Price Dispersion, Inflation, and Welfare

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Abstract

Several empirical studies have documented a positive relationship between the rate of inflation and the dispersion of consumer prices. We examine this relationship and the welfare costs of inflation in a monetary economy in which *ex ante* identical buyers search among prices posted by identical sellers. Under certain conditions, stationary monetary equilibria of our economy necessarily exhibit dispersion of real prices. If the degree of buyers' incomplete information about posted prices is fixed exogenously, both price dispersion and the average real price are increasing in the inflation rate. Money creation lowers welfare by increasing the market power of sellers and exacerbating the effects of the inflation tax. As the rate of inflation approaches the Friedman rule, price dispersion and welfare costs both vanish. If households choose the number of prices to observe, then the Friedman rule is not optimal. Rather, up to some point increased inflation lowers the average real price and raises welfare by inducing search and eroding sellers' market power.

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1. Introduction

Several empirical studies have documented a positive relationship between the rate of inflation and the dispersion of consumer prices. For example, using disaggregated data Lach and Tsiddon (1992) find that expected inflation increased the dispersion of the prices of foodstuffs in Israel over the period 1978-84. Similarly, Reinsdorf (1994) finds that price dispersion for 65 categories of foodstuffs in nine U.S. cities over the period 1980-82 is increasing in expected inflation but decreasing in unexpected inflation. Van Hoomissen (1988) and Tommasi (1993) also document positive relationships between inflation and price dispersion in Israel and Argentina, respectively.

In this paper, we study the effects of inflation on price dispersion and welfare using a model of monetary exchange in which dispersion arises from buyers' incomplete information regarding the prices offered by different sellers. In our model higher inflation always increases price dispersion but may or may not reduce welfare depending on whether sellers' overall market power is increased by the lowering of the value of fiat money or reduced by increased search.

Our model combines the price posting environment of Burdett and Judd (1983) with a random matching monetary model similar to those studied by Shi (1999) and Head and Shi (2002). The economy is populated by households comprised of measures of identical *sellers* who post fiat money prices at which they are willing to produce and sell consumption goods and *ex ante* identical *buyers* who hold fiat money, observe random numbers of price quotes posted by sellers, and may choose to buy at the lowest price they observe. As in the model of Burdett and Judd, if in equilibrium some buyers observe only a single price while others observe more than one, then the distribution of prices in equilibrium is necessarily non-degenerate and continuous.

With a fixed share of buyers observing a single price (*i.e.* with a fixed degree of incomplete information), in our model both price dispersion and the average real price increase with the inflation rate in a stationary monetary equilibrium[1]. Inflation lowers the expected future value of fiat money and increases overall market power, which differs across sellers in equilibrium. An increase in the rate of inflation raises the average real price and lowers household consumption by more than it would if information were sufficient to force prices to be concentrated at the level of marginal cost. Thus, price dispersion raises the welfare cost of inflation. Still, as the inflation rate approaches the Friedman rule, both price dispersion and welfare losses from inflation vanish.

If, however, households choose the number of prices to observe, then the Friedman rule is not optimal. By increasing price dispersion, inflation raises the gains to search, reduces the share of

[1] Throughout the paper, *real* prices will be *nominal* prices normalized by the money stock.

buyers observing a single price and weakens market power; an effect similar to that studied by Benabou (1988, 1992). If inflation is low, an increase in the rate of money creation may *raise* welfare if the reduction of market power due to increased search overcomes the increase in market power associated with the lowering of the value of fiat money. As the inflation rate rises, however, the former effect weakens and the latter eventually dominates. Thus, there is both an optimal inflation rate and a region within which small changes in inflation have little effect on the average real price and welfare.

A number of papers have studied relationships among inflation, price dispersion and welfare; and of these several consider search environments. Our paper, however, differs from these in several ways. To begin with, in this literature fiat money typically has value only by assumption, whereas in our model its value arises endogenously as it facilitates trade in the presence of search frictions. Also, many of these papers (*e.g.* Sheshinski and Weiss (1977), Caplin and Spulber (1987), Benabou (1988, 1992), Diamond (1993), and Tommasi (1994)) rely on exogenously imposed nominal rigidities and/or differences in search or production costs across agents to generate price dispersion in equilibrium. In contrast, in our model prices may be adjusted every period without cost and both buyers and sellers may be entirely symmetric, *ex ante*. These papers also typically focus on the pricing behavior of firms with consumer demand taken as given. For example, Tommasi (1994) models inflation as a shock to firms' cost and considers the degree to which, given a degree of market power, such cost movements are "passed-through" to consumers. In contrast, we study a general equilibrium economy with dynamic optimization by households and consider the effects of inflation on buyers' search and purchase decisions as well as on the pricing strategies of sellers.

As such, our paper is more closely related to those of Casella and Feinstein (1990), Tommasi (1999), and Fershtman, Fishman, and Simhon (2003). These papers consider overlapping generations models in which households have *less* incentive to search when inflation erodes the purchasing power of money at a sufficient rate. Price dispersion is not their primary focus, and to the extent that it exists in equilibrium it arises from *ex ante* heterogeneity among buyers (Casella and Feinstein (1990)) or sellers (Tommasi (1999) and Fershtman, Fishman, and Simhon (2003)). In contrast, our paper focuses on the link between price dispersion *per se* and the pricing and search strategies of homogeneous households. For sellers, price dispersion indicates differential market power and affects pricing in response to changes in the rate of inflation or the information of an average buyer. For buyers, price dispersion *increases* the incentive to search, an effect that is strongest at *low* levels of expected inflation. Thus, our paper focuses mainly on situations with low or moderate inflation rates, rather than on hyperinflations, which are the main concern of these other studies.

Our paper also contributes to the recent literature on price dispersion in search-theoretic monetary models. Soller-Curtis and Wright (2000) construct a model of price posting and exchange among *ex ante* identical agents in which consumers receive preference shocks and are thus heterogeneous *ex post*. In their model, if a stationary monetary equilibrium exists, then there is an equilibrium with price dispersion in which exchange takes place *only* at two discrete prices. In addition, for some parameter values their model also has a single price equilibrium. In contrast, our model typically has monetary equilibria *only* in which the distribution of fiat money prices is non-degenerate and continuous.

Peterson and Shi (2002) consider the effects of inflation on price dispersion and welfare in a model similar in some ways to ours, although they rely on heterogenous preferences to generate price dispersion and consider an environment in which prices are determined by bargaining. In their model the inflation rate affects the relative prices of goods which differ with regard to their desirability for a particular consumer. By increasing the relative price of more desirable goods, inflation induces a shift in consumption toward those that are less desirable, lowering welfare. We view their results as complementary to ours, as they arise in a setting in which prices are determined differently. In this paper we focus on posted price selling and and consider only price dispersion among identical households to isolate the effects of inflation on the market power of sellers.

The remainder of the paper is organized as follows. In Section 2, an environment with a fixed degree of incomplete information is presented. A stationary symmetric monetary equilibrium is defined for this environment in Section 3 and conditions under which a unique equilibrium of this type exists are characterized. In Section 4 the effects of inflation in this environment are considered. Households optimal search strategy is characterized in Section 5. The definition of equilibrium is modified accordingly and the effects of inflation are reconsidered with endogenous search. Conclusions and directions for further research are given in Section 6. Proofs and the numerical algorithm used in the computational examples are included in appendices.

2. The Economy

2.1. The Environment

Time is discrete and there is no aggregate uncertainty. Similar to the environments studied in Shi (1999) and Head and Shi (2002), the economy is comprised of large numbers (*i.e.* unit measures) of $H \geq 3$ different types of household. A type h household is distinguished by its ability to produce a non-storable consumption good of type h and the fact that it derives utility only from consumption of the type $h+1$ good, modulo H. Each household is comprised of large numbers (unit measures) of two different types of members, “buyers” and “sellers”. Household members do not have independent preferences. Rather, they share equally in the utility generated by *household* consumption.

A representative type h household receives utility $u(c)$ from consumption of c units of its preferred good in the current period, where $u(\cdot)$ is increasing and strictly concave, with $\lim_{c\to 0} u'(c)c = \infty$. Members of this household who are sellers can produce good h at constant marginal cost $\phi > 0$ utils per unit. The household maximizes the discounted sum of utility from consumption minus production costs over an infinite horizon:

$$U = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \phi x_t \right], \qquad \beta \in (0,1), \tag{2.1}$$

where β is a discount factor and x_t is total production by the household's sellers in period t.

Since a type h household produces good h and consumes good $h+1$, a double coincidence of wants between members of any two households is impossible. Moreover, households of a given type are indistinguishable and thus members of individual households cannot be relocated in the future following an exchange. As a result, all exchanges between members of different households must be *quid pro quo*. These exchanges are facilitated by the existence at time t of $M_t H$ units of perfectly durable and intrinsically worthless *fiat money*. A type h household may acquire some of this fiat money by having its producers sell output to a buyers of type $h-1$ households. This money may then be exchanged for type $h+1$ consumption good by the household's buyers in a future period.

At time t the *per household* stock of fiat money is given by M_t. In the initial period ($t = 0$) households of all types are endowed with M_0 units of this money. At the beginning of each subsequent period (*i.e.* for $t \geq 1$), they receive a lump-sum transfer, $(\gamma - 1)M_{t-1}$ with $\gamma > 0$, of new units of fiat money from a government that has no other purpose than to increase the stock of money at gross rate γ.

2.2. Household Optimization

At the beginning of period t a representative household of type h (for any h) has post-transfer money holdings m_t (in period 0, $m_0 = M_0$). These money holdings are divided equally among the household's unit measure of buyers. At this point the household's buyers and sellers split up for a “trading session”. Exchanges of goods for fiat money take place in bi-lateral matches between buyers and sellers, each of whom acts in accordance with household instructions. Sellers post prices, and buyers decide whether or not to purchase at the posted price. Following the trading session, buyers and sellers reconvene and the household consumes the goods purchased by its buyers. The sellers' sales revenue (in fiat money) and any remaining money unspent by the buyers are pooled and carried into the next period, when they are augmented with transfer $(\gamma - 1)M_t$ to become m_{t+1}.

The mechanism by which buyers and sellers are matched follows the "noisy sequential search" formulation of Burdett and Judd (1983). While households know the distribution of prices offered by sellers, each individual buyer receives only a random number of "price quotes" from prospective sellers at which he/she may buy. Let q_k, $k = 1, \ldots, K$, denote the probability that a randomly chosen buyer receives k price quotes (alternatively, q_k is the fraction of the household's buyers who receive k quotes)[2]. We assume that the allocation of money to buyers occurs *before* the number of quotes obtained by each individual buyer is known, so that the household has no reason *a priori* to treat buyers asymmetrically[3]. Let the distribution from which these price quotes are drawn (*i.e.* the distribution of prices posted by sellers of the appropriate type at time t) be described by the cumulative distribution function (*c.d.f.*) $F_t(p_t)$ on support $\mathcal{F}_t$. Denote the *c.d.f.* of the distribution of the *lowest* price quote received by a buyer at time t by $J_t(p_t)$, where

$$J_t(p_t) = \sum_{k=1}^{K} q_k \left[1 - [1 - F_t(p_t)]^k\right] \qquad \forall p_t \in \mathcal{F}_t. \tag{2.2}$$

Individual buyers are constrained to spend no more than the money distributed to them at the beginning of the period by the household. If buyer i purchases he/she does so at the lowest price observed, spending $\hat{m}_{it}(p_t)$ conditional on the price paid. Thus buyers face the exchange constraint

$$\hat{m}_{it}(p_t) \leq m_t \qquad \forall i, p_t. \tag{2.3}$$

All buyers, being identical, behave symmetrically if they receive the same lowest price quote (*i.e.* $\hat{m}_{it}(p_t) = \hat{m}_t(p_t)$ for all i, p_t). Thus, as there is no aggregate uncertainty, actual household consumption equals expected:

$$c_t = \int_{\mathcal{F}_t} \frac{\hat{m}_t(p_t)}{p_t} dJ_t(p_t). \tag{2.4}$$

2 The upper bound on the number of price quotes received, K, is of little importance. It could be made arbitrarily large, or as we will do later in our analysis of examples, it can be set K=2 with little effect on the overall results.

3 We assume that buyers cannot return to sellers from whom they have purchased in the past, and instead draw new price quotes from the distribution each period. This assumption enables price dispersion to persist in a stationary equilibrium of our model. Empirical evidence in Lach (2002) suggests that price dispersion is indeed persistent and that individual sellers change their prices frequently, limiting the ability of buyers to identify low price sellers for repeat purchases.

Sellers post prices as directed by the household. Expected sales for any seller who posts p_t are given by

$$x(p_t) = \frac{\hat{M}_t(p_t)}{p_t} \sum_{k=1}^{K} q_k k \left[1 - F_t(p_t)\right]^{k-1} \qquad i \in [0,1], \tag{2.5}$$

where $\hat{M}_t(p_t)$ is the quantity of money spent by the type $h-1$ buyer and $F_t(p_t)$ is used to denote the distribution of prices posted by sellers from other households of the same type (*i.e.* potential competitors). In (2.5), $\hat{M}_t(p_t)/p_t$ is the *quantity per sale* and the summation term is the *expected number of sales*. The latter term is number of *observations* of the seller's price multiplied by the probability that for each of these buyers it will be the lowest price observed. The number of observations of a seller's price is the ratio of the measures of buyers and sellers (in this case one) times the expected number of price observations for a randomly selected buyer, $\sum_k q_k k$. Given distribution $F_t(p_t)$, the probability that the other $k-1$ prices observed by a buyer all exceed the seller's price is $[1 - F_t(p_t)]^{k-1}$ for each k. Let the household specify a distribution, $\hat{F}_t(p_t)$, (which could be degenerate) on support, $\hat{\mathcal{F}}_t$, from which its sellers draw prices to post. The assumption of no aggregate uncertainty then implies that this household's total sales will be

$$x_t = \int_{\hat{\mathcal{F}}_t} x(p_t) d\hat{F}_t(p_t). \tag{2.6}$$

At time t, the state of the economy is summarized by the per household money stock, M_t. For a representative household of *any* type $h \in \{1, \ldots, H\}$, its own individual money holdings, m_t, are also a relevant state variable. The following Bellman equation represents the dynamic optimization problem of such a household:

$$v_t(m_t, M_t) = \max_{m_{t+1}, \hat{m}_t(p_t), \hat{F}_t(p_t)} \{u(c_t) - \phi x_t + \beta v_{t+1}(m_{t+1}, M_{t+1})\} \tag{2.7}$$

subject to

(2.3) (2.4) (2.5) (2.6)

$$m_{t+1} = m_t - \int_{\mathcal{F}_t} \hat{m}_t(p_t) dJ_t(p_t; M_t) + \int_{\hat{\mathcal{F}}_t} p_t x(p_t; M_t) d\hat{F}_t(p_t) + (\gamma - 1) M_t \tag{2.8}$$

$$M_{t+1} = \gamma M_t, \tag{2.9}$$

where the household takes the actions of other households as given. Here these actions are represented by $x_t(p_t; M_t)$ which from (2.5) can be seen to depend on $\hat{M}_t(p_t; M_t)$, and the distribution of exchange prices, $J_t(p_t; M_t)$, where the argument M_t has been added to $x_t(\cdot)$, $\hat{M}_t(\cdot)$, and $J_t(\cdot)$ to indicate that the behaviour of *other* households depends on the aggregate state, M_t.

