form or nominal exchange rate movements are passed through to consumer prices is declining in the average rate of inflation across countries. The model is also consistent with the observation that high average inflation is associated with a high inflation variance.

This version of the paper is preliminary and incomplete. Further work will consider endogenous money in response to cost and preference shocks. Further work will also focus more closely on the dynamics of inflation. In our model, sellers' pricing response to shocks generates a positive relationship between the variance and mean of inflation. Only in the case of monetary shocks, however, is endogenous inflation persistent.

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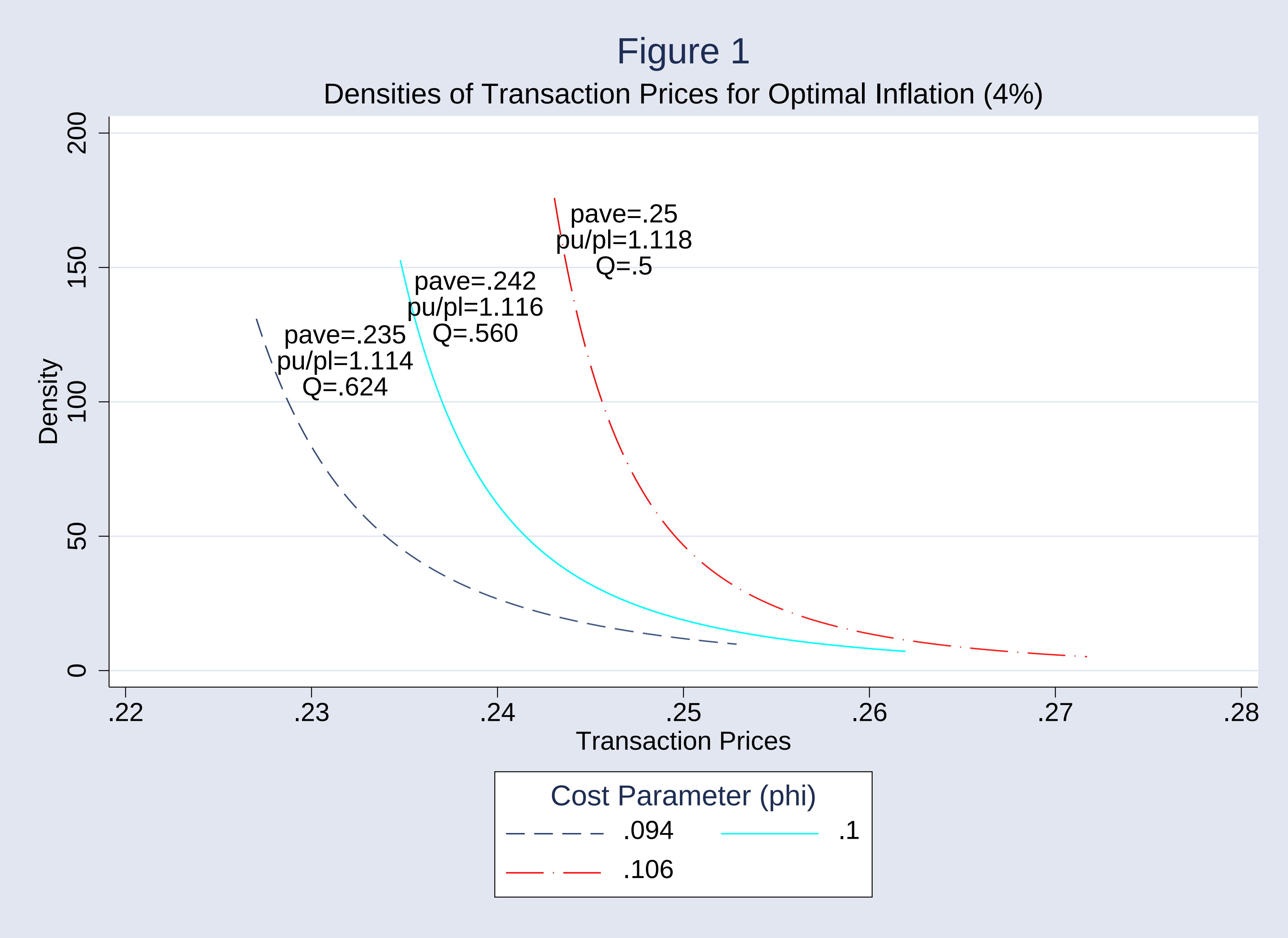


