Price-Posting, Price Dispersion, and Inflation in a Random Matching Model[1]

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Abstract

Linkages among the dispersion of prices, monetary growth, and inflation are examined in random matching monetary model. The model features price posting by identical sellers and noisy search by *ex ante* identical buyers among the posted prices. It is shown that if the probability that a buyer observes only one price is strictly between zero and one, then the distribution of real prices is necessarily non-degenerate and continuous in a stationary symmetric monetary equilibrium. In this environment, money creation results in constant inflation of nominal prices and an increase in both the dispersion and average level of real prices or purchasing power. Moreover, inflation lowers consumption by more in equilibria with price dispersion than in comparable cases where the price distribution is concentrated at the competitive price.

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1. Introduction

In this paper we develop a model to study linkages among the distribution of prices, monetary growth, and inflation in equilibrium. The model features price posting by identical sellers and noisy search by *ex ante* identical buyers among the posted prices. We find that if some buyers observe only one price but others observe strictly more than one, then in stationary symmetric equilbria with valued fiat money the distribution of prices is necessarily non-degenerate and continuous. We also find both price dispersion and the average real price level are increasing in the inflation rate in equilibria of this type. Inflation lowers welfare through the inflation tax, and this effect is larger with price dispersion than it would be in a competitive equilibrium.

Our interest in price-posting equilibria with dispersed fiat money prices stems from two sources. First, price-posting equilibria have proved somewhat difficult to square with the existence of valued fiat money in search environments. In a non-monetary model, Diamond (1971) showed that in an environment with identical buyers and sellers and in which sellers post prices, the unique equilibrium price distribution is degenerate at the buyers' reservation price. That is, sellers set prices to extract all gains from trade. In an environment with fiat money, however, this type of equilibrium typically cannot exist. Sales of goods for fiat money involve a producer or seller incurring costs in the present in exchange for money which can only be used in the future. If at that future time the buyer will realize no surplus from exchange, then he/she has no incentive in the present to exchange something valuable for fiat money. In a monetary economy with discounting this problem arises from the friction that gives rise to fiat money and does not require an explicit search cost. Only if there is a possibility that the agent will be able to purchase goods in the future at a price below his/her reservation price may fiat money have value in equilibrium.

Soller-Curtis and Wright (2000) construct a model of price-posting and exchange among *ex ante* identical agents in which fiat money can have value. In their environment, consumers receive preference shocks and are thus heterogeneous *ex post*. Sellers cannot extract all gains from trade from all buyers as doing so would entail pricing above some buyers' reservation prices. In this model, if a stationary monetary equilibrium exists, then there is an equilibrium with price dispersion. In this equilibrium, however, exchange will take place only at *two* different prices. With heterogeneity among buyers induced by random preference shocks, the reservation prices of *ex post* different types of buyers are proportional to each other and sellers cannot be indifferent between more than two prices. In addition, for some parameter values this model also has a single price equilibrium coexisting with the price dispersion equilibrium.

In this paper, we construct a model in which the distribution of fiat money prices in equilibrium will necessarily be non-degenerate and continuous. We do this by embedding the price posting environment of Burdett and Judd (1983) in a random matching monetary model similar to those studied by Shi (1999) and Head and Shi (2000). The economy is populated by identical households comprised of measures of identical *sellers* who post fiat money prices at which they are willing to produce and sell consumption goods, and *ex ante* identical *buyers* who hold fiat money, observe a random number of price quotes posted by sellers, and may choose to buy at the lowest price they observe. As in the model of Burdett and Judd, whether or not there is price dispersion depends on the probability that a representative buyer observes a single price. The *ex post* heterogeneity of buyers with regard to the number of price quotes they observe is the key factor generating price dispersion in equilibrium. In the model, if this probability is strictly between zero and one, then the distribution of prices in equilibrium is necessarily non-degenerate and continuous.

Our second source of motivation for this study is the literature on inflation and uncertainty about the price level. If inflation contributes to price dispersion, then it is possible that uncertainty about the return an individual agent will receive on his/her money holdings may contribute to the overall welfare costs of inflation. Several authors (for example, Sheshinski and Weiss (1977), Caplin and Spulber (1985, 1987), Benabou (1988) and Diamond (1993)) have modeled dispersion in real prices as stemming from menu costs. The Burdett-Judd mechanism employed here generates price dispersion in equilibrium without requiring assumptions such as menu costs or exogenously imposed nominal rigidities. Also, this framework relies on search frictions which naturally give rise to a role for fiat money without additional assumptions.

In our model inflation raises real prices and lowers welfare through the inflation tax. Money creation which erodes the value of fiat money over time affects the incentives of sellers to acquire fiat money and buyers to spend fiat money in the current period. We find that in equilibria with price dispersion, both the degree of price dispersion and the average real price level are increasing in the rate of inflation. Moreover, in response to an increase in the rate of inflation, the average transaction price rises and household consumption falls by more than they would in an environment in which prices are concentrated at the competitive level. Thus price dispersion of the type studied here raises the welfare costs of inflation.

This version of the paper is preliminary. The environment is presented in Section 2. A class of stationary symmetric monetary equilibrium is defined in Section 3. Also in this section existence of equilibria with price dispersion is established and equilibria are partially characterized. Section 4 considers the effects of inflation in the stationary symmetric monetary equilibrium with the focus in this version being on a numerical example. Section 5 concludes, and proofs of all propositions are contained in the appendix.

2. The Economy

Time is discrete and there is no aggregate uncertainty. Similar to the environments studied in Shi (1999) and Head and Shi (2000), the economy is comprised of a large number (measure H) of $H \geq 3$ different types of household. A type $h \in \{0, \ldots, H\}$ household is distinguished by its ability to produce a non-storable consumption good of type h and the fact that it derives utility only from consumption of the type $h+1$ good, modulo H. Each household in turn is comprised of large numbers (*i.e.* unit measures) of "buyers" and "sellers". Household members do not have independent preferences. Rather, they share equally in the utility generated by *household* consumption.

The a representative type h household receives utility $u(c_t)$ from consumption of c units of its preferred good at time t, where $u(\cdot)$ is strictly concave and increasing. Members of this household who are sellers can produce good h at a constant marginal cost of $\phi > 0$ units of utility per unit. The household maximizes the discounted sum of utility from consumption minus production costs over an infinite horizon:

$$U = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - x_t \right], \qquad \beta \in (0,1) \tag{2.1}$$

Here β is a discount factor and x_t represents total utility costs of production incurred by the household's sellers.