Prior to the trading session in period t, the household specifies rules to be followed by its buyers and sellers. That is, it chooses $\hat{m}_t(p_t; m_t, M_t)$ and $\hat{F}_t(p_t; m_t, M_t)$. Considering the buyers first (and suppressing the state vector (m_t, M_t)), the household gain to exchanging $\hat{m}_t(p_t)$ units of currency for consumption at p_t is given by the marginal utility of current household consumption times the quantity of consumption good purchased, $\hat{m}_t(p_t)/p_t$. The household cost of this exchange is the marginal value of money *next period* times the number of currency units given up, $\hat{m}_t$. Let ω_t denote the value to the household of a unit of currency spent by a buyer or acquired by a seller during the trading session of the current period. From the household Bellman equation, we have

$$\omega_t = \beta \frac{\partial v_{t+1}}{\partial m_{t+1}}. \tag{2.10}$$

The following lemma (proof in appendix A) characterizes the optimal spending rule for each p_t:

Lemma 1: *The optimal spending rule, $\hat{m}_t(p_t)$, has the following "reservation price" form:*

$$\hat{m}_t(p_t) = \begin{cases} m_t & p_t \leq \frac{u'(c_t)}{\omega_t} \\ 0 & p_t > \frac{u'(c_t)}{\omega_t}. \end{cases} \tag{2.11}$$

With regard to the choice of $\hat{F}_t(p_t)$, the expected return from having a seller post a particular price at time t depends on the distribution of prices posted by firms from *other* households of its type, $F_t(p_t)$, and the strategies of its prospective buyers, $\hat{M}_t(p_t)$. Let $\bar{p}_t$ denote the household's belief regarding the reservation price of its potential customers (all of whom are *ex ante* identical). The household will instruct no seller to post $p_t > \bar{p}_t$, as any seller who does so is expected to make no sales, generating an expected return to the household of zero.

The expected return to the household from having a seller post price $p_t \leq \bar{p}_t$ is given by

$$r(p_t) = \left[\omega_t \hat{M}_t(p_t) - \phi \frac{\hat{M}_t(p_t)}{p_t} \right] \sum_{k=1}^{K} q_k k \left[1 - F_t(p_t)\right]^{k-1}. \tag{2.12}$$

$r(p_t)$ equals the *return per sale* (in brackets) times the *expected number of sales* which is as described in the discussion of (2.5). The return per sale is the value of the currency units obtained minus the disutility of production. From this term it is clear that the return to posting a price lower than $p_t^* = \phi/\omega_t$ (the *marginal cost price*) is negative, and thus the household will instruct no seller to do so. The household will derive the same expected return from having its sellers post any price $p_t \in [p_t^*, \bar{p}_t]$ such that $p_t \in \underset{p_t}{\operatorname{argmax}}\, r(p_t)$.

Returning now to the household optimization problem given by (2.3)-(2.9) and making use of (2.10), (2.11), and (2.12) we have the following first-order condition,

$$u'(c_t)\frac{1}{p_t} - \lambda_t(p_t) - \omega_t = 0 \qquad \forall p_t, t, \tag{2.13}$$

where $\lambda_t(p_t)$ is a Lagrange multiplier on the buyers' exchange constraint, (2.3), conditional on p_t being the lowest price observed. We also have the envelope condition,

$$\frac{\partial v_t}{\partial m_t} = \int_{\mathcal{F}_t} \lambda_t(p_t) dJ_t(p_t) + \omega_t, \qquad \forall t. \tag{2.14}$$

Conditions (2.13) and (2.14) together with the buyers' acceptance rule, (2.11), and the requirement that all prices in $\hat{\mathcal{F}}_t$ satisfy $p_t \in \underset{p_t}{\operatorname{argmax}}\ r(p_t)$ characterize the household's optimal behaviour conditional on its state, (m_t, M_t), and beliefs regarding the actions of other households.

3. Equilibrium

3.1. Existence and Uniqueness

We restrict attention to equilibria that are stationary and symmetric. First, we require that all households behave symmetrically, have a common marginal valuation of money, Ω_t, and equal consumption, C_t, in each period[4]. Second, we require that equilibria be stationary in the sense that consumption remains constant over time (*i.e.* $C_t = C$ for all t), and in which all nominal prices grow at the rate of money creation, γ.

In a symmetric equilibrium all households have identical reservation prices: $\bar{p}_t = u'(C)/\Omega_t$. the common optimal spending rule (2.11) together with the definition of household consumption (2.4) thus give rise to a version of the simple quantity equation

$$C = M_t \int_{\mathcal{F}_t} \frac{1}{p_t} dJ_t(p_t) \qquad \forall t. \tag{3.1}$$

From (3.1), in a stationary monetary equilibrium, nominal transaction prices must on average grow at the gross rate of money creation, γ. If *all* nominal prices grow at rate γ, then there is a time-invariant distribution of real posted prices characterized by time-invariant support $\mathcal{F} \equiv \{p \mid p = p_t/M_t \text{ for some } p_t \in \mathcal{F}_t\}$ for all t, and *c.d.f*

$$F(p) \equiv F_t(p_t) \qquad p \in \mathcal{F} \quad \forall t. \tag{3.2}$$

If an $F(\cdot)$ satisfying (3.2) exists, we may think of buyers as observing real prices and define the *c.d.f* of the lowest *real* price observed in a manner analogous to (2.2):

$$J(p) = \sum_{k=1}^{K} q_k \left[1 - [1 - F(p)]^k\right] \qquad p \in \mathcal{F}, \quad \forall t. \tag{3.3}$$

[4] Where possible, capital letters (*e.g*, C_t, Ω_t, etc.) will be used to distinguish *per household* quantities from their counterparts for an individual household (c_t, ω_t etc.).

Similarly, in a symmetric stationary equilibrium with time-invariant distributions of posted and transactions prices, households' *nominal* money holdings, m_t, spending rule for buyers, $\hat{m}_t(p_t)$, and distribution of sellers' posted prices, $\hat{F}_t(p_t)$ will have time-invariant *real* counterparts obtained by dividing by the per household money stock: $m = m_t/M_t$, $\hat{m}(p) = \hat{m}_t(p_t)/M_t$, and $\hat{F}(p) = \hat{F}_t(p_t)$ for all $p \in \mathcal{F}$, for all t, respectively.

We then have the following definition:

Definition: A symmetric *stationary monetary equilibrium* (SME) is comprised of a collection of time-invariant individual household choices, m, $\hat{m}(p)$, $\hat{F}(p)$, common spending rule, $\hat{M}(p)$, and distribution of posted prices, $F(p)$, such that

1. Taking as given the price distribution, $F(p)$, and common spending rule, $\hat{M}(p)$, in all periods a representative household chooses $m_{t+1} = m$, $\hat{m}_t(p_t) = \hat{m}(p)$, and distribution $\hat{F}_t(p_t) = \hat{F}(p)$ for all $p \in \mathcal{F}$ to solve the optimization problem given by (2.3)—(2.9).

2. Individual choices equal per household quantities: $c = C$; and $\hat{m}(p) = \hat{M}(p)$ and $\hat{F}(p) = F(p)$ for all $p \in \mathcal{F}$.

3. Individual household money holdings equal the per household stock: $m = M$.

4. Consumption is constant over time: $C_t = C$.

5. Money has value: $C > 0$.

We now turn to establishing existence of and characterizing an SME. The key difference between the monetary model studied here and the model of Burdett and Judd (1983) is that whereas in the latter returns to sellers (firms) and buyers (consumers) conditional on transacting at a particular price are exogenous, here they depend on the endogenous sequence of households' marginal valuations of money, $\{\Omega_t\}_{t=0}^{\infty}$. The stationarity conditions imposed here require that in equilibrium $\Omega_{t+1} = \Omega_t/\gamma$ for all t. Thus the existence of an SME hinges on the existence of an $\Omega = \gamma^t\Omega_t$ for all t which is consistent with equilibrium conditions. To begin with, however, conditional on households having a well-defined reservation price, $\bar{p}$, and probability distribution over the number of price quotes received by each buyer, $\{q_k\}_{k=1}^{K}$, the potential price distributions in any SME may be restricted. To this end we have the following proposition corresponding to and largely following from Theorem 4 of Burdett and Judd (1983).

Proposition 1: *Given* $\{q_k\}_{k=1}^{K}$ and a common, finite, buyers' reservation price $\bar{p}$,

i. *if* $q_1 = 1$, *then the only possible price distribution in a SME is concentrated at the reservation price,*

$$\bar{p} = \frac{\bar{p}_t}{\gamma^t} = \frac{u'(C)}{\Omega} \qquad \forall t. \tag{3.4}$$

ii. *If* $q_1 = 0$, *then the only possible price distribution in an SME is concentrated at the "marginal cost" price,*

$$p^* = \frac{p_t^*}{\gamma^t} = \frac{\phi}{\Omega}, \qquad \forall t. \tag{3.5}$$

iii. *If* $q_1 \in (0, 1)$, *there is a unique dispersed price distribution that may be a component of an SME. This distribution is continuous with connected support.*

Proposition 1 does not establish existence of an SME. Rather it establishes that depending on q_1 there are restrictions on the possible distributions of offered prices, $F(p)$, that may arise as a component of a SME. If all buyers observe a single price ($q_1 = 1$), then the only price that maximizes sellers expected return is the buyers' common reservation price, and so the only possible price distribution in equilibrium is concentrated at this price. This is a version of the well-known result due to Diamond (1971). If all buyers receive *more* than one price quote, the return to offering a slightly lower price is always positive if the posted price is greater than p^*, and so the only possible price distribution is concentrated at this price. The only remaining possibility is that some buyers receive one price quote while others receive more. In this case the proposition establishes that contingent on the reservation price, $\bar{p}$ (and hence the marginal valuation of money, Ω), there is a unique continuous distribution of real offered prices, $F(p)$, that may be a component of an SME.

We now turn to the determination of Ω. First, consider the link between the marginal valuation of money, Ω, and consumption in an SME. Imposing stationarity, in an SME (2.10), (2.13), and (2.14) imply

$$\Omega_t = \beta u'(C) \int_{\mathcal{F}_{t+1}} \frac{1}{p_{t+1}} dJ_{t+1}(p_{t+1}) = \frac{1}{\gamma^{t+1} M_0} \beta u'(C) \int_{\mathcal{F}} \frac{1}{p} dJ(p). \tag{3.6}$$

With $M_0 = 1$ (a normalization) and $\Omega = \gamma^t \Omega_t$, (3.6) becomes

$$\Omega = \frac{\beta}{\gamma} u'(C) \int_{\mathcal{F}} \frac{1}{p} dJ(p). \tag{3.7}$$

Combining (3.7) with (3.1) we have

$$\Omega = \frac{\beta}{\gamma} u'(C) C, \tag{3.8}$$

an expression relating households' marginal valuation of money to the level of consumption. In an SME the current marginal value of an additional unit of currency in the trading session of the next period is equal to the quantity it is expected to buy, C/γ, evaluated at the household's marginal utility $u'(C)$, discounted as utility is not realized until the next period.

As in Shi (1999) and for the same reasons, if $\gamma < \beta$ there can be no stationary equilibrium in which money has value and with $\gamma = \beta$ the equilibrium is indeterminate. In the former case households will never spend money and in the latter they are indifferent between spending it and holding it for the future. In addition, for the case of $\gamma = \beta$, we have the following:

Proposition 2: *If $\gamma = \beta$, then there is no SME with a dispersed distribution of prices.*

From this point on, we constrain the rate of money creation, γ, to be greater than the discount factor, β. The following proposition contains our main existence results for these cases.

Proposition 3: *Given $\{q_k\}_{k=1}^{K}$ and $\gamma > \beta$*

i. *if $q_1 = 1$, then there is no SME.*

ii. *if $q_1 = 0$, then there is a unique SME with*

$$J(p) = F(p) = \begin{cases} 1 & p \geq p^* = \phi/\Omega \\ 0 & p < p^*. \end{cases} \tag{3.9}$$

iii. *if $q_1 \in (0,1)$, then there exists an SME with a dispersed and continuous distribution of real prices, $F(p)$, implicitly characterized by*

$$\left[\Omega - \frac{\phi}{p}\right] \sum_{k=1}^{K} q_k k \left[1 - F(p)\right]^{k-1} = \left[1 - \frac{\phi}{u'(C)}\right] \Omega q_1 \qquad p \in \mathcal{F}, \tag{3.10}$$

where $\mathcal{F} = [\underline{p}, \bar{p}]$, with

$$\underline{p} = \frac{\phi}{\Omega}\left[1 - \left(1 - \frac{\phi}{u'(c)}\right) \frac{q_1}{\sum_{k=1}^{K} q_k k}\right]^{-1} \qquad \text{and} \qquad \bar{p} = \frac{u'(C)}{\Omega} = \frac{\gamma}{\beta C}. \tag{3.11}$$

In this case the densities of posted and transactions prices are given, respectively, by

$$f(p) \equiv F'(p) = \frac{\phi}{p^2}\left[\frac{\sum_{k=1}^{K} q_k k \left[1 - F(p)\right]^{k-1}}{\left[\Omega - \phi/p\right] \sum_{k=2}^{K} q_k k(k-1) \left[1 - F(p)\right]^{k-2}}\right] \qquad p \in \mathcal{F} \tag{3.12}$$

and

$$j(p) \equiv J'(p) = \sum_{k=1}^{K} q_k k \left[1 - F(p)\right]^{k-1} f(p) \qquad p \in \mathcal{F} \tag{3.13}$$

In case (i.), we know from Proposition 1 that if all buyers observe only one price, the only possible distribution of prices in an SME is concentrated at the buyers' reservation price. Proposition 3 establishes that there can be no SME with this property. In this case the return to acquiring money is insufficient to induce sellers to accept it in exchange for goods at any finite price. This result obtains even in the absence of an explicit search cost because this is a model of exchange involving fiat money. Households must be compensated for the fact that acquiring fiat money in the current period yields returns only in the future. These returns are discounted, and the return on fiat money cannot be sufficient to overcome this discounting if $\gamma > \beta$.