Figure 1
Densities of Transaction Prices for Optimal Inflation (4%)
Density
0
50
100
150
200
.22
.23
.24
.25
.26
.27
.28
Transaction Prices
pave=.235
pu/pl=1.114
Q=.624
pave=.242
pu/pl=1.116
Q=.560
pave=.25
pu/pl=1.118
Q=.5
Cost Parameter (phi)
.094
.1
.106

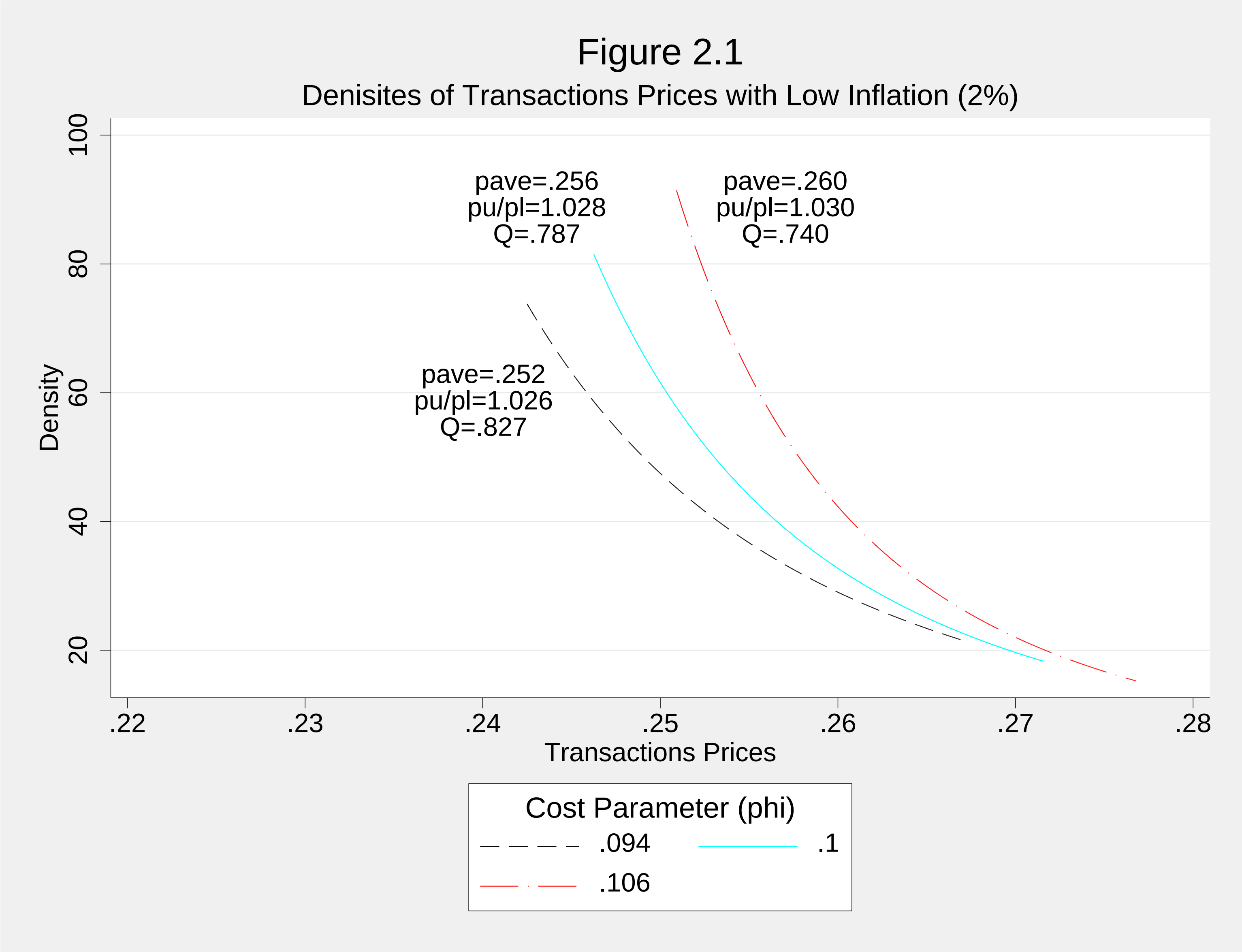


Figure 2.1
Denisites of Transactions Prices with Low Inflation (2%)
pave=.256
pu/pl=1.028
Q=.787
pave=.260
pu/pl=1.030
Q=.740
pave=.252
pu/pl=1.026
Q=.827
Density
100
80
60
40
20
.22
.23
.24
.25
.26
.27
.28
Transactions Prices
Cost Parameter (phi)
.094
.1
.106