Households of a given type are indistinguishable and thus cannot be relocated in the future following an exchange. As a result, all exchanges between members of different households must be *quid pro quo*. Since a type h household produces good h and consumes good $h+1$, a double coincidence of wants between members of any two households is impossible. Exchange is facilitated by the existence at time t of $M_t H$ units of perfectly storable and intrinsically worthless *fiat money* at time t. A type h household may acquire some of this fiat money by having its producers sell output to a buyers of type $h-1$ households. This money may then be exchanged for type $h+1$ consumption good by the household's buyers in a future period.

Let the *per household* stock of fiat money be given by, M_t. In the initial period ($t = 0$) households of all types are endowed with M_0 units of this money. At the beginning of each subsequent period (*i.e.* for $t \geq 1$), they receive a lump-sum transfer, $(\gamma - 1)M_{t-1}$, of new units of fiat money from a government that has no other purpose than to increase the stock of money at gross rate $\gamma \geq 1$.

At the beginning of period t a representative type h household (for any h) has post-transfer money holdings, m_t (in period 0, $m_0 = M_0$ for all households). These money holdings are divided equally among the household's unit measure of buyers. At this point the household's buyers and sellers split up for a "trading session". Buyers receive a random number of price quotes posted by type $h+1$ households. Following household instructions, the buyers decide whether and how much to buy at the lowest price quote that they receive. Meanwhile, each of the household's sellers posts a price at which it is willing to sell consumption good to a buyer from a type $h-1$ household. It then sells to all households that receive and accept its price offer. At the end of the period, buyers and sellers reconvene. The consumption good purchases of the buyers, and the sales receipts (in units of fiat money) are pooled. The household consumes the goods purchases and carries its sales receipts into the next period, where they are augmented by transfer $(\gamma-1)M_t$ to become m_{t+1}.

The mechanism by which sellers post prices and buyers choose among the price quotes they receive follows the "noisy sequential search" formulation of Burdett and Judd (1983). Each buyer receives a random number of price quotations from prospective sellers. Let q_k, $k = 1, \ldots, \infty$, denote the probability that a randomly chosen buyer receives k price quotations, (alternatively, q_k is the fraction of the household's B buyers who receive k quotations). Denote the distribution of prices from which these quotations are drawn (*i.e.* the distribution of prices posted by sellers from type $h+1$ households at time t) be denoted $F_t(p_t)$. Let the distribution of the *lowest* price quote received by buyers at time t be denoted $J_t(p_t)$, where

$$J_t(p_t) = \sum_{k=1}^{\infty} q_k \left[1 - [1 - F_t(p_t)]^k\right] \qquad \forall p_t \tag{2.2}$$

If each buyer chooses to purchase at the lowest price quote it receives, and spends $\hat{m}_t(p_t)$ conditional on the price paid, then expected purchases of consumption good for a randomly selected buyer will be given by

$$c_{it} = \int_{p_t} \frac{\hat{m}_{it}(p_t)}{p_t} dJ_t(p_t) \qquad i \in [0,1], \tag{2.3}$$

Each buyer is constrained to spend only the money distributed to them at the beginning of the period by the household:

$$\hat{m}_{it}(p_t) \le m_t \qquad \forall i, p_t. \tag{2.4}$$

As there is no aggregate uncertainty, actual household consumption equals expected:

$$c_t = \int_i \int_{p_t} \frac{\hat{m}_{it}(p_t)}{p_t} dJ_t(p_t) di = \int_{p_t} \frac{\hat{m}_t(p_t)}{p_t} dJ_t(p_t). \tag{2.5}$$

where the second inequality in (2.5) assumes that all buyers, being identical, behave identically if faced with the same lowest price (*i.e.* $\hat{m}_{it}(p_t) = \hat{m}_t(p_t)$ for all p_t).

Each seller is directed by the household to post a particular price, p. If a buyer from a type $h-1$ household chooses to purchase at this price, then the seller incurs ϕ for each unit of good produced. Thus, the expected utility cost incurred by a seller who posts p is given by

$$x_i(p_t) = \phi \frac{\hat{M}_t(p_t)}{p_t} \sum_{k=1}^{\infty} q_k k \left[1 - F_t(p_t)\right]^{k-1} \qquad i \in [0,1], \tag{2.6}$$

where $\hat{M}$ is the quantity of money spent by the type $h-1$ buyer and here $F_t(p_t)$ is used to denote the distribution of prices posted by other *type* h households at time t. If the household instructs all of its sellers to post the same price, then total utility cost is given by $x(p_t)$. If the household specifies a distribution $\hat{F}_t(p_t)$ from which sellers choose prices to post, then the assumption of no aggregate uncertainty implies

$$x_t = \int_{p_t} x(p_t) d\hat{F}_t(p_t). \tag{2.7}$$

For a representative household of type h, the state at time t is given by its individual money holdings, m_t, and the aggregate money stock per household, M_t. The following Bellman equation represents the household's dynamic optimization problem.

$$v_t(m_t, M_t) = \max_{m_{t+1}, \hat{m}_t(p_t), \hat{F}_t(p_t)} \left\{ u(c_t) - x_t + \beta v_{t+1}(m_{t+1}, M_{t+1}) \right\} \tag{2.8}$$

subject to

(2.4) (2.5) (2.6) (2.7)

$$m_{t+1} = m_t - \int_{p_t} \hat{m}_t(p_t) dJ_t(p_t) + \int_{p_t} \hat{M}_t(p_t) \sum_{k=1}^{\infty} q_k k \left[1 - F_t(p_t)\right]^{k-1} d\hat{F}_t(p_t) + (\gamma - 1) M_t \tag{2.9}$$

$$M_{t+1} = \gamma M_t, \tag{2.10}$$

where the household takes the actions of other households, represented here by the sequences of functions, $\hat{M}_t(p_t)$, and distributions, $F_t(p_t)$, as given.