In case (ii.) we know from Proposition 1 that if all buyers observe strictly more than one price, then the only possible distribution of prices in an SME is concentrated at the sellers' minimum, or marginal cost price, p^*. Proposition 3 establishes that there is a unique equilibrium in which buyers extract all surplus from trade with sellers. As in Burdett and Judd (1983), complete information is not required for this equilibrium in which all trade takes place at a price equal to sellers' marginal cost. Even if all buyers observe only two prices, this outcome will obtain[5] .

When some buyers observe exactly one price and others observe more (case iii.), Proposition 1 establishes that in this case any SME necessarily exhibits dispersion of real prices. Proposition 3 establishes that under certain conditions there exists a positive marginal valuation of money, Ω, supporting such an equilibrium. The price dispersion equilibrium constructed here contrasts with that of Soller-Curtis and Wright (2000) in two ways. Firstly, in their model, for some parameter values a single price equilibrium may coexist with the dispersed price equilibrium. Secondly, in the dispersed price equilibria that exist in their model, generically the equilibrium price distributions are concentrated on exactly two discrete prices.

With $q_1 \in (0,1)$ the SME may not be unique (see the discussion in the appendix following the proof of Proposition 3). For somewhat stronger conditions on utility, however, we have

Proposition 4: *Let the utility function have the constant relative risk aversion (CRRA) form,*

$$u(c) = \frac{c^{1-\alpha} - 1}{1 - \alpha} \tag{3.14}$$

[5] Cases (i.) and (ii.) are reminiscent of random matching monetary models (*e.g.* Shi (1995) and Trejos and Wright (1995)) in which prices are determined by bargaining. Non-existence of an SME in case (i.) corresponds to the lack of monetary equilibria in these models if sellers make take-it-or-leave-it offers to buyers. Case (ii.) corresponds to the situation in these models where *buyers* make take-it-or-leave-it offers. In this case, the promise of future returns to fiat money future induces sellers to accept money in exchange for goods.

with

$$\alpha > \frac{q_1 + \sum_{k=2}^{K} q_k k}{\sum_{k=2}^{K} q_k k} > 1 \tag{3.15}$$

then, if $q_1 \in (0,1)$ *there exists a unique SME characterized by (3.10)-(3.13).*

Proposition 4 identifies a sufficient condition for the uniqueness of the SME. Computational experiments indicate that many economies which fail to satisfy this condition do nevertheless have a unique SME. We now turn to the analysis of one such example.

3.2. An Illustrative Example

Clearly, many aspects of the SME hinge on the probability of observing exactly one price, as opposed to observing at least two. For this reason, in constructing a tractable example it is useful to assume that buyers observe either one price (with probability $q \in (0,1)$) or two prices (with probability $1-q$) *only.* In this case the support of the distribution of posted prices, $F(\cdot)$, is given by $\mathcal{F} = [\underline{p}, \bar{p}]$ where,

$$\underline{p} = \frac{\phi}{\Omega}\left[1 - \left(1 - \frac{\phi}{u'(c)}\right)\frac{q}{2-q}\right]^{-1} \qquad \text{and} \qquad \bar{p} = \frac{\gamma}{\beta C}. \tag{3.16}$$

The distribution $F(\cdot)$ is

$$F(p) = \frac{\left[\Omega - \frac{\phi}{p}\right](2-q) - \left[1 - \frac{\phi}{u'(C)}\right]\Omega q}{\left[\Omega - \frac{\phi}{p}\right]2(1-q)} \qquad p \in \mathcal{F}, \tag{3.17}$$

and the densities of posted and transactions prices are

$$f(p) = \frac{\phi}{p^2}\left[\frac{q + 2(1-q)[1-F(p)]}{[\Omega - \phi/p]2(1-q)}\right] \qquad p \in \mathcal{F}, \tag{3.18}$$

and

$$j(p) = [q + 2(1-q)(1-F(p))]\, f(p) \qquad p \in \mathcal{F} \tag{3.19}$$

respectively, where Ω depends on equilibrium consumption, C, as given by (3.8).

Here it can be easily shown $f'(p) < 0$ and $j'(p) < 0$. That is, the densities of both posted and transactions prices are monotonically decreasing for any choice of parameters ϕ and β, for any rate of money creation, $\gamma > \beta$, for any admissible utility function, and for any $q \in (0,1)$. This, however, is not a robust feature of the equilibrium and stems from the fact that *all* sellers are assumed to have the same production cost, ϕ. Alternatively, we could posit a distribution of costs, $G(\phi)$, over sellers. Judicious choice of $G(\cdot)$ would then enable us to produce price distributions of many different shapes, an issue that has been addressed in the empirical literature on applications of the model of Burdett and Mortensen (1998) to wage dispersion (*e.g.* Bontemps, Robin, and van den Berg (2000)). Since here we are not attempting to match an empirical distribution of prices, and since computations and proofs are dramatically simpler with symmetric costs, we maintain this assumption throughout the paper.

We now consider the effects of various parameters on the SME, concentrating on the properties of the distribution of real transactions prices, $J(\cdot)$. The analysis is complicated by the fact that the marginal valuation of money, Ω, depends on the equilibrium level of consumption, C, and so complete analytic solutions are not possible. We therefore assign specfic values to the economy's parameters and and compute SME's numerically using an algorithm described in appendix B.

We restrict utility to the CRRA class, (3.14). In this case, from (3.8) we have $\Omega = (\beta/\gamma)C^{1-\alpha}$. For a baseline scenario we set $\alpha = 1$ (log utility)[6] so that $\Omega = \beta/\gamma$ and set $\beta = .95$ and $\phi = .1$. We begin by considering three values for the probability of observing exactly one price, $q \in \{.4, .5, .6\}$, and will return to changes in the values of other parameters later. Issues regarding inflation will be postponed to the next section; for now we set $\gamma = 1$ so that the money stock is constant and "nominal" prices are equal to real prices and do not grow over time.

Figure 1 depicts densities of transactions prices for SME's of baseline economies with $q \in \{.4, .5, .6\}$. As suggested by (3.17) and (3.19), for each q $j(\cdot)$ is monotone decreasing in p. In the figure it is clear that as q increases, the average transaction price,

$$P \equiv \int_{\mathcal{F}} p \, dJ(p), \tag{3.20}$$

increases as well. Moreover, the support $\mathcal{F}$ shifts to the right and the range of transaction prices increases in the sense that $\bar{p}/\underline{p}$ rises. Overall the distribution of transactions prices becomes more disperse as q increases in the sense that coefficient of variation increases from .0135 with $q = .4$ to .0161 with $q = .5$ and .0186 with $q = .6$.

Intuition for the rightward shift of distribution of real prices in response to an increase in q is straightforward. An increase in q implies that a larger fraction of buyers observes only one price. This reduction in the information of buyers effectively increases the market power of sellers, enabling them to post higher prices. Note that an increase in $\bar{p}$ requires equilibrium consumption to fall (from (3.11)) so that an increase in q is unambiguously welfare reducing.

The increase in price *dispersion* arises because an increase in q affects the market power of sellers differentially. Considering the densities in Figure 1 it is apparent that as the fraction of

[6] While log utility does not satisfy $\lim_{c\to 0} u'(c)c = \infty$ ($\alpha > 1$ for the CRRA case) so that neither Proposition 3 nor Proposition 4 apply, an SME may nonetheless be demonstrated numerically to exist and be unique.

buyers observing a single price rises the range of real prices increases and the upper tail spreads rightward, increasing the variance. From a seller's point of view, as q increases, the likelihood that a prospective buyer also has another price quote at which he/she could buy falls. This is particularly important to sellers posting prices at the upper end of the distribution who expect that a relatively high percentage of their sales will be made to buyers with no alternative. In response to an increase in q these sellers raise their price by a relatively large increment, as the decline in the number of sales that results will be small. In contrast, sellers pricing at the lower end of the distribution are counting on a large volume of sales, expecting to sell to many buyers in spite of the existence of a competitor. They will raise their price by a relatively small increment in response to an increase in q so as not to lose customers. Since this latter group of sellers is large (given the shape of the density of transaction prices), an increase in q results in a larger increase in the variance of prices than of the average price.

We now briefly discuss the effects of changes in other parameters on the distribution of real prices in an SME[7]. Increases in the parameter α raise real prices and increase price dispersion. An increase in α lowers households' intertemporal elasticity of substitution, thus increasing sellers' market power for a fixed q. The effects of an increase in α are then qualitatively similar to those of an increase in q. Increases in ϕ shift the distribution of real prices to the right, lower consumption, and increase price dispersion. This occurs as those sellers posting high prices and selling to buyers with no alternative can pass a large share of the cost increase on to their custormers. In contrast, those at the lower end face competitors for a large fraction of their sales and cannot pass the cost through by raising their price without losing a significant share of these sales.

4. Inflation

We now consider the effects of money creation on the distribution of real prices in equilibrium. Restricting attention to equilibria with a constant distribution of real transaction prices, it is a straightforward consequence of (3.1) that the gross rate of inflation equals the rate of money creation.

With $q_1 = 0$ the distribution of real prices is concentrated at the "marginal cost" price, p^*, as established by Propositions 1 and 3. In this case the effects of changes in the inflation rate can be summarized in the following proposition:

[7] The discount factor, β, affects the distribution of real prices because it determines the cost of having to acquire fiat money in the current period for use in the future. Thus, it is a key determinant of market power overall. Its effects, however, are symmetric to those of inflation, so we consider it in the next section.

Proposition 5: *If $\gamma \geq \beta$ and $q_1 = 0$, then an increase in γ raises the price and lowers both consumption and welfare in an SME.*

Thus, money is neutral but not superneutral. Monetary injections through lump-sum transfers erode the purchasing power of fiat money, acting as an inflation tax. When all buyers observe at least two prices ($q_1 = 0$), the steady-state effects of anticipated inflation at a constant rate are essentially the same in our search model as in models with Walrasian markets such as that of Cooley and Hansen (1989). In particular, we have:

Corollary 5.1: *If $q_1 = 0$ and $\gamma = \beta$, then $C = {u'}^{-1}(\phi)$ in an SME.*

That is, the Friedman Rule attains the efficient allocation in an SME provided that there is no market power.

For the case in which $q_1 \in (0, 1)$, Proposition 5 may be partially generalized to

Proposition 6: *If there exists a unique SME with price dispersion, then* ceteris paribus *an increase in the rate of inflation (γ) causes an increase in the average transaction price, P; a reduction in equilibrium consumption, C; and an increase in the support of the distributions of posted and transaction prices, $F(\cdot)$ and $J(\cdot)$, in the sense that $\bar{p}/\underline{p}$ rises.*

Higher inflation raises the average real price and lowers consumption in an SME with price dispersion through the inflation tax, just as it does in the marginal cost price equilibrium for an economy with $q_1 = 0$. With $q_1 > 0$, however, higher inflation also increases the market power of sellers by raising buyers' reservation price; an effect similar to that stressed by Casella and Feinstein (1990). Overall, the magnitude of the real price increase (or of the decrease in consumption) may be substantially higher in an economy with price dispersion than in an economy with $q_1 = 0$. This will be demonstrated in computational experiments below.

It is *differences* in market power across sellers that account for the increase in the range of real prices resulting from higher inflation. Those sellers pricing in the top regions of the distribution tend to have fewer competitors and thus can extract a very large share of the decline in the value of money. Those pricing in the lower regions are constrained in their price increases by the fact that they stand to lose a substantial volume of sales to competitors.

With regard to welfare, equilibria with price dispersion are always suboptimal owing to the effects of market power generated by incomplete information. Regardless of the degree of incomplete information, however, both price dispersion and welfare losses diminish as the economy approaches the Friedman Rule:

Proposition 7: *Fix $q_1 > 0$ and suppose that there is a unique SME. Let $\bar{p}(\gamma)$ and $C(\gamma)$ denote the upper support of the price distribution and level of consumption in the SME as functions of the gross inflation rate, γ. Similarly, define $p^*(\gamma) \equiv \phi/[u'(C(\gamma)C(\gamma)]$, the marginal cost price, as a function of γ. Then,*

$$\lim_{\gamma \to \beta} \frac{\bar{p}(\gamma)}{p^*(\gamma)} = 1 \tag{4.1}$$

and

$$\lim_{\gamma \to \beta} C(\gamma) = {u'}^{-1}(\phi). \tag{4.2}$$

According to (4.1), as the economy approaches the Friedman Rule ($\gamma \longrightarrow \beta$) the upper bound of the price distribution (which by Proposition 1 is equal to buyers' reservation price) converges to the marginal cost price, p^*, so that price dispersion vanishes. From (4.2) it can be seen that in this case consumption also converges to its optimum level. Recall, however, from Proposition 2 that the equilibrium described by (3.10)-(3.13) breaks down *at* the Friedman Rule, and that with $\gamma = \beta$ the SME is indeterminate.

The result that price dispersion vanishes and consumption approaches the optimum as the economy approaches the Friedman Rule is again due to the link between the rate of inflation and sellers' degree of market power. The deviation of the rate of money creation from the discount factor ($\gamma > \beta$) imposes on households a cost of waiting until the next period in hopes of observing a lower price. As γ approaches β this costs falls, making households more willing to wait for a better price and eroding the market power of sellers. In the limit, sellers have no market power and both price dispersion and the distortion of equilibrium consumption disappear[8].

It is now useful to return to the parametric example introduced in the previous section in order to characterize more fully the effects of inflation on the distribution of real prices in equilibrium. Let the parameters of the baseline economy other than γ remain as specified earlier: $\beta = .95$, $\alpha = 1$ (log utility), and $\phi = .1$. For illustrative purposes fix the probability of observing exactly one price at $q = .5$, and consider differing values of the money creation rate such that $\gamma \in [.96, 1.5]$ (net inflation rates ranging from -4% to 50%) where the upper bound of this interval is arbitrary[9].

[8] If households faced an explicit search cost, then both Propositions 2 and 7 would continue to hold in slightly modified form as long as the fraction of buyers observing a single price is exogenous. In the next section, we consider the implications of allowing households to choose the number of price quotes observed by their buyers.

[9] In this economy, a reduction in the discount factor, β, is roughly equivalent to an increase in the inflation rate, γ, as it is the ratio, β/γ that is important. For this reason, we consider only the effects of changes in the inflation rate and do not directly analyze changes in β.

Figure 2 contains transaction price densities for the SME's of our parametric example with $\gamma = 1$, 1.05, and 1.10. The implications of Proposition 6 are evident here: increases in the inflation rate are associated with increases in both the range of transactions prices and the average transaction price, P. Moreover, the increase in P is large relative to what it would be if $q = 0$. If $q = 0$, then with log utility, $P = p^* = \gamma\phi/\beta$ for all γ. In this case the change in P is proportional to the change in γ. In contrast, for the three cases depicted in Figure 2, the average price increases by 9.6% as γ rises from 1 to 1.05, and by an additional 8.8% as it rises from 1.05 to 1.1; an increase in P of roughly 1.9% for every percentage point increase in the inflation rate. Figure 3 plots the average real transaction price, P, against the gross inflation rate with $q = 0$ and with $q = .5$. In the picture it is clear that P increases by more as the inflation rate rises for the case of $q = .5$.