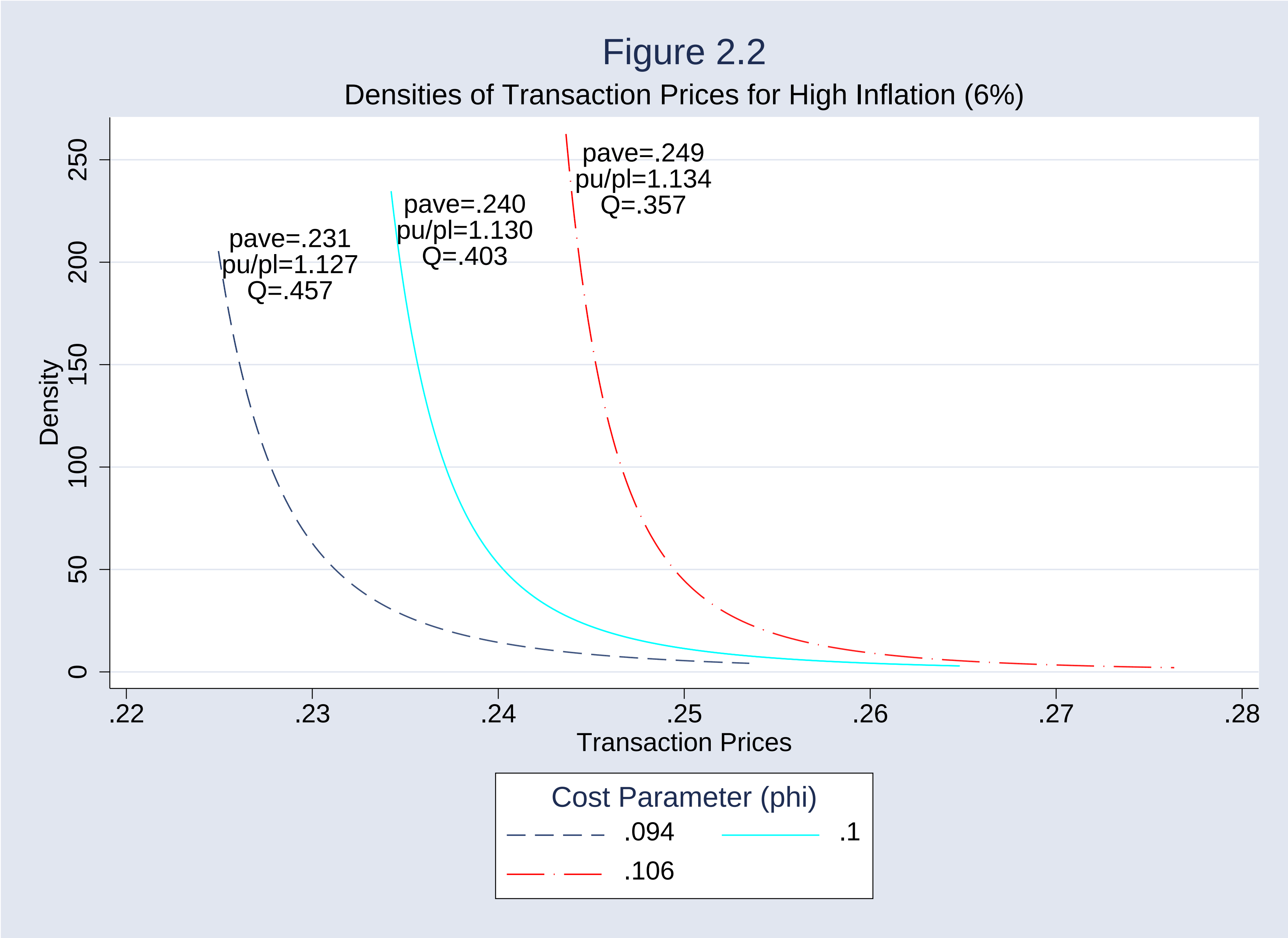


Figure 2.2
Densities of Transaction Prices for High Inflation (6%)
Density
0
50
100
150
200
250
.22
.23
.24
.25
.26
.27
.28
Transaction Prices
pave=.231
pu/pl=1.127
Q=.457
pave=.240
pu/pl=1.130
Q=.403
pave=.249
pu/pl=1.134
Q=.357
Cost Parameter (phi)
.094
.1
.106