Prior to the trading session in period t, the household must specify rules by which its buyers and sellers act. Considering the buyers first, the household specifies a function, $\hat{m}_t(p_t)$, which tells the buyer how much of its currency holdings to exchange for consumption good if the lowest price quote it receives is p_t. The household gain to exchanging at $\hat{m}_t$ units of currency for consumption at p_t is given by the marginal utility of current household consumption times the quantity of consumption good purchased, $\hat{m}_t/p_t$. The household cost of this exchange is the marginal value of money *next period* times the number of currency units given up, $\hat{m}_t$. Let ω_t denote the value to the household of a unit of currency spent by a buyer or acquired by a seller during the trading session of the current period. From the household Bellman equation, we have

$$\omega_t = \beta \frac{\partial v_{t+1}}{\partial m_{t+1}}. \tag{2.11}$$

For each p_t then, the optimal spending rule is characterized in the following lemma:

Lemma 1: *The optimal spending rule, $\hat{m}_t(p_t)$, has the following "reservation price" form:*

$$\hat{m}_t(p_t) = \begin{cases} m_t & p_t \leq \frac{u'(c_t)}{\omega_t} \\ 0 & p_t > \frac{u'(c_t)}{\omega_t}. \end{cases} \tag{2.12}$$

Proof: For all p_t, the optimal spending rule satisfies

$$\hat{m}_t(p_t) \in \underset{\hat{m}_t}{\operatorname{argmax}} \left[\frac{u'(c_t)}{p_t} - \omega_t \right] \hat{m}_t \tag{2.13}$$

subject to

$$\hat{m}_t \leq m_t. \tag{2.14}$$

Since neither household consumption, c_t, nor the household's marginal valuation of money, ω_t, is affected by the spending of a particular buyer in a match, the household will instruct its buyers to spend their entire money holdings as long as the bracketed term in (2.12) is positive. That is, the household reservation price, $\bar{p}_t$, is given by $u'(c_t)/\omega_t$. If the lowest price that a buyer observes at time t is *greater* than $\bar{p}_t$, then the buyer returns to the household with its money holdings unspent and the household carries the money into period $t+1$.

The household also specifies a distribution of prices from which its sellers posted prices are drawn. The expected return from having a seller post a particular price at time t depends on the distribution of prices posted by firms from other households of its type, $F_t(p_t)$, and the strategies of its prospective buyers, $\hat{M}_t(p_t)$. Let $\tilde{p}_t$ denote the household's belief regarding the reservation price of its potential customers. The household will instruct no seller to post $p_t > \tilde{p}_t$, as any such seller is expected to make no sales, generating an expected return to the household of zero. The expected return to the household from having a seller post price $p_t \leq \tilde{p}$ is given by

$$r(p_t) = \left[\omega_t \hat{M}_t(p_t) - \phi \frac{\hat{M}_t(p_t)}{p_t} \right] \sum_{k=1}^{\infty} q_k k \left[1 - F_t(p_t)\right]^{k-1}. \tag{2.15}$$

Using the potential buyers' optimal spending rules, (2.15) becomes

$$r(p_t) = \left[\omega_t - \frac{\phi}{p_t} \right] M_t \sum_{k=1}^{\infty} q_k k \left[1 - F_t(p_t)\right]^{k-1}. \tag{2.16}$$

The household will derive the same expected return from having its sellers post any price greater than or equal to $p^* = \phi/\omega_t$ such that $p_t \in \underset{p_t}{\operatorname{argmax}}\ r(p_t)$. From (2.16) it is clear that the return to posting a price lower than p^* is negative, and thus the household will instruct no seller to do so.

3. Equilibrium

We restrict attention to a equilibria that are stationary and symmetric. First, we require that all households behave symmetrically, and that they all have a common marginal valuation of money, Ω_t, and equal consumption, C_t in each period. Second, we require that equilibria be stationary in the sense that consumption remains constant over time (*i.e.* $C_t = C$ for all t). Note that throughout, capital letters (*e.g*, C_t, X_t, Ω_t, etc.) will be used to distinguish *per household* quantities from their counterparts for an individual household (c_t, x_t, ω_t, etc.).

Noting that in a symmetric equilibrium all households have identical reservation prices ($\bar{p}_t = \tilde{p}_t$), the common optimal spending rule (2.12) together with the definition of household consumption (2.5) gives rise to a version of the simple quantity equation

$$C = M_t \int_p \frac{1}{p_t} dJ_t(p_t). \tag{3.1}$$

Thus, in a stationary monetary equilibrium, the average nominal price must grow at the gross rate of money creation, γ. We conjecture that such an equilibrium is characterized by a stationary distribution of real offer prices, F, constructed from the sequence of nominal price distributions as follows:

$$F(p) \equiv F_t(p_t) \qquad \text{where} \quad p \equiv \frac{p_t}{\gamma^t} \quad \forall t. \tag{3.1}$$

Similarly, we have

$$J(p) = \sum_{k=1}^{\infty} q_k \left[1 - [1 - F(p)]^k\right] \qquad \forall p = \frac{p_t}{\gamma^t}, \quad \forall t. \tag{3.2}$$

Defining $\hat{F}(p) \equiv \hat{F}_t(p_t)$, $\hat{m}(p) \equiv \hat{m}_t(p_t)/\gamma^t$, and $\hat{M}(p) \equiv \hat{M}_t(p_t)/\gamma^t$, writing individual and per household money holdings in the current period as $m \equiv m_t/\gamma^t$ and $M \equiv M_t/\gamma^t$ respectively, and using a superscript "$\,'$", we may eliminate the time subscripts from the household Bellman equation.

A symmetric stationary monetary equilibrium (SSME) is defined as a collection of functions $v(m, M)$, $m'(m, M)$, $\hat{m}(m, M)$, $\hat{M}(M)$, and price distributions $\hat{F}(m, M)$ and $F(M)$, (where the argument (p) of the functions $\hat{m}$, $\hat{M}$, $\hat{F}$, and F has been suppressed) such that

1. Given the price distribution, F, and strategies of other households' buyers, $\hat{M}$, the value function $v(m, M)$ satisfies the Bellman equation given by (2.4),(2.5), and (2.7)-(2.10), with $m'(m, M)$, $\hat{m}(m, M)$, and distribution $\hat{F}(m, M)$ the associated policy functions.