The changes in the densities as γ increases (Figure 2) also suggest an increase in price dispersion, and this is indeed borne out in the coefficients of variation of the respective distributions. The coefficient of variation rises from .0161 with a constant money stock to .0312 and .0455 with inflation rates of 5% and 10% respectively. Figure 4 plots the coefficient of variation of transactions prices in the SME against the inflation rate. Clearly, by this measure dispersion of transactions prices increases as the inflation rate rises, again due to the differential market power of sellers.

We now consider the welfare costs generated by inflation in the model. As a measure of the welfare cost of inflation, consider the quantity of consumption, as a percentage of optimum consumption $\bar{C} = u'^{-1}(\phi)$, by which a representative household would have to be compensated to give them the same period utility in the SME as they would receive at the optimum. In particular let $\delta(q, \gamma)$ be the amount, written as a function of the measure of buyers observing a single price and the gross inflation rate, by which consumption must be increased to give the household the same period utility as they would receive at the optimum: $u\left(C + \delta(q, \gamma)\right) - \phi C = u(\bar{C}) - \phi\bar{C}$. Then,

$$\Delta(q, \gamma) \equiv \left[\delta(q, \gamma)/\bar{C}\right] \times 100 \tag{4.3}$$

will be our measure of the welfare cost of inflation due to the effects of inflation at gross rate γ and the search friction represented by q. Using this notation, $\Delta(0, \gamma)$ is a measure of the welfare costs due only to inflation:

$$\Delta(0, \gamma) = \frac{u^{-1}\left(u(\bar{C}) - \phi\left[\bar{C} - C^*\right]\right)}{\bar{C}} \times 100, \tag{4.4}$$

where $C^* = \Omega/\phi$, the consumption that obtains when $q = 0$ and the price distribution is concentrated at the marginal cost price.

Figure 5 plots $\Delta(0, \gamma)$ and $\Delta(.5, \gamma)$ for γ between .96 and 1.5. Clearly, the welfare cost of a given rate of inflation is higher with incomplete information and price dispersion than without. This is unsurprising as Figure 3 demonstrated that price dispersion raises the average real transaction price associated with a particular rate of inflation. It is also the case that the ratio, $\Delta(.5, \gamma)/\Delta(0, \gamma)$, is *decreasing* in γ. With $\gamma = .96$, $\Delta(.5, \gamma)/\Delta(0, \gamma) \approx 4$, and this ratio goes to infinity as $\gamma \longrightarrow \beta$ (.95 in this parameterization). Thus the marginal cost of price dispersion *per se* is especially large (in a relative sense) at low levels of inflation.

Overall, for our parametric example with $q \in (0, 1)$, an increase in the gross rate of money creation, γ, results (for the unique SME) in

i. *an increase in the real average transaction price and a reduction in consumption;*

ii. *an increase in the dispersion (measured by the the coefficient of variation) of transactions prices;*

iii. *a larger reduction in consumption than would occur if $q = 0$, ceteris paribus.*

While these experiments consider only this particular example with log utility and specific values of ϕ and β, computational experiments have shown them to be robust to changes in parameter values, including changes in the coefficient of relative risk aversion (α). In general, an increase in the rate of inflation raises the average price and increases price dispersion, and for a given level of inflation, price dispersion increases the welfare cost of inflation.

5. Endogenous Search

To this point, we have treated the probability with which a buyer observes a single price as a parameter. We now allow the household to choose this probability by adapting the endogenous non-sequential search formulation of Burdett and Judd (1983) to our monetary economy.

Suppose that a household chooses the number of price quotes that its buyers observe and pays fixed cost $\mu > 0$ for each price quote received by a buyer. In order to preserve symmetry across members within a household, we assume that households choose *measures* of its buyers who observe particular numbers of price quotes (or, alternatively, a probability distribution over the number of quotes observed by a representative buyer), rather than the number of quotes observed by a particular member. In this way, the household will again have no incentive to allocate different money balances to individual buyers on the basis of the number of quotes they will observe.

We again restrict attention to symmetric SME's. Taking the distribution of real posted prices, $F(\cdot)$ as given, each household chooses probability distribution $\mathcal{Q} \equiv \{q_k\}_{k=1}^K$ over the number of price quotes observed by each of its buyers to maximize the net return,

$$\mathcal{R}(\mathcal{Q}) = u\left(c(\mathcal{Q})\right) - \mu \sum_{k=1}^{K} k q_k \qquad (5.1)$$

subject to

$$c(\mathcal{Q}) = \sum_{k=1}^{K} q_k c_k \tag{5.2}$$

$$q_k \geq 0 \quad k = 1, \ldots, K \qquad \text{and} \qquad \sum_{k=1}^{K} q_k = 1, \tag{5.3}$$

where c_k is the expected consumption purchases of those buyers who observe k price quotes from a given distribution, $F(\cdot)$, and $\mu \sum_{k=1}^{K} k q_k$ is total search costs. The optimal spending rule for buyers remains characterized by (2.11) and we have

$$c_k = m \int_{\mathcal{F}} \frac{1}{p} dJ_k(p) \qquad k = 1, \ldots, K \tag{5.4}$$

where $J_k(\cdot)$ is the distribution of the lowest price quote received by a buyer who observes k prices drawn from distribution $F(\cdot)$:

$$J_k(p) = 1 - [1 - F(p)]^k \qquad k = 1, \ldots, K \quad p \in \mathcal{F}. \tag{5.5}$$

The problem of optimizing (5.1) subject to (5.2) and (5.3) generates the following first-order and complementary slackness conditions for choice of $\mathcal{Q}$:

$$u'\left(c(\mathcal{Q})\right) c_k \leq \mu k + \upsilon, \qquad q_k \geq 0, \qquad q_k \left[u'(c(\mathcal{Q}))c_k - \mu k - \upsilon\right] = 0 \qquad k = 1, \ldots, K, \tag{5.6}$$

where υ is a Lagrange multiplier associated with the constraint that the q_k's add to one in (5.3). Making use of (5.4) and (5.5) we then have:

Lemma 2: *In the optimal $\mathcal{Q}$, $q_k > 0$ for at most two values, k and $k+1$, $1 \leq k \leq K-1$.*

As we focus exclusively on stationary monetary equilibria, we consider only cases in which a positive measure of household buyers observe a single price ($q_1 > 0$). As it turns out, this is not restrictive. Moreover, if the search cost, μ, is strictly positive, then the economy with endogenous search may have an SME *only if* strictly positive measures of household buyers observe one and two prices.

To see this, reconsider Propositions 1 and 3 from the analysis of the noisy search economy. Fixing a common probability distribution, $\mathcal{Q}$, for all households, the economy here can be viewed as one with noisy search. In this case, from Proposition 3 we know that there can be no SME with $q_1 = 1$. That is, there is no possibility for money to have value in equilibrium if sellers can extract all the exchange surplus from buyers. Endogeneity of search does not affect the logic of this proposition, since it hinges on households' pricing strategy for sellers. From Proposition 1 we also know that for the case in which $q_1 = 0$, the only possible price distribution in an SME is concentrated at the marginal cost price. Therefore, there is no possibility of an SME with price dispersion if $q_1 = 0$. Moreover, we have

Proposition 8: *With $\mu > 0$ and $\mathcal{Q}$ chosen optimally, there is no SME in which $q_1 = 0$.*

Proposition 8 is reminiscent of a result due to Benabou (1988). If all households observe more than one price, then from Proposition 1 we know that the only price distribution consistent with optimal price-posting by sellers is concentrated at the marginal cost price, p^*. If the price distribution is concentrated at a single price, however, then there can be no return to an individual household from having its buyers observe more than one quote. Moreover, it is costly to do so (a household that has measure $q_2 > 0$ of its buyers observe a second price quote must pay additional search cost μq_2). Therefore, it must be the case that in any SME a positive measure of buyers observes a single price. A direct implication of Proposition 8 is

Corollary 8.1: *If $\mu > 0$, then in any SME with endogenous non-sequential search, positive measures of buyers observe exactly one and two prices only.*

Given that we may restrict attention to cases in which positive measures of buyers observe one and two prices only, we cast the household's search decision as choice of a single variable, q: the measure of buyers observing exactly one price. In this case we rewrite the optimization problem described by (5.1)-(5.3) as

$$q^*(Q) = \underset{q \in [0,1]}{\operatorname{argmax}} \; u\big(qc_1 + (1-q)c_2\big) - (2-q)\mu. \tag{5.7}$$

In (5.7), we write the optimal choice q^* as a function of Q, the average (or per household) measure of buyers observing exactly one price, assuming that the distribution of posted prices $F(\cdot)$ and thus both c_1 and c_2 are as specified by equations (3.16)—(3.19); *i.e.* are consistent with the SME of a *noisy* search economy with $q = Q$. Sufficient conditions for an SME of this form to exist and be unique are given by Propositions 3 and 4, respectively.

We now define an SME for the economy with endogenous search in terms of a per household measure, Q, of buyers observing a single price, and the optimal choice on the part of an individual household in that situation, $q^*(Q)$. To this end the definition of an SME for the noisy search economy given in section 3.1 need only be modified by

i. adding the measure Q to the list of components of an SME;

ii. adding the requirement that Q be consistent with households' optimal choice: $q^*(Q) = Q$.

Thus, an SME of an economy with endogenous search is equivalent to one of an economy with noisy search in which no household has incentive to deviate by choosing a measure of its buyers to observe a single price different from Q, given the search cost μ.

We now turn to establishing existence of and characterizing an SME. Consider a class of noisy search economies differing only with regard to the fraction of buyers observing a single quote, Q. For each economy in this class, write the distribution of posted prices as $F(\cdot; Q)$ and let $c_1(Q)$ and $c_2(Q)$ denote the expected purchases of buyers observing one and two prices, respectively. In light of Corollary 8.1 we restrict attention to $Q \in (0, 1)$, as for such Q $F(\cdot; Q)$ is non-degenerate and thus $c_2(Q) - c_1(Q) > 0$. We may then characterize the household's optimal search strategy as a function of Q given search cost μ.

For $Q \in (0, 1)$ define $\Phi_1(Q)$ and $\Phi_2(Q)$ as follows:

$$\Phi_1(Q) = u'(c_1(Q)) \left[c_2(Q) - c_1(Q)\right] \tag{5.8}$$

$$\Phi_2(Q) = u'(c_2(Q)) \left[c_2(Q) - c_1(Q)\right] \tag{5.9}$$

$\Phi_1(Q)$ is the marginal return to the household of increasing the measure of buyers observing a second price quote when all buyers are currently observing just one. Similarly, $\Phi_2(Q)$ is the marginal return to increasing this measure when all buyers are currently observing two. Since by assumption $u'(\cdot)$ is monotone decreasing, from (5.7) we have the household's optimal search strategy:

$$q^*(Q) = \begin{cases} 1, & \text{if } \Phi_1(Q) < \mu; \\ \frac{1}{c_1(Q) - c_2(Q)} \left[u'^{-1}\left(\frac{\mu}{c_2(Q) - c_1(Q)}\right) - c_2(Q)\right], & \text{if } \Phi_2(Q) \leq \mu \leq \Phi_1(Q); \\ 0, & \text{if } \Phi_2(Q) > \mu; \end{cases} \tag{5.10}$$

Equations (5.8)-(5.10) illustrate the importance of search costs for the household's search strategy and ultimately for the existence of an SME. For any $Q \in (0, 1)$ with $\Phi_1(Q) < \mu$, the household has no incentive to have any of its buyers observe a second price quote, and so it deviates by setting $q^*(Q) = 1$. Thus, there can be no SME in which the proportion of buyers observing a single price is Q. Similarly, with $\Phi_2(Q) > \mu$ households deviate by setting $q^*(Q) = 0$ and again there can be no SME. Existence of an SME is possible *only if* the search cost is such that there exists $Q \in (0, 1)$ such that $\Phi_2(Q) \leq \mu \leq \Phi_1(Q)$. Intuitively, the cost of search must be low enough that households are willing to have all of their buyers observe at least one quote and to have a *positive* measure observe two. It must be high enough that households do not want to have all of their buyers observed two quotes.

We do not derive an explicit parameter restriction on the search cost μ guaranteeing the existence of a Q (or a range of $Q's$) such that $\Phi_2(Q) \leq \mu \leq \Phi_1(Q)$. This would be algebraically complex and the resulting expression would depend on the exact form and parameterization of the utility function. Instead we a establish existence of an SME for an economy with endogenous search by showing that any $Q \in (0,1)$ may be a component of an SME of such an economy for appropriate choice of search cost μ:

Proposition 9: *Under the conditions guaranteeing existence of an SME for an economy with noisy search and $Q \in (0,1)$ (i.e. those of Proposition 3, part iii.), search cost μ may be chosen so that $q^*(Q) = Q$. Thus, for appropriate choice of μ an SME exists for the economy with endogenous non-sequential search.*

We now modify our parametric example (with logarithmic utility) to include endogenous search. Rather than fixing Q and solving for μ as suggested by Proposition 9, we choose a specific value for μ, and hold it constant in experiments with different inflation rates. We choose $\mu = .015$; a value for which our economy has a SME with price dispersion for gross inflation rates between 1 and 1.5. Figure 6 depicts (5.10) for the baseline economy with $\mu = .015$ and $\gamma = 1.05$, illustrating that in this case the economy has a *unique* SME with $Q = .3538$.

The uniqueness of the SME here distinguishes our economy from that of Burdett and Judd (1983). In their model, generically, if there is an interior equilibrium with endogenous search, then there are two. This difference stems the fact that in our model buyers' reservation price is endogenous as it depends on the value of fiat money. First, our economy has no non-autarkic equilibrium with $Q = 1$ since in this case sellers would never expend effort to acquire fiat money irrespective of search costs. Second, in our economy it may be the case that $\lim_{Q\to 1} q^*(Q) = 0$ rather than 1, as is always the case in Burdett and Judd (1983). In our economy, as the fraction of buyers observing a single price approaches one, the distribution of posted prices collapses and the average price converges to the buyers' reservation price, $\bar{p} = u'(C)/\Omega$, which depends on the value of fiat money. Thus, as $Q \longrightarrow 1$, $\bar{p}$ tends to infinity and consumption to 0. Returning to (5.8), it can be seen that it is possible to have $\lim_{Q\to 1} q^*(Q) = 0$ as long as $c_1(Q)$ goes to zero sufficiently more quickly than $c_2(Q)$. This happens in our example, but is not a possibility in the economy of Burdett and Judd (1983) where the buyers' reservation price is exogenously fixed[10].

[10] In our economy uniqueness is a possibility; we do not claim that it is a general result. Computational experiments indicate that $q^*(\cdot)$'s like that depicted in Figure 6 are typical for economies with CRRA utility. Greater curvature, as is required for Proposition 4 to hold, makes uniqueness more likely.