Figure 3

Pass−Through Coefficient and Average Q

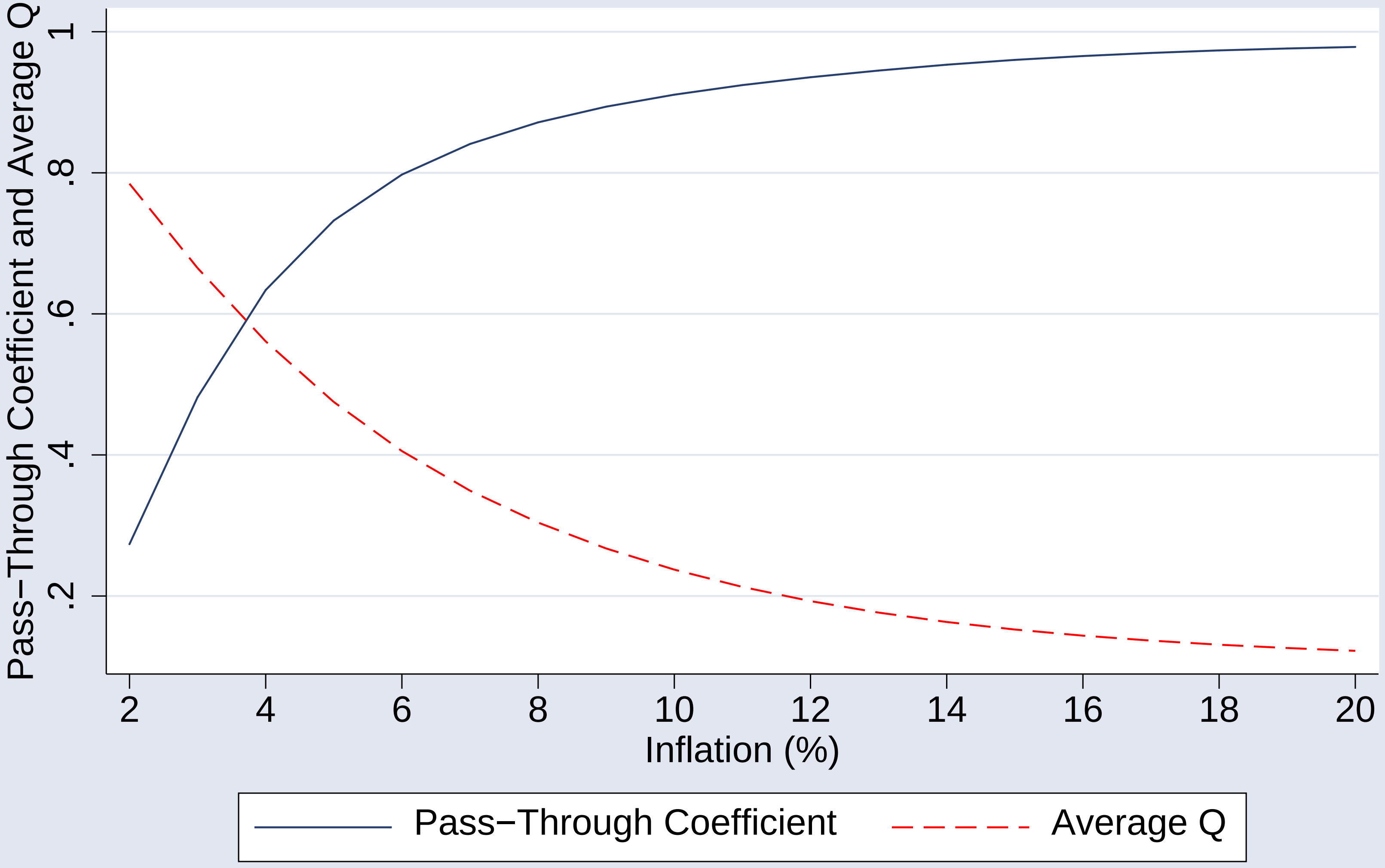