2. Individual choices equal their per household counterparts: $m = M$, $\hat{m}(m, M) = \hat{M}(M)$, and $\hat{F}(m.M) = F(M)$.

3. Consumption is constant over time: $C_t = C$.

We now turn to establishing existence of and characterizing a SSME. The key difference between the monetary model studied here and the model of Burdett and Judd (1983) is that whereas in the latter returns to sellers (firms) and buyers (consumers) conditional on transacting at a particular price are exogenous, here they depend on households' marginal valuation of money, Ω, a variable determined in equilibrium. To begin with, however, conditional on households having a well-defined reservation price, $\bar{p}$, and a probability distribution, $\{q_k\}_{k=1}^{\infty}$, over the number of price quotes received by each buyer, the potential price distributions in any SSME may be restricted. The following proposition corresponds to and largely follows from Theorem 4 of Burdett and Judd (1983) and characterizes the possible distributions of posted prices in a SSME.

Proposition 1: *Given* $\{q_k\}_{k=1}^{\infty}$ and a common, finite, buyers' reservation price $\bar{p}$,

i. *if* $q_1 = 1$, *then the only possible price distribution in a SSME is concentrated at the reservation price,*

$$\bar{p} = \frac{\bar{p}_t}{\gamma^t} = \frac{u'(C)}{\Omega} \qquad \forall t, \tag{3.3}$$

where $\Omega = \gamma^t \Omega_t$ for all t in an SSME.

ii. *If* $q_1 = 0$, *then the only possible price distribution in a SSME is concentrated at the producer's minimum price,*

$$p^* = \frac{p_t^*}{\gamma^t} = \frac{\phi}{\Omega}, \qquad \forall t. \tag{3.4}$$

iii. *If* $q_1 \in (0, 1)$, *then, given a reservation price* $\bar{p}$ *there is a unique dispersed price distribution that may be a component of a SSME.*

Note that Proposition 1 does not establish existence of an SSME. Rather it establishes that depending on q_1 there are restrictions on the possible distributions of offered prices, $F(p)$, that may arise as a component of a SSME. If all buyers observe a single price ($q_1 = 1$), then the proposition establishes that since the only price that maximizes sellers expected return is the buyers' common reservation price, the only possible price distribution in equilibrium is concentrated at this price. This is the well-known result due to Diamond (1971). If all buyers receive *more* than one price quote, the return to offering a slightly lower price is always positive if the posted price is greater than p^*, and so the only possible price distribution is concentrated at this price. The only remaining possibility is that some buyers receive one price quote, and others receive more than one. In this case the proposition establishes that contingent on the reservation price, $\bar{p}$ (and hence the marginal valuation of money, Ω), there is a unique continuous distribution of real offered prices, $F(p)$, that may be a component of an SSME.

We now return to the household optimization problem given by (2.4)—(2.10). In addition to (2.11) we have the following first-order condition,

$$u'(c_t)\frac{1}{p_t}j(p_t) - \lambda_t(p_t) - \omega_t = 0 \qquad \forall p_t, t, \tag{3.5}$$

where $\lambda_t(p_t)$ is a Lagrange multiplier on the buyers' exchange constraint, (2.4). We also have the envelope condition,

$$\frac{\partial v_t}{\partial m_t} = \int \lambda_t(p_t) dJ_t(p_t) + \omega_t, \qquad \forall t. \tag{3.6}$$

Combining (2.11), (3.5), and (3.6), in an SSME we have

$$\Omega_t = \beta u'(C) \int \frac{1}{p_{t+1}} dJ_{t+1}(p_{t+1}) = \frac{1}{\gamma^{t+1}} \beta u'(C) \int \frac{1}{p} dJ(p). \tag{3.7}$$

with $\Omega = \gamma^t \Omega t$, (3.7) becomes

$$\Omega = \frac{\beta}{\gamma} u'(C) \int \frac{1}{p} dJ(p). \tag{3.8}$$

Normalizing $M_0 = 1$, (3.8) becomes

$$\Omega = \frac{\beta}{\gamma} u'(C) C. \tag{3.9}$$

We now have two propositions (which are proved in the appendix) that establish existence of and partially characterize the SSME.

Proposition 2: *If $\gamma = \beta$, then there is no SSME with a dispersed distribution of prices.*

As in Shi (1999), if $\gamma < \beta$ there can be no stationary equilibrium in which money has value, and with $\gamma = \beta$ the equilibrium is indeterminate. In the former case households will never spend money and in the latter they are indifferent between spending it and holding it for spending in the future leading to multiple stationary monetary equilibria. From this point on, we restrict attention to cases in which $\gamma > \beta$. The following proposition collects results for these cases.

Proposition 3: *If $\gamma > \beta$, given $\{q_k\}_{k=1}^{\infty}$,*

i. *if $q_1 = 1$, then there is no SSME.*

ii. *if $q_1 = 0$, then there is a unique SSME with*

$$J(p) = F(p) = \begin{cases} 1 & p \geq p^* \\ 0 & p < p^*. \end{cases} \tag{3.10}$$

iii. *if $q_1 \in (0,1)$, then there exists an SSME with a dispersed and continuous distribution of real prices, $F(p)$, implicitly characterized by*

$$\left[\Omega - \frac{\phi}{p}\right] \sum_{k=1}^{\infty} q_k k \left[1 - F(p)\right]^{k-1} = \left[1 - \frac{\phi}{u'(C)}\right] \Omega q_1 \qquad p \in \mathcal{F}, (3.11)$$

where $\mathcal{F}$ is the support of $F(.)$, $\mathcal{F} = [\bar{p}, \underline{p}]$, with $\bar{p} = u'(C)/\Omega$ and $\underline{p}$ determined by

$$\left[\Omega - \frac{\phi}{\underline{p}}\right] \sum_{k=1}^{\infty} q_k k = \left[1 - \frac{\phi}{u'(C)}\right] \Omega q_1. \tag{3.12}$$

In case **i.**, we know from Proposition 1 that if all buyers observe only one price, the only possible distribution of prices in an SSME is concentrated at the buyers' reservation price. In this case the return to acquiring money is insufficient to induce sellers to accept it in exchange for goods at any finite price. In case **ii.** we know from Proposition 1 that if all buyers observe strictly more than one price, then the only possible distribution of prices in an SSME is concentrated at the sellers' minimum price, p^*. Proposition 3 establishes that there is a unique equilibrium in which buyers extract all surplus from trade with sellers. As in Burdett and Judd (1983), complete information is not required for this equilibrium, at which all trade takes place at the same price that arise in a "competitive" equilibrium. Even if all buyers observe only two prices, this outcome will obtain.