Figure 7 illustrates the relationship between the share of buyers observing a single price, Q, in the SME and the gross rate of inflation γ. Clearly, Q is decreasing in γ and the relationship is strongly convex. Recall (see Figure 4) that for a fixed share of buyers observing a single price (one-half, in the figure), an increase in inflation raises price dispersion. The greater the degree of price dispersion, *ceteris paribus*, the greater the return to having buyers observe more than one price. Thus, households respond to the increase in price dispersion generated by a higher inflation rate by lowering the measure of their buyers who observe only one price. This result is similar to that obtained by Benabou (1988,1992), in partial equilibrium with exogenous nominal rigidities. Inflation, by increasing the market power of sellers differentially, increases price dispersion. This in turn induces increased search intensity and may weaken market power overall.

To understand the convexity of the relationship depicted in Figure 7, note that the effect of an increase in the inflation rate on price dispersion rises with the fraction of buyers observing a single price. When this fraction is high, a relatively large share of sales go to buyers who have no alternative, and thus sellers are able to raise their prices by more in response to a fall in the value of fiat money. In Figure 7, as γ increases, price dispersion increases at a decreasing rate because of the reduction in Q. This in turn causes the response of Q to an increase in the inflation rate to become smaller and smaller.

With endogenous search, inflation affects consumption and the average real price through two channels. First, it erodes the purchasing power of money, increases market power, and reduces consumption by acting as a tax on the medium of exchange. Second, it generates price dispersion and induces increased search intensity, weakening the market power of sellers stemming from incomplete information; an effect that tends to *lower* the average price and *raise* consumption. Figure 8 illustrates that at low levels of inflation the latter effect may dominate and increases in the inflation rate may reduce the average transaction price. At higher levels of inflation, since further increases have only small effects on Q, they raise the average transaction price as the increase in market power due to the inflation tax dominates.

Given that an increase in inflation has conflicting effects on market power and consumption, it is not clear that it necessarily lowers welfare. To obtain a measure of the welfare costs of inflation in the economy with endogenous search, we modify (4.3):

$$\Delta(\gamma) = \frac{u^{-1}\big(u(\bar{C}) - \phi\left[\bar{C} - C\right] + [2 - Q]\,\mu\big) - C}{\bar{C}} \times 100. \tag{5.11}$$

Here C and Q are consumption and the measure of buyers observing a single price, respectively, in the SME with gross inflation rate γ. Comparing (5.11) and (4.3), note that the welfare cost of inflation now includes another component, the search cost associated with obtaining $2 - Q$ price quotes per buyer on average. Thus, the fact that inflation induces search adds two dimensions to its effect on welfare. First, the search induced by inflation comes at cost. Second, because it induces search, inflation increases information thus weakening market power.

Figure 9 depicts $\Delta(\gamma)$ (labelled (5.11)), $\Delta(0,\gamma)$ (labelled (4.4); the same curve as depicted in Figure 5), and a measure of the welfare costs of inflation in the economy with endogenous search associated *only* with the effect of inflation on consumption:

$$\hat{\Delta}(\gamma) = \frac{u^{-1}\big(u(\bar{C}) - \phi\left[\bar{C} - C\right]\big) - C}{\bar{C}} \times 100 \tag{5.12}$$

(labelled (5.12) in the figure). Both $\hat{\Delta}(\gamma)$ and $\Delta(0,\gamma)$ are net of search costs (in the economy with noisy search there are none) and thus isolate the consumption costs of inflation.

Figure 9 illustrates that at low rates of inflation, the reduction of market power resulting from increased search in response to an increase in γ may be sufficient to lower welfare losses, even if search costs are taken into account. At sufficiently high rates of inflation, however, increases in the inflation rate have little effect on search, and thus the effects of the inflation tax dominate.

Focusing on only the consumption costs ($\hat{\Delta}$) and comparing Figures 9 and 5, it can be seen that the increase in search intensity causes the cost of inflation to be lower in the endogenous search economy than in the model with noisy search for gross inflation rates lower than 1.04, the cases in which Q is less than .5, the level at which it is fixed in Figure 5. When search costs are added, however, increased inflation may lower welfare even if it raises consumption. In the example, an increase of the gross inflation rate from 1.01 to 1.02 raises consumption from 8.10 to 8.45, but lowers welfare because of the increase in search costs required to drive the fraction of buyers observing a single price down from .730 to .596.

Overall, the effects of inflation in the economy with endogenous search may be summarized:

1. *An increase in the inflation rate has two conflicting effects on sellers' market power:*
   i.) it reduces buyers' valuation of their currency holdings increasing market power;
   ii.) it increases price dispersion, inducing search and reducing market power.

2. *At low levels of inflation the latter effect dominates: an increase in the money creation rate lowers the average real price and raise consumption. At high levels, the former effect dominates—an increase in money creation raises real prices and lowers consumption.*

3. *Welfare varies non-monotonically with inflation and the optimal rate exceeds that prescribed by the Friedman Rule.*

While we demonstrate these results here only for a specific parametric example, computational experiments show them to be robust qualitatively to a wide range of changes in parameter values.

6. Conclusion

This paper has considered a monetary economy in which price posting by identical sellers and search by *ex ante* identical buyers may lead to a stationary symmetric monetary equilibrium in which the distribution of real prices is non-degenerate and continuous. With fixed search intensity, money creation acts as an inflation tax, increasing both the level and dispersion of real prices and lowering welfare. Moreover, the effect on welfare of a given increase in the inflation rate is larger than it would be if prices were concentrated at the marginal cost price. With endogenous search, an increase in the inflation rate induces increased search intensity, weakening the market power of sellers. While this effect tends to reduce the costs associated with the inflation tax, it also results in higher search costs. Overall, the effect of an increase in the inflation rate on welfare is non-monotonic: at low levels of inflation the marginal cost of an increase in the money creation rate may be negative, but for higher rates it will become positive.

The model studied here generates price dispersion associated with anticipated inflation without imposing nominal rigidities exogenously. All sellers are free to change their prices from period to period, without cost. Even so, prices may react "sluggishly" to changes in the rate of money creation. In the version of the model with endogenous search, the increase in the average real price will be less (as a percentage) than the permanent increase in the inflation rate that causes it. In particular, for some changes in the inflation rate the real price actually falls and for others (*e.g* consider an increase in γ from 1.04 to 1.05 in Figure 8) there will be almost no change in the average real transaction price in response to an increase in the inflation rate. Further research will consider the extent to which this mechanism limits *nominal* price movements in response to temporary shocks to the rate of money creation.

Appendix A: Proofs

Proof of Lemma 1: For all p_t, the optimal spending rule satisfies

$$\hat{m}_t(p_t) \in \underset{\hat{m}_t}{\text{argmax}} \left[\frac{u'(c_t)}{p_t} - \omega_t \right] \hat{m}_t \tag{A.1}$$

subject to

$$\hat{m}_t \leq m_t. \tag{A.2}$$

Since neither household consumption, c_t, nor the household's marginal valuation of money, ω_t, is affected by the spending of a particular buyer in a match, the household will instruct its buyers to spend their entire money holdings as long as the bracketed term in (2.11) is positive. That is, the household reservation price is given by $u'(c_t)/\omega_t$. If the lowest price that a buyer observes at time t is *greater* than this reservation price, then the buyer returns to the household with its money holdings unspent and the household carries the money into period $t+1$. ∎

Proof of Proposition 1: Note that in an SME, the distribution of posted prices, $F(p)$, satisfies $F(p) = \hat{F}(p)$ where $\hat{F}(p)$ is chosen by a representative household so that $p \in \text{argmax}_{p \in \mathcal{F}}\, r(p)$, with $\mathcal{F}$ the support of $F(\cdot)$. Thus, an SME is associated with a pair, $(F(\cdot), \Pi)$, where $\Pi = \max_{p \in \mathcal{F}}\, r(p)$. The proof of the Proposition then follows directly from Lemmas 1 and 2 of Burdett and Judd (1983, pp.959-61) with $\bar{p}$ and p^* in our notation corresponding to $\tilde{p}$ and r, respectively, in theirs. ∎

Proof of Proposition 2: Suppose that $\gamma = \beta$ and there is an SME with price dispersion. From Proposition 1 we know that such an equilibrium can occur only if $q_1 \in (0,1)$ and that the distribution of posted prices in this equilibrium, $F(\cdot)$, is non-degenerate and continuous with connected support. Let $J(\cdot)$ be the distribution of transactions prices associated with $F(\cdot)$ according to (2.2). Using (3.6), we have

$$\Omega = u'(C) \int_{\mathcal{F}} \frac{1}{p} dJ(p) \tag{A.3}$$

With $\bar{p} = u'/\Omega$, (A.3) is

$$\frac{1}{\bar{p}} = \int_{\mathcal{F}} \frac{1}{p} dJ(p). \tag{A.4}$$

With $\bar{p}$ the upper support of $J(\cdot)$, (A.4) implies

$$J(p) = \begin{cases} 1 & p \geq \bar{p} \\ 0 & p < \bar{p}. \end{cases} \tag{A.5}$$

This contradicts the premise that there is price dispersion (*i.e.* that $J(\cdot)$ is non-degenerate). ∎

Proof of Proposition 3:

i. Suppose that an SME exists with $\gamma > \beta$ and $q_1 = 1$. In any SME we have

$$\bar{p} = \frac{\gamma}{\beta C}. \tag{A.6}$$

From Proposition 1, with $q_1 = 1$, the only possible distribution of posted prices (and therefore of transactions prices as well) is concentrated at the reservation price, $\bar{p}$, so that $C = 1/\bar{p}$. In this case (A.6) becomes

$$\bar{p} = \left[\frac{\gamma}{\beta}\right] \bar{p}. \tag{A.7}$$

With $\gamma > \beta$, (A.7) can be satisfied by no finite, non-zero price. This contradicts the existence of an SME in this case and establishes part **i.** of the proposition.

ii. From Proposition 1, with $q_1 = 0$ the only possible distribution of transactions prices is concentrated at the marginal cost price, p^*, where in an SME,

$$p^* = \frac{\gamma \phi}{\beta u'(C) C}. \tag{A.8}$$

Since $p^* = 1/C$, (A.8) becomes

$$u'(C) = \frac{\gamma \phi}{\beta}. \tag{A.9}$$

With $u(\cdot)$ increasing and strictly concave, $u'(\cdot)$ is monotone and (A.9) has a unique solution. This establishes part **ii.** of the Proposition.

iii. In any SME, the marginal value of fiat money, Ω, is a function of consumption, C, as specified by (3.8). Thus (3.11) and (3.10) implicitly express the upper and lower bounds of the support of the price distribution (which must be connected by Proposition 1), and the *c.d.f.* $F(\cdot)$ itself as functions of consumption, Making use of the dependence of the price distribution on consumption, define the function $T : \Re_{++} \rightarrow \Re$ as follows:

$$T(C) = \int_{\mathcal{F}(C)} \frac{1}{p} dJ(p; C) \tag{A.10}$$

where in (A.10) consumption C has been added as an argument to $\mathcal{F}$ and $J(\cdot)$ to make explicit the dependence of these components of an SME on household consumption. Note that the *c.d.f* of the distribution of transactions prices, $J(\cdot)$ depends on C through $F(\cdot)$ via (2.2). We will associate an SME with a fixed point of (A.10). That is, given a consumption level that satisfies $T(C) = C$, when the marginal value of a unit of fiat money is $\Omega(C) = \beta/\gamma u'(C)C$, the price distribution given by (3.10) with $\Omega = \Omega(C)$ is the distribution of posted prices in a stationary monetary equilibrium.

Let $\bar{C} = {u'}^{-1}(\phi)$, denote the "unconstrained optimum" level of consumption. It is straightforward to show that

$$T(\bar{C}) = \frac{\beta}{\gamma}\bar{C} < \bar{C}. \tag{A.11}$$

We now prove the following claim:

Claim 1: There exists $\hat{C} \in (0, \bar{C})$ such that $T(\hat{C}) > \hat{C}$.

Suppose the contrary: For all $C \in (0, \bar{C}]$, $T(C) \leq C$. Then

$$\lim_{C \to 0} T(C) = 0. \tag{A.12}$$

Since $\lim_{C\to 0} u'(C)C = \infty$, we have $\lim_{C\to 0} \bar{p}(C) = \infty$ and $\lim_{C\to 0} \underline{p}(C) = 0$. Fix $\epsilon > 0$. Then, there exists $\tilde{C}$ such that for all $C < \tilde{C}$, $1/\epsilon \in \mathcal{F}(C)$. With price $1/\epsilon$ in the support of the price distribution, measure $J(1/\epsilon; C)$ buyers each make individual purchases strictly greater than ϵ, for all $C < \tilde{C}$. Thus,

$$T(C) > J(1/\epsilon; C)\epsilon \qquad \forall C \in (0, \tilde{C}), \tag{A.13}$$

as the expression on the right hand side of the inequality in (A.13) is how much these buyers would purchase in total if they each purchased only ϵ units of consumption good, when in fact they all buy strictly more. Since ϵ is fixed, in order for (A.12) to hold given (A.13), it must be the case that

$$\lim_{C \to 0} J(1/\epsilon; C) = 0 \qquad \forall \epsilon > 0. \tag{A.14}$$

Using (3.10) we may write

$$F(1/\epsilon; C) = \frac{[\Omega(C) - \phi\epsilon]\left[2q_2 + \sum_{k=3}^{K} q_k k[1 - F(1/\epsilon; C)]^{k-1}\right] + \frac{q_1\phi(\beta C - \gamma\epsilon)}{\gamma}}{[\Omega(C) - \phi\epsilon]\, 2q_2}. \tag{A.15}$$

For fixed ϵ, since $\lim_{C\to 0} \Omega(C) = \infty$, $\lim_{C\to 0} F(1/\epsilon; C) = 1$. Since $J(p; C) \geq F(p; C)$ for all C and $p \in \mathcal{F}(C)$, $\lim_{C\to 0} J(1/\epsilon; C) \geq \lim_{C\to 0} F(1/\epsilon; C)$. Thus

$$\lim_{C \to 0} J(1/\epsilon; C) = 1, \tag{A.16}$$

a contradiction to (A.14). This proves *Claim 1.*

From *Claim 1*, there exists a $\hat{C} > 0$ such that $T(\hat{C}) > \hat{C}$. Choosing such a $\hat{C}$, restrict the domain of $T(\cdot)$ to the closed interval $[\hat{C}, \bar{C}]$. It is clear from (A.10), (3.10), (3.11), and (3.8) that $T(C)$ is continuous on this interval. With $T(\hat{C}) > \hat{C}$ and $T(\bar{C}) < \bar{C}$, there must exist a $C \in [\hat{C}, \bar{C}]$ such that $T(C) = C$.