Figure 4

The Variance and Mean of Inflation with Cost Shocks

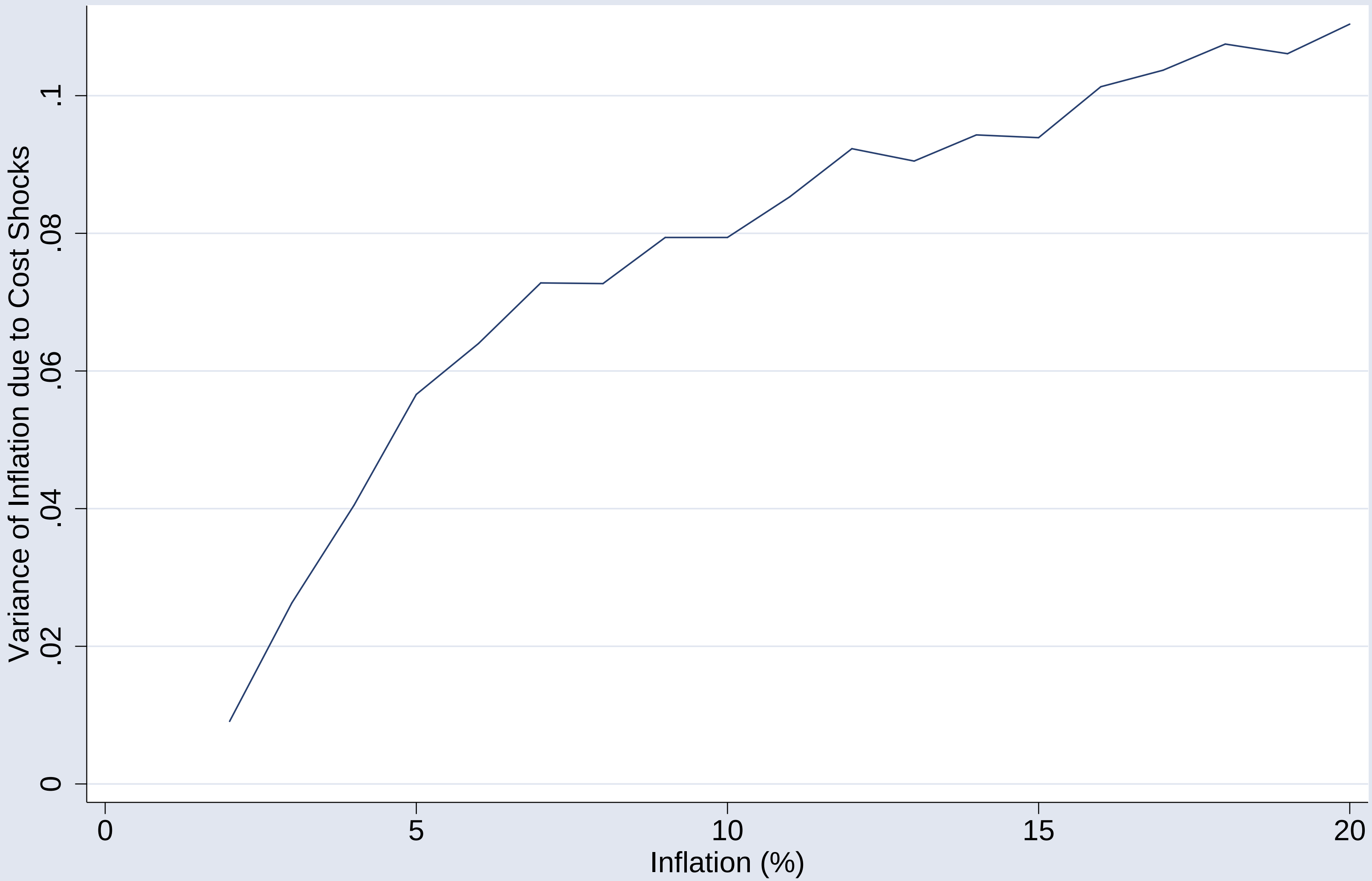
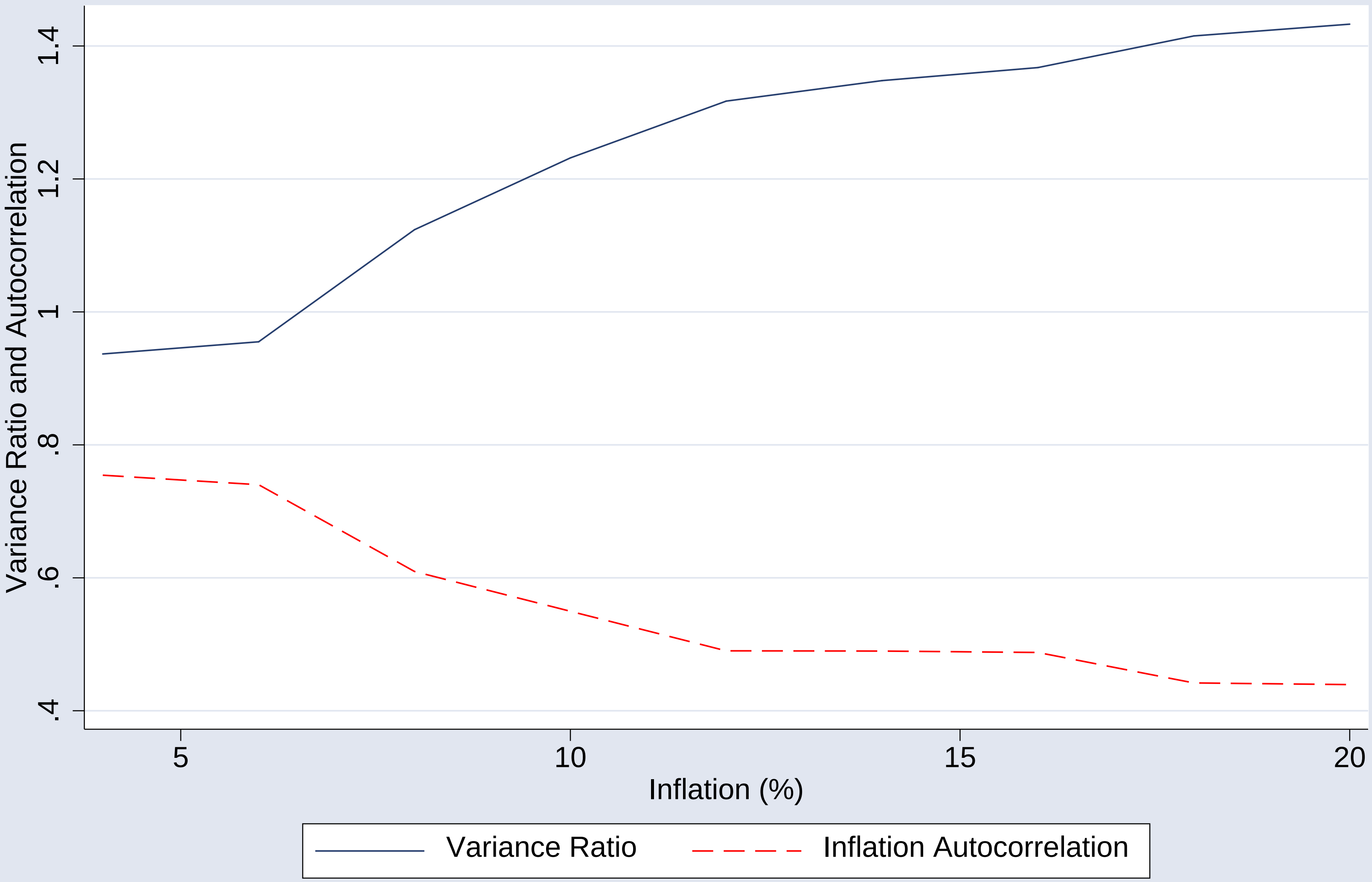