These results are reminiscent of random matching monetary models such as Shi (1995) and Trejos and Wright (1995) in which the terms at which money is exchanged for goods is determined by bargaining. In these models fiat money will not have value in equilibrium if sellers make take-it-or-leave-it offers to buyers. This corresponds to case **i.** of Proposition 3. Case **ii.** corresponds to the situation in bargaining models where *buyers* make take-it-or-leave-it offers to sellers. In this case, the promise of returns to the household when its buyers spend fiat money in the future induces sellers to accept money in exchange for goods.

Case **iii.** covers situation in which some buyers observe exactly one price and other observe more. In this case the unique SSME exhibits dispersion of real prices. Moreover, the distribution of prices is necessarily continuous with connected support. These results contrast with the findings of Soller-Curtis and Wright (2000) in two ways. Firstly, in their model, for some parameter values a single price equilibrium may coexist with the dispersed price equilibrium. Secondly, in the dispersed price equilibria that exist in their model, generically the equilibrium price distributions are concentrated on exactly two discrete prices.

To further describe prices in the SSME of the model studied here, it is useful to derive the densities of posted and transactions prices. Using (3.11) the former is given by

$$f(p) \equiv F'(p) = \frac{\phi}{p^2}\left[\frac{\sum_{k=1}^{\infty} q_k k\,[1-F(p)]^{k-1}}{[\Omega-\phi/p]\sum_{k=1}^{\infty} q_k k(k-1)\,[1-F(p)]^{k-2}}\right] \qquad p \in \mathcal{F}. \tag{3.13}$$

The distribution of transactions prices in equilibrium is given by (3.2), and has density

$$j(p) = \sum_{k=1}^{\infty} q_k k\,[1-F(p)]^{k-1} f(p) \qquad p \in \mathcal{F}. \tag{3.14}$$

Restricting attention to the case in which buyers observe only either one or two prices ($q_k = 0$ for $k > 2$), (3.13) becomes

$$f(p) = \frac{\phi}{p^2}\left[\frac{q+2(1-q)[1-F(p)]}{[\Omega-\phi/p]2(1-q)}\right] \qquad p \in \mathcal{F}, \tag{3.15}$$

where q is the probability of observing one price. In this case it can be easily shown that the densities of posted and transactions prices, $f(p)$ and $j(p)$, respectively, are monotonically decreasing.

Deriving the effects of changes in various parameters on the price distributions is complicated by the fact that the marginal valuation of money, Ω, affects not only the densities at each point, but also the upper and lower supports of the distributions. Ω, itself, owing to the effectively "complete insurance" inherent in the household structure, depends on the average transaction price, $P \equiv \int p dJ(p)$. In the next section, we consider the relationships among the gross rate of inflation, γ, the distribution of prices, and consumption in equilibrium.

4. Inflation

We now consider the effects of money creation on the distribution of real prices in equilibrium. In the case of $q_1 = 0$, when the distribution of prices is concentrated at the same price that would arise in a competitive equilibrium (p^*), the effects of the inflation rate are straightforward and can be summarized in the following proposition:

Proposition 4: *If $\gamma > \beta$ and $q_1 = 0$, then an increase in γ raises the price and lowers consumption in the SSME.*

Unsurprisingly, money creation effected by lump-sum transfers acts in the usual way as an inflation tax. This lowers the households' marginal valuation of money, Ω, in the SSME and raises p^*, which is given by (3.3). That is, while money is neutral it is not super-neutral. Since with $q_1 = 0$ the distribution of transactions prices is given by (3.10), household consumption is given by $C^* = 1/p^*$, and thus falls in the SSME as price rises. Period utility in the SSME is given by $u(C^*) - \phi C^*$. This also falls as inflation rises since it is an increasing function of consumption as long as $u'(C) - \phi > 0$, which is required for fiat money to be valued in a stationary equilibrium.

For cases in which $q_1 \in (0,1)$, the effects of inflation on the average price, $P = \int p dJ(p)$, and consumption, $C = \int \frac{1}{p} dJ(p)$, are more complicated. Changes in γ entirely change the distributions F and J, including both their upper and lower supports. For this reason, the effects of inflation on prices, consumption and welfare for the case of $q_1 \in (0,1)$ are examined in a numerical example. The results of this example, prompt the following conjecture, an analog to Proposition 4. for this case.

Conjecture 1: *If $\gamma > \beta$ and $q_1 \in (0,1)$, then in an SSME, an increase in the gross rate of money creation, γ,*

i. *increases the coefficient of variation of the distribution of transactions prices;*

ii. *raises the real average price and lowers consumption;.*

iii. *reduces consumption by more than it would if $q_1 = 0$, ceteris paribus.*

An Example with Logarithmic Utility

Let $u(c) = \ln c$, the discount factor $\beta = .95$ and the marginal utility cost of production $\phi = .1$. For this functional form and parameter values, consider economies with gross inflation rates ranging from -4% to 50% (*i.e.* $\gamma \in [.96, 1.5]$) and fractions of buyers observing a single price ranging from 0 to 90% (*i.e.* $q_1 \in [0, .9]$). Assume that all buyers observe either one or two prices so that $q_k = 0$ for $k > 2$, and let q and $1 - q$ denote the fractions observing one and two prices respectively.

For $q = 0$, Proposition 3 establishes that the unique SSME of this economy is a single price equilibrium with that price being the producers minimum price, p^*. For the logarithmic case this equilibrium is easily calculated and can be characterized by

$$\Omega = \frac{\beta}{\gamma}, \qquad p^* = \frac{\gamma\phi}{\beta}, \qquad C^* = \frac{\beta}{\gamma\phi}. \tag{4.1}$$

The implications of Proposition 4. are obvious here: an increase in the inflation rate lowers household's marginal valuation of money, raises the price level and lowers household consumption. This is a standard illustration of an inflation tax in which its size for a given inflation rate is determined by how much is sacrificed in the present for future use of the medium of exchange, ϕ/β.