Setting $\Omega = \Omega(C)$ as per (3.8), (3.10) and (3.11) yield a distribution of posted prices, $F(\cdot)$, for an SME with consumption equal to C. This establishes the proposition. ∎

Note: In general, the function $T(\cdot)$ may not be monotone on $[\hat{C}, \bar{C}]$. For this reason it is very difficult to rule out the possibility of multiple equilibria. For *CRRA* utility, however, computational experiments suggest that while T may be non-monotonic, it nevertheless has a unique fixed point between 0 and $\bar{C}$. Proposition 4 (see below) establishes that with enough curvature T is monotone decreasing, so that the SME which exists by Proposition 3 is unique.

Proof of Proposition 4: If $u(c)$ has the *CRRA* form (3.14), then α must be greater than one for $\lim_{c \to 0} u'(c)c = \infty$ to be satisfied. In this case, Proposition 3 obtains and there exists an SME. Specifically, as proved above, $T(\bar{C}) < \bar{C}$ and there exists $\hat{C} \in (0, \bar{C}]$ with $T(\hat{C}) > \hat{C}$. Thus, if it can be shown that T is monotonically decreasing in C over $(0, \bar{C}]$, then the SME that exists is unique. Note that there can be no SME with $C > \bar{C}$, as can be seen from (2.13). In this case, households would not be willing to accept fiat money in exchange for production.

Integrating (A.10) by parts and rearranging we may write:

$$T(C) = \frac{1}{\bar{p}(C)} + \int_{\mathcal{F}(C)} \frac{1}{p^2} J(p; C) dp. \tag{A.17}$$

Given that utility is continuously differentiable (it has the *CRRA* form), differentiation of $T(\cdot)$ is straightforward, though algebraically complex. Applying Leibnitz's Rule, differentiating of (A.17) with respect to C and combining terms yields

$$T_c = \int_{\mathcal{F}(C)} \frac{1}{p^2} J_c(p) dp, \tag{A.18}$$

where $J_c(p)$ is the derivative of $J(p; C)$ with respect to C, for each p. Thus, a *sufficient* condition for $T_c < 0$ is $J_c(p) < 0$ *for all* p. Differentiating both sides of (3.10) with respect to C, we obtain

$$- \sum_{k=2}^{K} q_k k(k-1)[1 - F(p)]^{k-2} F_c(p) =$$

$$\frac{(\beta/\gamma) q_1}{[\Omega - (\phi/p)]^2} \left[\left(\Omega - \frac{\phi}{p} \right) [u''(c)c + u'c - \phi] - \Omega \left(1 - \frac{\phi}{u'(c)} \right) [u''(c)c + u'(c)] \right] \qquad \forall p. \tag{A.19}$$

Noting that with the given utility function, $\alpha = \frac{-u''(c)c}{u'(c)}$, and (A.19) may be written

$$\sum_{k=2}^{K} q_k k(k-1)[1-F(p)]^{k-2} F_c(p) =$$

$$\frac{(\beta/\gamma)q_1}{[\Omega - (\phi/p)]^2}\left[\left(\Omega - \frac{\phi}{p}\right)\left(\alpha + \frac{\phi}{u'(c)} - 1\right) + \Omega\left(1 - \frac{\phi}{u'(c)}\right)[1-\alpha]\right] \qquad \forall p. \tag{A.20}$$

From (A.20) we have that for each p, $F_c(p) < 0$ if and only if

$$\left[\Omega - \frac{\phi}{p}\right]\left(\alpha + \frac{\phi}{u'(c)} - 1\right) < [\alpha - 1]\left[1 - \frac{\phi}{u'(c)}\right]. \tag{A.21}$$

Clearly, if (A.21) holds for $\underline{p}$, then it holds for all p. Using the expression for $\underline{p}$ in an SME, (3.11), we have that $F_c(p) < 0$ for all p if

$$\alpha > \frac{q_1\phi + u'(c)\sum_{k=2}^{K} q_k k}{u'(c)\sum_{k=2}^{K} q_k k}. \tag{A.22}$$

The right hand side of (A.22) is increasing in C, so it is sufficient to evaluate it at the maximal C, $\bar{C} = u'^{-1}(\phi)$. In this case we have

$$\alpha > \frac{q_1 + \sum_{k=2}^{K} q_k K}{\sum_{k=2}^{K} q_k k}; \tag{3.15}$$

which is the required condition. Thus, if (3.15) is satisfied, $T(\cdot)$ is decreasing on $(0, \bar{C})$ and there exists a *unique* SME. ∎

Proof of Proposition 5: In an SME with $q_1 = 0$, the distribution of real transactions prices satisfies (3.9). Therefore, consumption satisfies $C = 1/p^*$ and we have

$$\frac{1}{C} = p^* = \frac{\phi}{\Omega} = \frac{\phi}{(\beta/\gamma)u'(C)C}. \tag{A.23}$$

Rearranging, we have

$$u'(C) = \frac{\gamma\phi}{\beta}. \tag{A.24}$$

with $u(\cdot)$ increasing and strictly concave, the proposition follows. Note also that with $\gamma = \beta$, $u'(C) = \phi$, and C equals $\bar{C}$, the optimal level of consumption. ∎

Proof of Proposition 6: Recall the definition of $T(\cdot)$, (A.10). This function was defined for a fixed $\gamma > \beta$. Clearly, for each $C \in (0, \bar{C}]$ (the domain of T) we may define a function $\hat{T} : (\beta, \bar{\gamma}] \to \Re$ as follows:

$$\hat{T}(\gamma; C) \equiv \int_{\mathcal{F}(\gamma)} \frac{1}{p} dJ(p; \gamma, C) \qquad \gamma > \beta, \tag{A.25}$$

where for any $\gamma \in (\beta, \bar{\gamma}]$, $\mathcal{F}(\gamma; C)$ refers to the support, $[\underline{p}(\gamma; C), \bar{p}(\gamma; C)]$ of the distribution $F(\gamma; C)$ defined by (3.10) and (3.11) with consumption fixed at C.

The proof of Proposition 3iii establishes the existence of an SME by showing that $T(\cdot)$ must cross the 45^o line from above. If the equilibrium is unique, then this must be true at the only equilibrium level of consumption. Therefore, in order to show that an increase in the inflation rate, γ, reduces consumption, it suffices to show that an increase in γ reduces $\hat{T}(\cdot; C)$ for any $C \in (0, \bar{C}]$. To this end, we differentiate $\hat{T}(\cdot; C)$ for arbitrary C, and show that this derivative, $\hat{T}_\gamma(\cdot; C)$, is less than zero for all C in the domain of $T(\cdot)$. From (A.25), (3.10), (3.11), and (3.8) it is clear that $\hat{T}(\cdot; C)$ is differentiable with respect to γ for any fixed $C \in (0, \bar{C}]$. Moreover, calculation of $\hat{T}_\gamma(\cdot; C)$ is straightforward.

Integrating by parts to obtain an expression analogous to (A.17), differentiating this expression via Leibnitz's rule and combining terms, we have

$$\hat{T}_\gamma(\gamma; C) = \int_{\mathcal{F}(\gamma;C)} \frac{1}{p^2} J_\gamma(p; \gamma, C) dp \qquad C \in (0, \bar{C}] \tag{A.26}$$

For any C, a sufficient condition for $\hat{T}_\gamma(\gamma; C)$ to be negative is $F_\gamma(p; \gamma, C) < 0$, (and as a result $J_\gamma(p; \gamma, C) < 0$) for all $p \in \mathcal{F}(\gamma; C)$.

Differentiating both sides of (3.10) with respect to γ and making use of (3.8), we have for all $C \in (0, \bar{C}]$ and for all $p \in \mathcal{F}(\gamma; C)$:

$$F_\gamma(p; \gamma, C) = \frac{-1}{\sum_{k=2}^K q_k k(k-1)[1 - F(p; \gamma, C)]^{k-2}} \left[\frac{[u'(C)C - \phi C]\beta q_1}{[\Omega - (\phi/p)]} \right] \frac{\phi}{p}. \tag{A.27}$$

Since $u'(C) > \phi$ for all $C \in (0, \bar{C})$, (A.27) implies $F_\gamma(p; \gamma, C) < 0$ for all p over the domain of $T(\cdot)$. Thus an increase in γ shifts $T(\cdot)$ downward, reducing consumption and increasing the average price, P, in the SME.

Since in the SME $\bar{p} = \gamma/\beta C$, it is clear that an increase in γ leads to a more than proportionate increase in the upper support of $J(p)$. Making use of the expression for the lower support of the distribution, (3.11), we may write

$$\frac{\bar{p}}{\underline{p}} = \frac{q_1 \phi + u'(C) \sum_{k=2}^K q_k k}{\phi \sum_{k=1}^K q_k k}. \tag{A.28}$$

With $u(\cdot)$ increasing and strictly concave, an increase in γ must therefore raise $\bar{p}/\underline{p}$ in the SME. ∎

Note: From inspection of (3.11) it is clear that it is not possible to sign the change in the lower support of the price distribution, $\underline{p}$, as this depends on the *magnitude* of the change in household consumption.

Proof of Proposition 7: With $q_1 > 0$, for any $\gamma > \beta$, $\bar{p}(\gamma)$ in the SME is given by (3.11). Thus,

$$\lim_{\gamma \to \beta} \bar{p}(\gamma) = \frac{1}{C(\beta)}, \tag{A.29}$$

where

$$C(\beta) \equiv \lim_{\gamma \to \beta} C(\gamma) = \lim_{\gamma \to \beta} \int_{\mathcal{F}(\gamma)} \frac{1}{p} dJ(p; \gamma). \tag{A.30}$$

From (A.29), as $\gamma \longrightarrow \beta$ the measure of buyers purchasing at prices strictly less than $\bar{p}(\gamma)$ must go to zero. That is, given $\xi > 0$, for any $\epsilon > 0$

$$J(\bar{p}(\gamma) - \xi) < \epsilon \tag{A.31}$$

for γ sufficiently close to β. Thus, defining $\bar{p}(\beta) \equiv 1/C(\beta)$, the sequences of distributions of both transaction prices and posted prices converge to degenerate distributions concentrated at $\bar{p}(\beta)$. Next, we prove the following claim:

Claim 2: As $\gamma \longrightarrow \beta$ the upper support of the distribution of posted prices converges to the marginal cost price. That is,

$$\lim_{\gamma \to \beta} \frac{\bar{p}(\gamma)}{p^*(\gamma)} = 1. \tag{A.33}$$

.

Proof of Claim 2: Suppose the contrary, then since for all $\gamma > \beta$ the return $r(p)$ (see (2.13)) to posting a price below the marginal cost price $p^*(\gamma)$ is negative, we have

$$\lim_{\gamma \to \beta} \bar{p}(\gamma) > \lim_{\gamma \to \beta} p^*(\gamma). \tag{A.34}$$

Given (A.34) it is possible to choose $\xi > 0$ such that for all $\gamma > \beta$, $p^*(\gamma) < \bar{p}(\gamma) - \xi < \bar{p}(\gamma)$. From (A.31) for any $\epsilon > 0$, for γ sufficiently close to β

$$F(\bar{p}(\gamma) - \xi) < \epsilon. \tag{A.35}$$

Consider then the return to a household from having a seller post price $\bar{p}(\gamma) - \xi$ for any γ in the range over which (A.35) holds:

$$\begin{aligned} r(\bar{p}(\gamma) - \xi) &= \left[\omega - \frac{\phi}{\bar{p}(\gamma) - \xi}\right] \sum_{k=1}^{K} q_k k [1 - F(\bar{p}(\gamma) - \xi)]^{k-1} \\ &> \left[\omega - \frac{\phi}{\bar{p}(\gamma) - \xi}\right] \sum_{k=1}^{K} q_k k [1 - \epsilon]^{k-1}. \end{aligned} \tag{A.36}$$

Here the inequality in the second line of (A.36) is obtained by combining (A.31) with the equation in the first line. Recall that the return from posting price $\bar{p}(\gamma)$ may be written

$$r(\bar{p}(\gamma)) = \left[\omega - \frac{\phi}{\bar{p}(\gamma)}\right] q_1. \tag{A.37}$$

Combining (A.36) and (A.37) we have

$$r(\bar{p}(\gamma) - \xi) - r(\bar{p}(\gamma)) > \left[\omega - \frac{\phi}{\bar{p}(\gamma) - \xi}\right] \sum_{k=1}^{K} q_k k [1-\epsilon]^{k-1} - \left[\omega - \frac{\phi}{\bar{p}(\gamma)}\right] q_1. \tag{A.38}$$

For ϵ sufficiently close to zero, (A.38) may be re-written:

$$r(\bar{p}(\gamma) - \xi) - r(\bar{p}(\gamma)) > \omega \sum_{k=2}^{K} q_k k - \frac{\phi}{\bar{p}(\gamma) - \xi} \sum_{k=1}^{K} q_k k + \frac{\phi}{\bar{p}(\gamma)} q_1. \tag{A.39}$$

With $p^*(\gamma) < \bar{p}(\gamma) - \xi < \bar{p}(\gamma)$, (A.39) is clearly continuous in ξ and satisfies

$$\lim_{\xi \to 0} \left[\omega \sum_{k=2}^{K} q_k k - \frac{\phi}{\bar{p}(\gamma) - \xi} \sum_{k=1}^{K} q_k k + \frac{\phi}{\bar{p}(\gamma)} q_1\right] = \left[\omega - \frac{\phi}{\bar{p}(\gamma)}\right] \sum_{k=2}^{K} q_k k > 0. \tag{A.40}$$

Thus, a small enough price deviation, ξ, generates a positive return to the household. Households will therefore not be willing to drive the measure of their sellers pricing above $\bar{p}(\gamma) - \xi$ to one for γ sufficiently close to β that $J(p(\gamma) - \xi) < \epsilon$ as $\epsilon \longrightarrow 0$. This contradicts (A.34) since if it holds, such a price deviation is always available, and thus establishes Claim 2.

From Claim 2, we have

$$\lim_{\gamma \to \beta} \bar{p}(\beta) = p^*(\beta) \equiv \frac{\phi}{u'(C(\beta))C(\beta)}. \tag{A.41}$$

Combining (A.41) with (A.29) we have

$$u'(C(\beta)) = \phi \qquad \text{or} \qquad \lim_{\gamma \to \beta} C(\gamma) = u'^{-1}(\phi). \tag{A.42}$$

This establishes Proposition 7. ∎

Proof of Lemma 2: First, note that given (5.4) and (5.5) we have that $c_{k+1} - c_k$ is declining in k. Next, we prove the following claim:

Claim 3: If $q_k > 0$ and $q_{k'} > 0$ with $1 \leq k', k \leq K$ and $k' > k$. Then, $q_{k+1} > 0$.