Figure 5

Densities of Transactions prices: Money shocks

Figure 6

Relative Inflation Variance and Inflation Autocorrelation



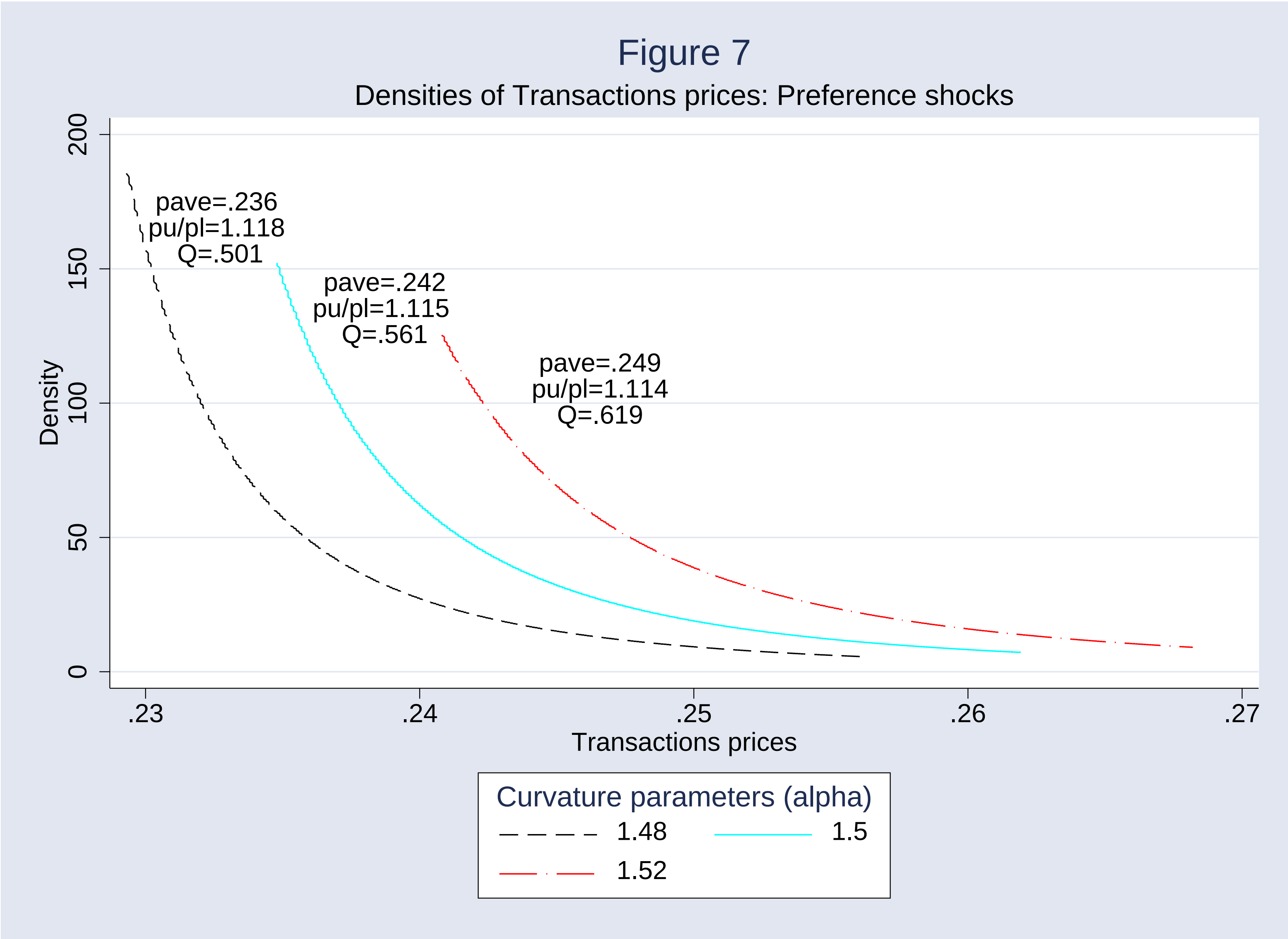


Figure 7
Densities of Transactions prices: Preference shocks
Density
200
150
100
50
0
.23
.24
.25
.26
.27
Transactions prices
pave=.236
pu/pl=1.118
Q=.501
pave=.242
pu/pl=1.115
Q=.561
pave=.249
pu/pl=1.114
Q=.619
Curvature parameters (alpha)
1.48
1.5
1.52


Figure 8
Pass−Through Coeffieicnt and Average Q
Pass−Through Coefficient and Average Q
0
1
2
3
4
2
4
6
8
10
12
Inflation (%)
Pass−Through Coefficient
Average Q