With $q > 0$, it is not possible to derive the unique SSME analytically. It is nonetheless straightforward to compute it numerically. For cases with $\gamma \in \{.96, .97, \ldots, 1.49, .1.50\}$ and $q \in \{.1, .2, \ldots, .9\}$ we compute the SSME of the economy using the following algorithm:

1. Set an initial level for the upper support of the equilibrium price distribution,

$$\bar{p}_1 = \frac{\gamma}{\beta C^*}, \tag{4.2}$$

   where C^* is given by (4.1) and we have used the fact that $\Omega = \beta/\gamma$.

2. Compute the lower support of the distribution implied by $\bar{p}_1$, $\underline{p}_1$, using (3.12), and the density of the offer distribution using (3.15) for all $p \in [\underline{p}_1, \bar{p}_1]$. Call this density $f_1(p)$.

3. Recover the distribution of transactions prices, $j_1(p)$, implied by $f_1(p)$ from (3.14), and compute the price level, $P_1 = \int p dJ_1(p)$, and household consumption, $C_1 = \int (1/p) dH_1(1/p)$ using the density $j_1(p)$.

4. Let $\hat{p} = \frac{\gamma}{\beta C_1}$ and check $|\hat{p} - \bar{p}_1|$. If the difference is sufficiently small, stop. An approximation to the SSME is given by $j_1(p)$, P_1 and C_1. Otherwise, let $\bar{p}_2 = (\hat{p} - \bar{p}_1)/2$ and return to step 2.

We make no claims for the efficiency of this algorithm. It does, however, converge quickly to an SSME in all cases that we consider.

The effects of changes in the inflation rate and the fraction of buyers observing exactly one price quote on the distribution of real transactions prices in the SSME are illustrated in Figures 1, 2, and 3. Figure 1 summarizes the effects of these variables on the average transaction price, P, and its coefficient of variation, a measure of real price dispersion. The two panels on the left-hand side of the figure illustrate the effects of increasing the inflation rate from -4% to 50% (in 1% increments) with the share of buyers observing a single price held constant at 50%. In the upper left panel it can be seen that P rises monotonically in the inflation rate, with the relationship being very close to linear. The bottom left panel illustrates that the dispersion of real prices is also increasing in the inflation rate. Interestingly, the increase in price dispersion is greatest at low levels of inflation.

In thinking about the relationships depicted in the left-hand panels of Figure 1, it is useful to consider the densities of transactions prices for some selected cases. Figure 2 depicts transactions price densities for inflation rates of -4%, 5%, 15%, and 50% with $q = .5$. For each density, the prices labelled are the upper and lower supports, and the average price. The competitive price, p^*, that would prevail if all buyers observed two prices (*i.e.* if $q = 0$), is listed for each case as well. In all cases the densities have a similar shape, and of course all are monotonically decreasing as expected. As the rate of inflation increases *ceteris paribus*, in the SSME the distribution of real prices shifts to the right, with both the upper and lower supports increasing. The range of the distribution increases in absolute terms, and the lower support rises in greater proportion than does the competitive price, p^*. Thus, the average price rises both absolutely and relative to p^*. An increase in the inflation rate raises real prices by more than they would increase in a competitive market.

Intuition for the fact that prices rise by more in cases with price dispersion than in the competitive case with $q = 0$ is aided by considering the expression for buyers' reservation price in the logarithmic case,

$$\bar{p} = \frac{\gamma u'(C)}{\beta}. \tag{4.3}$$

As the inflation rate rises, the marginal value of a unit of currency in the SSME falls, raising $\bar{p}$ along with p^* (see (4.1)). As γ rises, however, the marginal effect of additional increases inflation rises relative to the competitive case because $u'(C)$ rises as consumption falls, whereas the marginal utility cost of production, ϕ, does not.

Figure 1 also depicts the effects of increasing the share of buyers observing a single price while holding the inflation rate constant at 5%. The upper and lower right-hand panels depict the effects on P and the coefficient of variation of prices, respectively. P is monotonically increasing in q and the relationship is strongly convex. The lower right panel shows that the coefficient of variation is also increasing in q, with the relationship appearing to be slightly concave, especially at low q's.

Figure 3 depicts densities of prices for the three cases or 10%, 50%, and 90% of buyers observing a single price. Here can be seen that an increase in q has a dramatic effect on the shape of the density. With relatively few buyers observing a single price, transactions prices are concentrated near the competitive price, $p^* = .1105$. The distribution is very skewed with the mean close to the lower support. As q increases, the range of the distribution not only becomes wider, but the mass of the distribution shifts toward the upper support, raising the average transaction price relative both to the lower support of the distribution and relative to the competitive price, p^*. The intuition for this effect is straightforward. If relatively few buyers observe only a single price, then the number of sales made decreases rapidly in the posted price. Sellers tend to post prices near the competitive price and transactions prices are clustered in this region. As the share of buyers observing a single price increases, the drop in expected sales associated with posting a higher price falls, more sellers post higher prices, and more buyers find their lowest price observed in the upper part of the posted price distribution. This raises the return to sellers of posting a relatively high price and shifts transactions prices toward the buyers' reservation price.

Figures 4 and 5 depict the effects of increases in γ and q, respectively, on consumption in the SSME. In Figure 4 the lower line (symbol '+') illustrates the change in consumption in the SSME with $q = .5$, relative to the unconstrained optimum consumption level ($u'(C) = \phi$) as the inflation rate ranges from -4% to 50%. The upper line is the *additional* consumption loss relative to the competitive case. Thus the lower line is the total loss due to the combination of the pure effect of the inflation tax in a competitive equilibrium and the elevation of real prices due to market power exhibited by price setting households. The upper line subtracts out the former and is just the effect associated with price setting and dispersion. Note that the reduction in consumption associated with price setting is substantial and that it increases most rapidly at low levels of inflation. With the inflation rate constant at 5%, the effect on consumption of increasing the share of buyers observing one price from 10% to 90% is depicted in Figure 5. Mirroring the effect of q on the average price, the consumption costs are relatively low at small percentages of buyers observing a single price. As q rises, the costs rise slowly at first, and later at a rapidly rising rate.