Proof of Claim 3: Suppose not, so that $q_{k+1} \leq 0$. Since both q_k and $q_{k'}$ are strictly positive

$$u'(C)c_k = \mu k + \upsilon \tag{A.43}$$

$$u'(C)c_{k'} = \mu k' + \upsilon \tag{A.44}$$

$q_{k+1} \leq 0$ implies

$$u'(C)c_{k+1} \leq \mu(k+1) + \upsilon \tag{A.45}$$

so that

$$u'(C)[c_{k+1} - c_k] \leq \mu \tag{A.46}$$

or

$$c_{k+1} - c_k \leq \frac{\mu}{u'(C)}. \tag{A.47}$$

Since $c_{k+1} - c_k$ is declining in k, we have that

$$c_{k'} - c_k < \frac{\mu}{u'(C)}[k' - k], \tag{A.48}$$

or

$$u'(C)c_{k'} < \mu k' + \upsilon. \tag{A.49}$$

Given (5.6), (A.49) contradicts $q_{k'} > 0$. This establishes Claim 3.

Having established Claim 3, note that (A.43) together with a strict equality in (A.47) imply (A.49) for any $k' > k+1$. This establishes the lemma. ∎

Proof of Proposition 8: Suppose that an SME exists for an economy with $\mu > 0$ and $q_1 = 0$. From Proposition 1 we know that with $q_1 = 0$, the distribution of transactions prices is given by (3.9). In this case, given (5.4) and (5.5) we know that $c_k = c_1$ for all $k > 1$. With $\mu > 0$, (5.6) thus implies $q_k = 0$ for all $k > 1$. (5.3) then implies $q_1 = 1$, a contradiction. ∎

Given Lemma 2, Proposition 8 directly implies that if $\mu > 0$, then in any SME with non-sequential search, strictly positive measures of buyers observe one and two prices only.

Proof of Proposition 9: Fix $Q \in (0, 1)$. By Proposition 3, part iii, there exists an SME for an economy with noisy search in which the fraction of buyers observing a single price is given by Q. Moreover, this economy has a distribution of posted prices given by (3.16) and (3.17) with $q = Q$ and a distribution of transactions prices given by (2.2) with $q_1 = Q$ and $q_2 = 1 - Q$. Given these price distributions, the quantities purchased by buyers observing one and two prices are given by (5.4) for $k = 1, 2$. Aggregate consumption in this equilibrium, $C(Q)$, may be written

$$C(Q) = Qc_1(Q) + (1-Q)c_2(Q) > 0 \qquad \text{where} \qquad c_2(Q) > c_1(Q). \tag{A.50}$$

From (A.50) we know that $u'(C(Q))[c_2(Q) - c_1(Q)]$ is strictly positive and finite. Thus we may choose search cost $\mu > 0$ such that

$$u'\left(C(Q)\right)\left[c_2(Q) - c_1(Q)\right] = \mu. \tag{A.51}$$

Combining (A.50), (A.51), (5.8) and (5.9) we have $\Phi_2(Q) < \mu < \Phi_1(Q)$ so that (5.10) may be written

$$q^*(Q) = \frac{1}{c_1(Q) - c_2(Q)}\left[u'^{-1}\left(\frac{\mu}{c_2(Q) - c_1(Q)}\right) - c_2(Q)\right]. \tag{5.10}$$

Using (A.51) to substitute for $\mu/[c_2(Q) - c_1(Q)]$ in (5.10) we have $q^*(Q) = Q$. That is, with μ chosen to satisfy (A.51) no household may gain by choosing a measure of its buyers to observe a single price different from Q. Thus, with μ chosen to satisfy (A.51), there exists an SME for the economy with endogenous non-sequential search. ∎

Appendix B: Numerical Algorithm

1. Set an initial level for "C" equal (or close) to the unconstrained optimum $\bar{C} = u'^{-1}(\phi)$. For the examples we consider this is given by $C_0 = \phi^{-\frac{1}{\alpha}} - \epsilon$, where a positive ϵ may be chosen to speed up computations in some cases.

2. For this fixed level of C, compute $\Omega(C_0)$, $\bar{p}(C_0)$, $\underline{p}(C_0)$ using (3.8) and (3.16), and a density of transactions prices, $J(p;C_0)$, using (3.17)-(3.19).

3. Let

$$\tilde{C} = \int_{\underline{p}(C_0)}^{\bar{p}(C_0)} \frac{1}{p} dJ(p;C_0) \tag{B.1}$$

   and check $|\tilde{C} - C_0|$.

   i. If $|\tilde{C} - C_0| < \xi$ for ξ small, stop and let the SME be characterized by transactions price distribution $J(p;\tilde{C})$ and consumption level $\tilde{C}$.

   ii. If $|\tilde{C} - C_0| > \xi$, let $C_1 = (\tilde{C} + C_0)/2$, return to step 2. and continue.

We make no claims for the efficiency of this algorithm. It does, however, converge quickly to an SME in all cases that we consider. With speed of convergence not much of a problem, we compute equilibria of the economy with non-sequential search by using this algorithm to map out $q^*(Q)$ directly (applying (5.4) and (5.10)).

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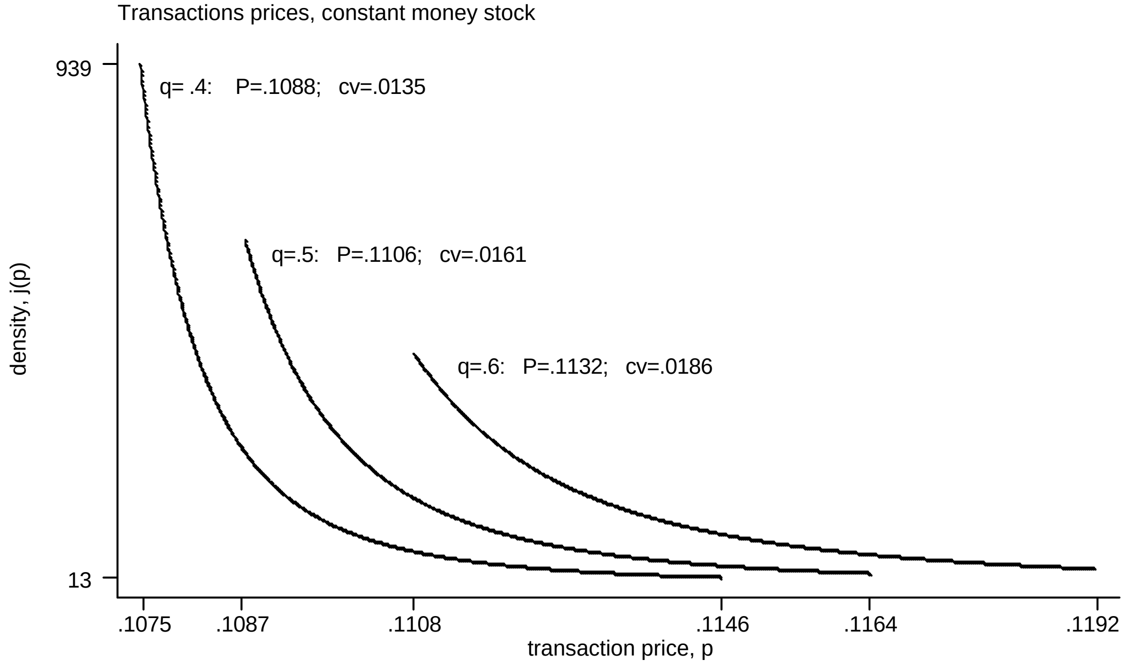

Transactions prices, constant money stock
939
13
density, j(p)
q= .4:   P=.1088;   cv=.0135
q=.5:   P=.1106;   cv=.0161
q=.6:   P=.1132;   cv=.0186
.1075
.1087
.1108
.1146
.1164
.1192
transaction price, p

Figure 1

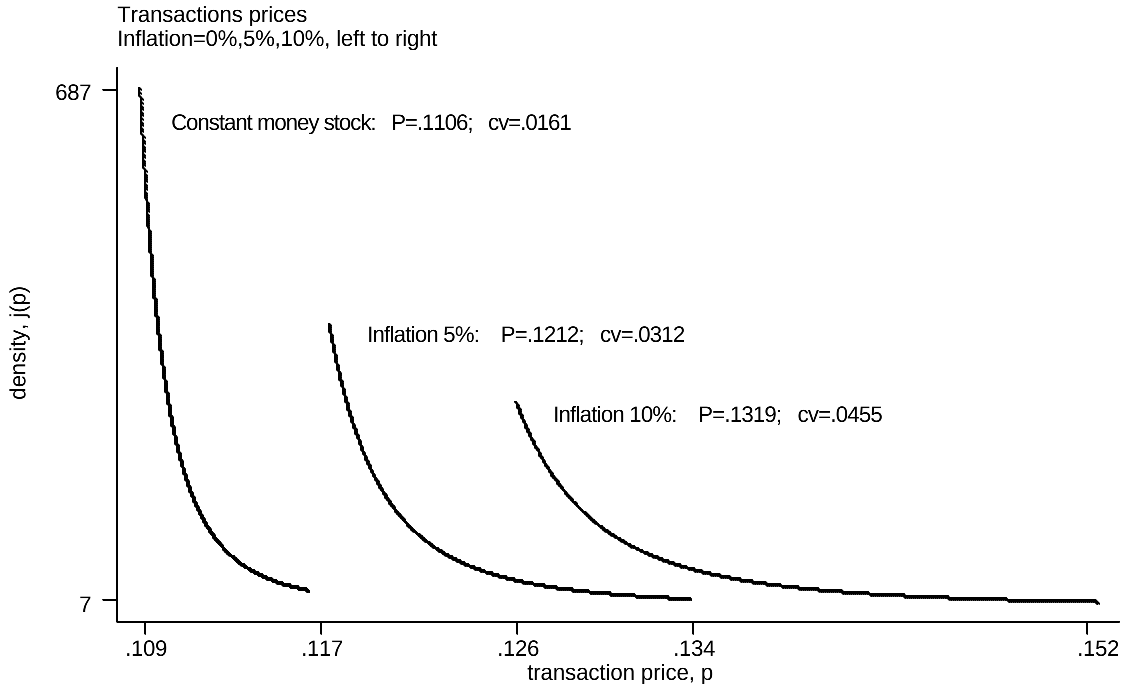

Transactions prices
Inflation=0%,5%,10%, left to right
687
7
density, j(p)
Constant money stock: P=.1106; cv=.0161
Inflation 5%: P=.1212; cv=.0312
Inflation 10%: P=.1319; cv=.0455
.109
.117
.126
.134
.152
transaction price, p

Figure 2

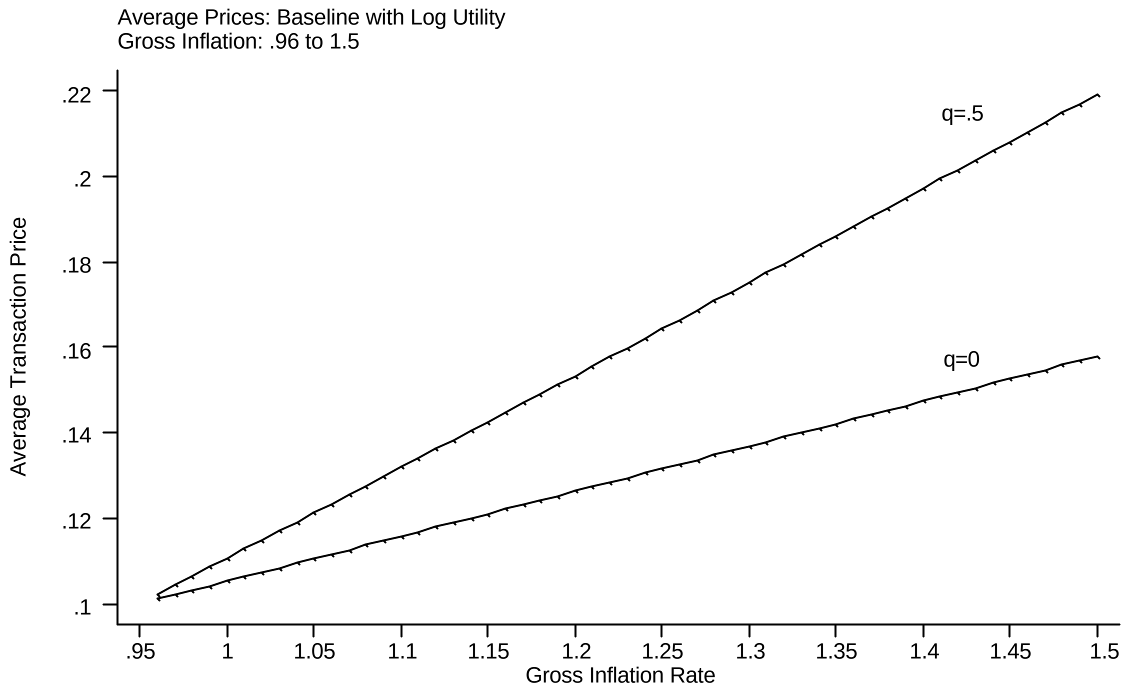

Average Prices: Baseline with Log Utility
Gross Inflation: .96 to 1.5
q=.5
q=0
Average Transaction Price
.22
.2
.18
.16
.14
.12
.1
.95
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
Gross Inflation Rate

Figure 3

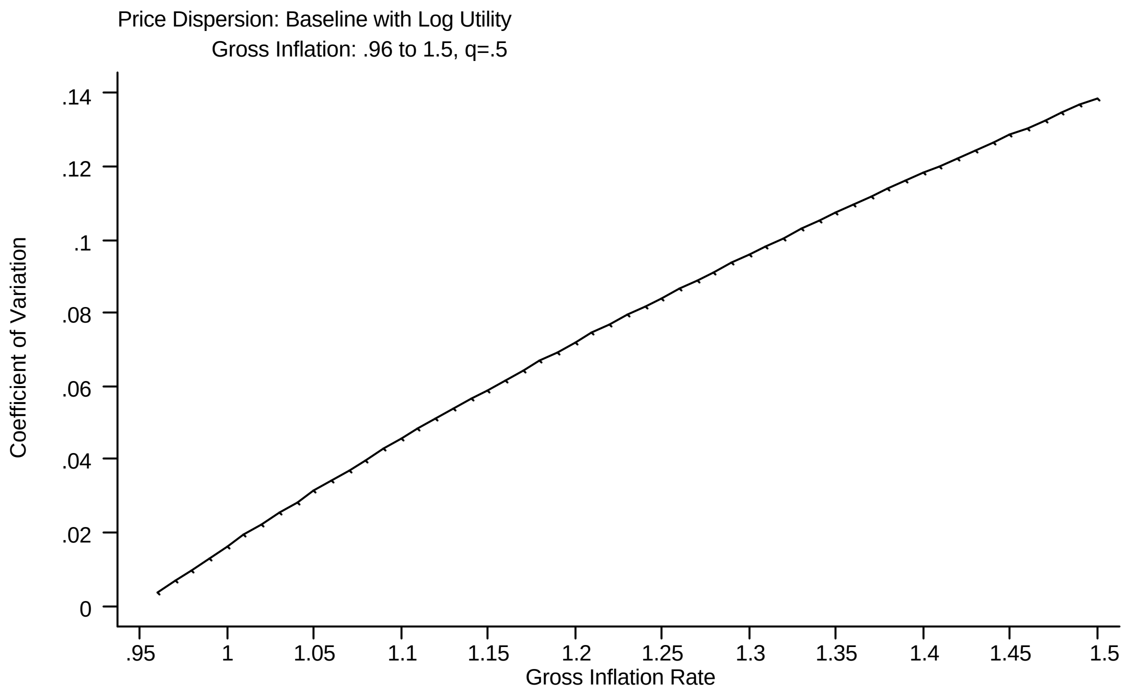

Price Dispersion: Baseline with Log Utility
Gross Inflation: .96 to 1.5, q=.5
Coefficient of Variation
.14
.12
.1
.08
.06
.04
.02
0
.95
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
Gross Inflation Rate