Overall, the results from the logarithmic example are consistent with what would be expected based on Conjecture 1. Inflation raises the average real price and lowers consumption in the SSME. Moreover, these effects increase as we move farther from the competitive equilibrium by raising the share of buyers who observe only one price.

5. Conclusion

This paper has presented a random matching monetary model in which price posting by *ex ante* identical sellers and noisy search by *ex ante* identical buyers may lead to a stationary symmetric monetary equilibrium in which the distribution of real prices is non-degenerate and continuous. These findings contrast with those of Soller-Curtis and Wright (2000) who analyze a different economy and find equilibria in which price dispersion takes only a very special form. Specifically, in their model exchange takes place at exactly two distinct prices in equilibrium.

In addition to presenting a model in which continuous price dispersion arises in equilibrium, this model analyzes the effects of money creation on the distribution of real prices and the level of per household consumption in the stationary equilibrium. When all buyers observe more than one price, there is no price dispersion in equilibrium and the effect of inflation is to lower the marginal value of money, raise the "competitive" price and lower consumption. An example illustrates that when some buyers observe only one price, the effects of inflation on real prices and consumption are larger in the resulting stationary equilibrium with price dispersion than they would be in a competitive setting.

Further work will consider the generality of the findings of the logarithmic example, and explore extensions of the basic environment. In particular, it would be interesting to examine cases in which the probabilities of observing different numbers of price quotes are endogenous, either through a sequential search specification similar to that studied by Burdett and Judd (1983) or by introducing a technology through which the household could raise the probability of its buyers observing more than one price quote at a cost. In such a setting inflation would in general affect the share of buyers observing a single price. In general this may be expected to affect the relationships among inflation, prices, and welfare which are the focus of this research.

Appendix

Proof of Proposition 1.

Note that in an SSME, the distribution of posted prices, $F(p)$, satisfies $F(p) = \hat{F}(p)$ where $\hat{F}(p)$ is chosen by a representative household so that $p \in \text{argmax}_{p\in\mathcal{F}}\, r(p)$, with $\mathcal{F}$ the support of $F(\cdot)$. Thus, an SSME is associated with a pair, $(F(\cdot), \Pi)$, where $\Pi = \max_{p\in\mathcal{F}}\, r(p)$. The proof of the proposition then follows directly from Lemmas 1 and 2 of Burdett and Judd (1983, pp.959-61) with $\bar{p}$ and p^* in our notation corresponding to $\tilde{p}$ and r, respectively, in theirs.

Proof of Proposition 2.

Suppose that $\gamma = \beta$ and there is an SSME with price dispersion. From proposition 1 we know that such an equilibrium can only occur with $q_1 \in (0, 1)$ and that the distribution of posted prices, $F(\cdot)$ is non-degenerate and continuous with connected support. Let $J(\cdot)$ be the distribution of transactions prices associated with $F(\cdot)$ according to (2.2). Using (3.8), we have

$$\Omega = u'(C) \int \frac{1}{p} dJ(p) \tag{A.1}$$

With $\bar{p} = u'/\Omega$, (A.1) is

$$\frac{1}{\bar{p}} = \int \frac{1}{p} dJ(p). \tag{A.2}$$

With $\bar{p}$ the upper support of $J(\cdot)$, (A.2) implies

$$J(p) = \begin{cases} 1 & p \geq \bar{p} \\ 0 & p < \bar{p}. \end{cases} \tag{A.3}$$

This contradicts the premise that there is price dispersion (*i.e.* that $J(\cdot)$ is non-degenerate).

Proof of Proposition 3.

i. Suppose that an SSME exists with $\gamma > \beta$ and $q_1 = 1$. In any SSME we have

$$\bar{p} = \frac{\gamma}{\beta C}. \tag{A.4}$$

From Proposition 1, with $q_1 = 1$, the only possible distribution of posted prices (and therefore of transactions prices as well) is concentrated at the reservation price, $\bar{p}$, so that $C = 1/\bar{p}$. In this case (A.4) becomes

$$\bar{p} = \left[\frac{\gamma}{\beta}\right] \bar{p}. \tag{A.5}$$

With $\gamma > \beta$, (A.5) can be satisfied by no finite, non-zero price. This contradicts the existence of an SSME in this case and establishes part **i.** of the proposition.

ii. From proposition 1, with $q_1 = 0$ the only possible distribution of transactions prices is concentrated at the competitive price, p^*, where in an SSME,

$$p^* = \frac{\gamma\phi}{\beta u'(C^*)C^*}. \tag{A.6}$$

Since $p^* = 1/C^*$, (A.6) becomes

$$u'(C^*) = \frac{\gamma\phi}{\beta}. \tag{A.7}$$

With $u(\cdot)$ strictly concave, $u'(\cdot)$ is monotone decreasing and (A.7) has a unique solution. This establishes part **ii.** of the proposition.

iii. Existence follows from Proposition 1 and Theorem 4 of Burdett and Judd (1983). Details and uniqueness to follow...

Proof of Proposition 4.

In the SSME with $q_1 = 0$, the distribution of real transactions prices satisfies (3.10). Therefore, consumption satisfies $C^* = 1/p^*$, and we have

$$\frac{1}{C^*} = p^* = \frac{\phi}{\Omega} = \frac{\phi}{(\beta/\gamma)u'(C^*)C^*}. \tag{A.8}$$

Rearranging, we have

$$u'(C^*) = \frac{\gamma\phi}{\beta}. \tag{A.9}$$

with $u(\cdot)$ strictly concave, the proposition follows.