Figure 4

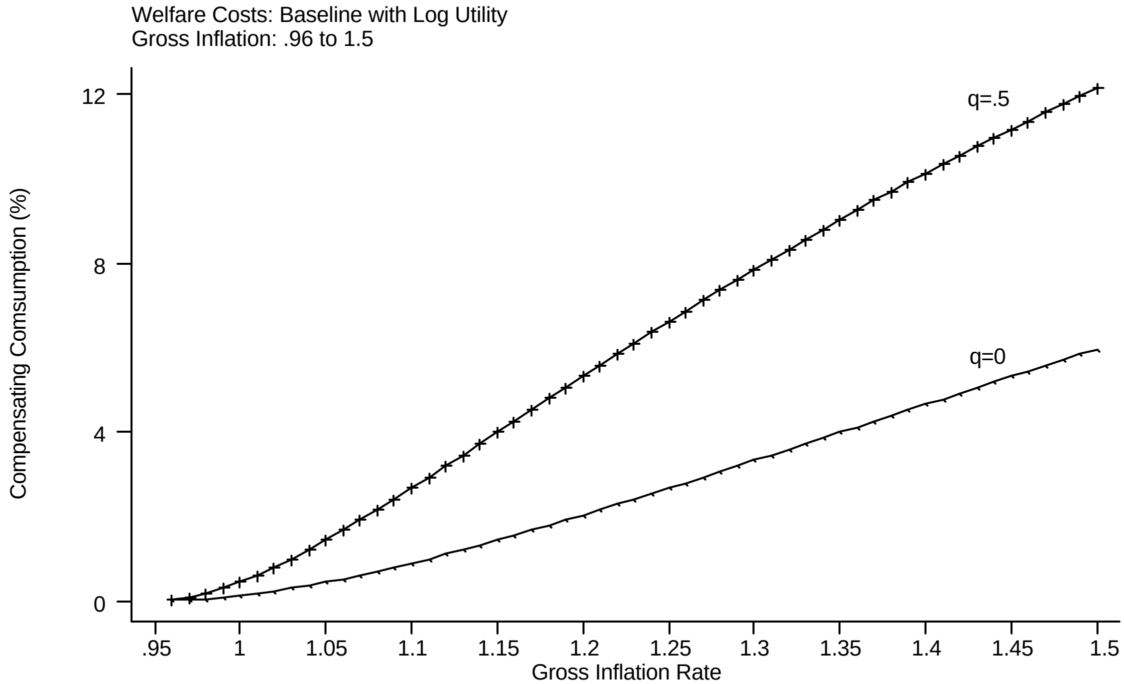
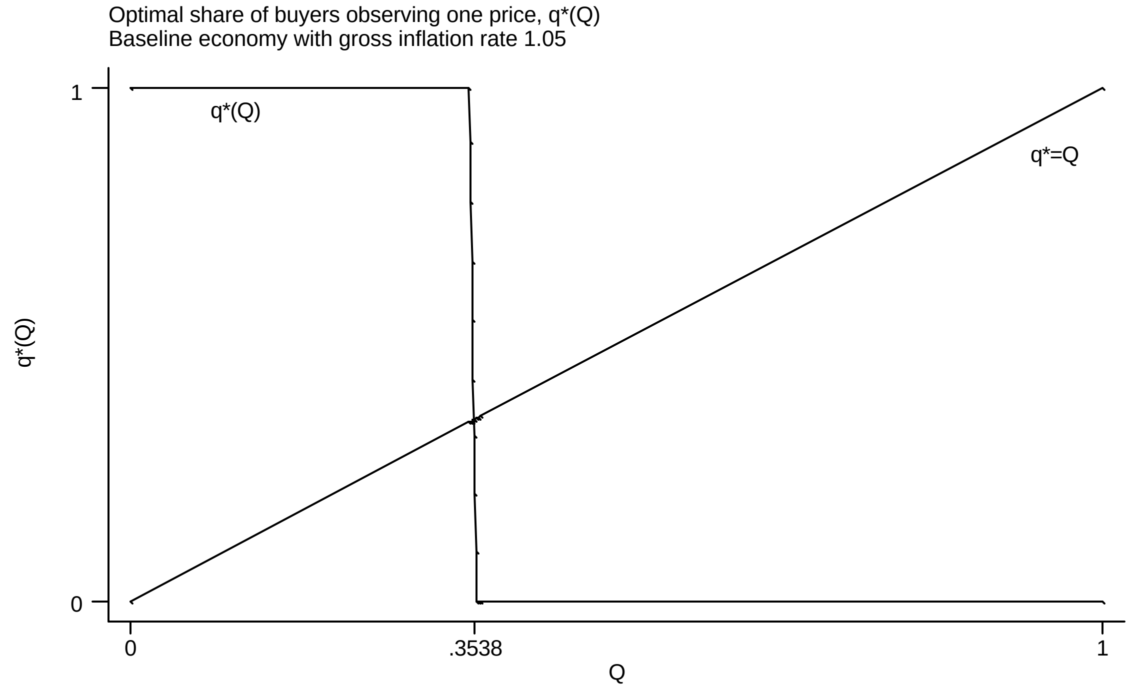
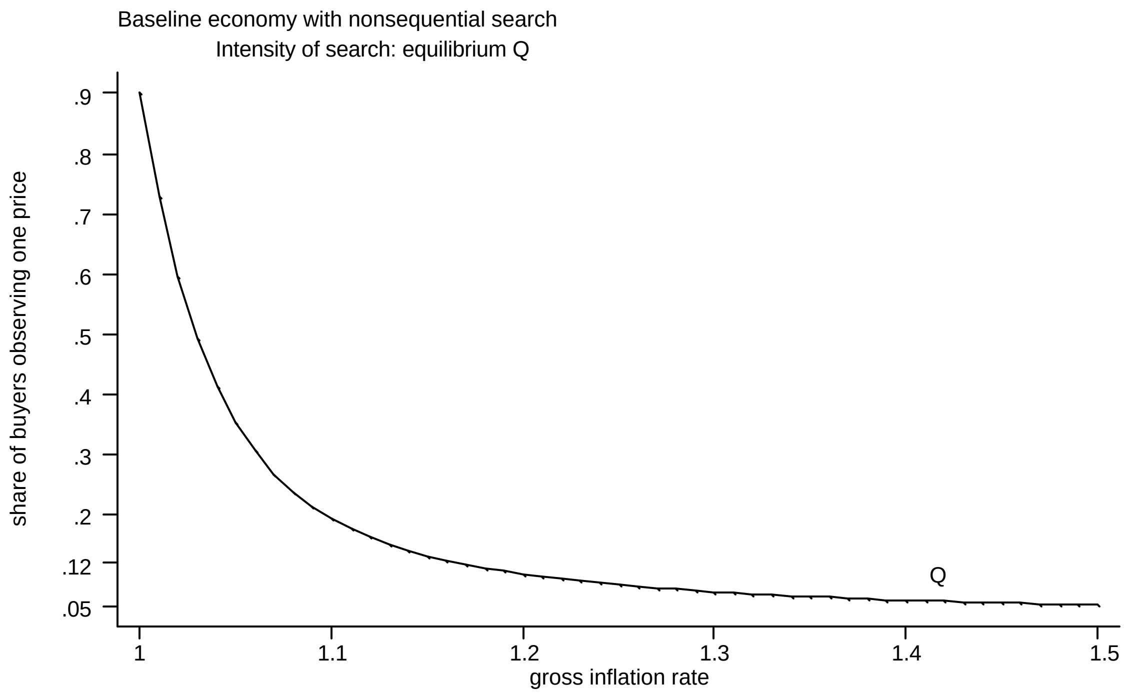

Welfare Costs: Baseline with Log Utility
Gross Inflation: .96 to 1.5
Compensating Comsumption (%)
12
8
4
0
q=.5
q=0
.95
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
Gross Inflation Rate

Figure 5



Optimal share of buyers observing one price, q*(Q)
Baseline economy with gross inflation rate 1.05
1
0
q*(Q)
q*=Q
q*(Q)
0
.3538
1
Q

Figure 6



Baseline economy with nonsequential search
Intensity of search: equilibrium Q
share of buyers observing one price
.9
.8
.7
.6
.5
.4
.3
.2
.12
.05
Q
1
1.1
1.2
1.3
1.4
1.5
gross inflation rate

Figure 7

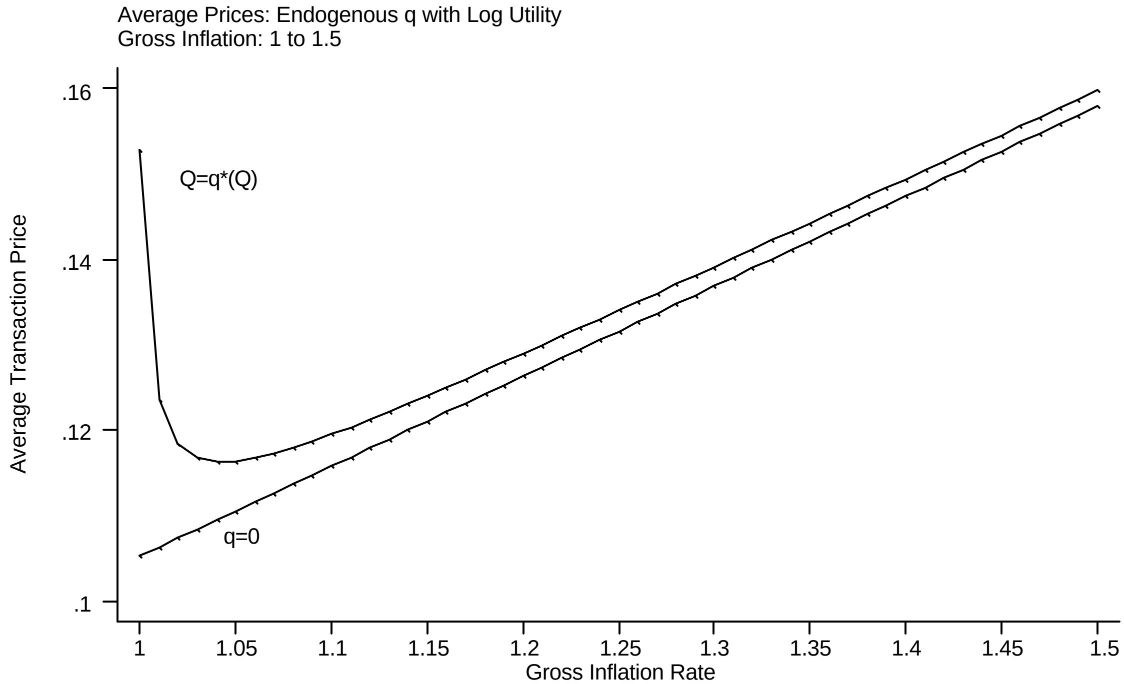

Average Prices: Endogenous q with Log Utility
Gross Inflation: 1 to 1.5
Average Transaction Price
.16
.14
.12
.1
Q=q*(Q)
q=0
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
Gross Inflation Rate

Figure 8

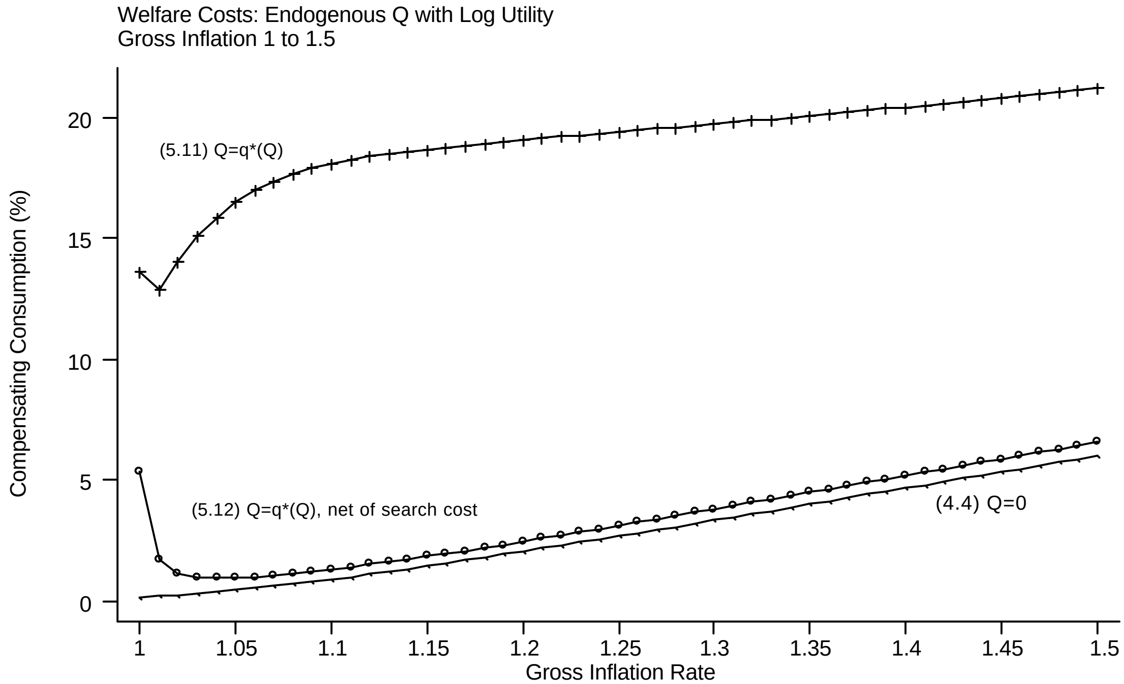

Welfare Costs: Endogenous Q with Log Utility
Gross Inflation 1 to 1.5
Compensating Consumption (%)
(5.11) Q=q*(Q)
(5.12) Q=q*(Q), net of search cost
(4.4) Q=0
Gross Inflation Rate
0
5
10
15
20
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5

Figure 9