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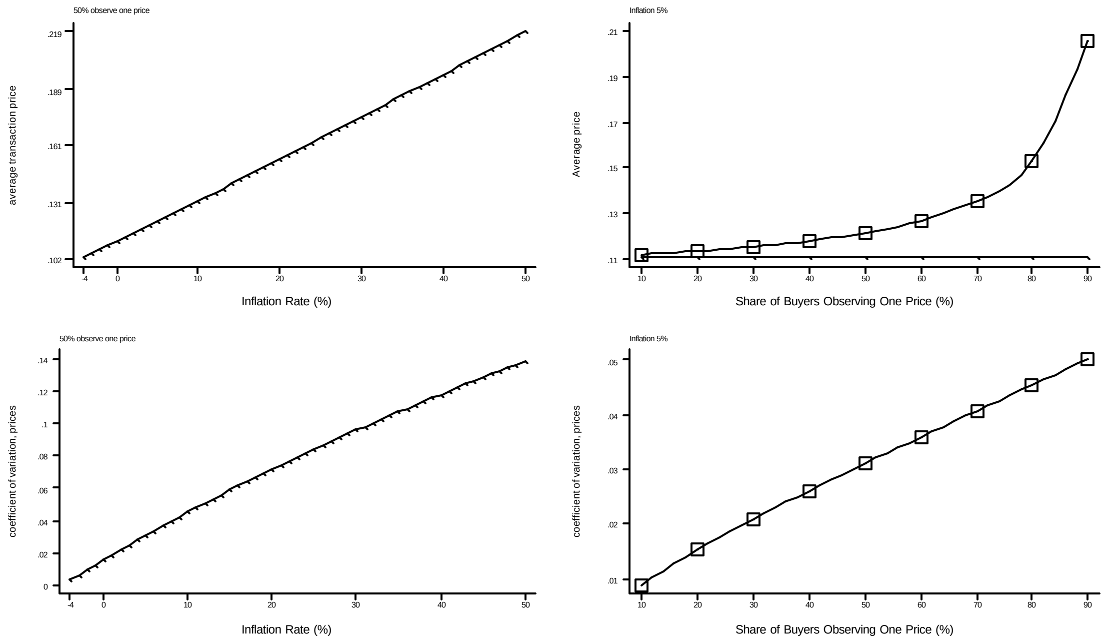
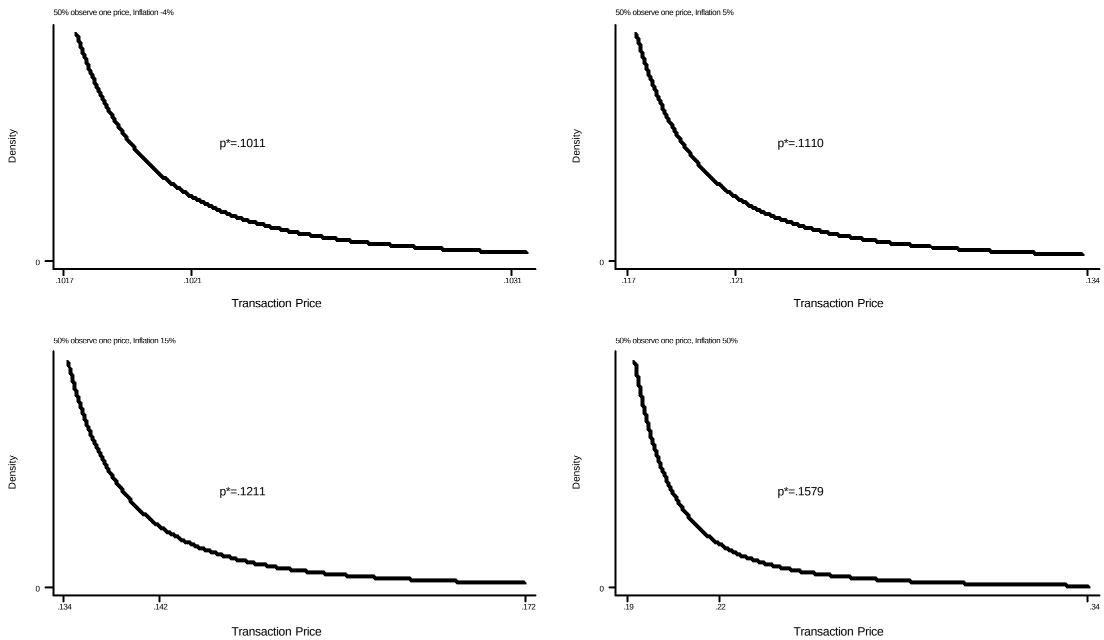


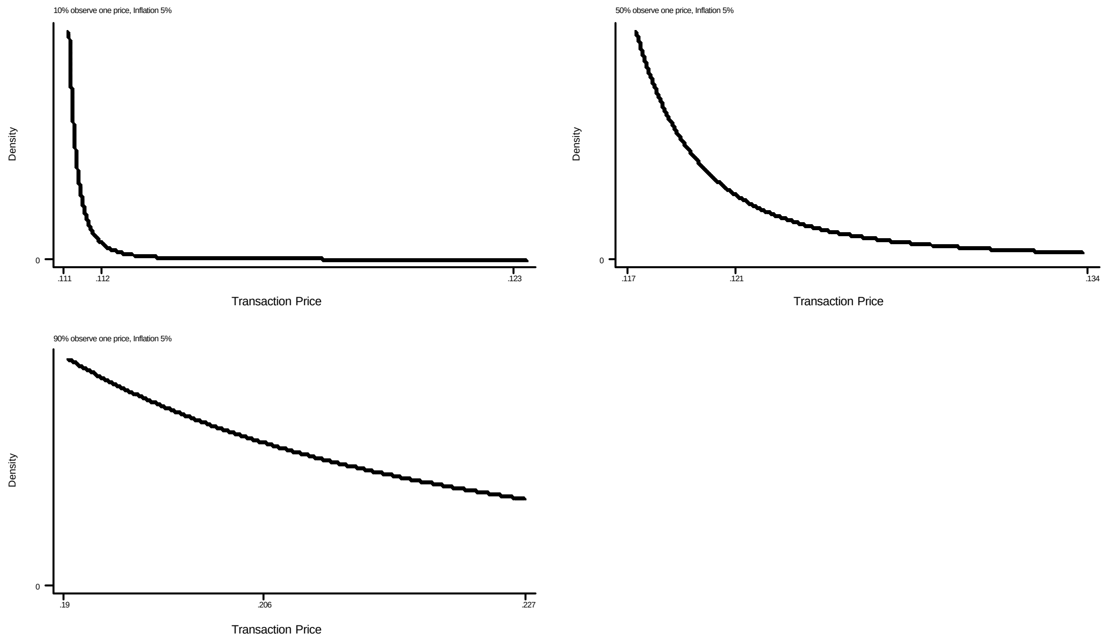
Average Transactions Prices and Dispersion

Figure 1



Densities of Transactions Prices

Figure 2



Densities of Transactions Prices

Figure 3

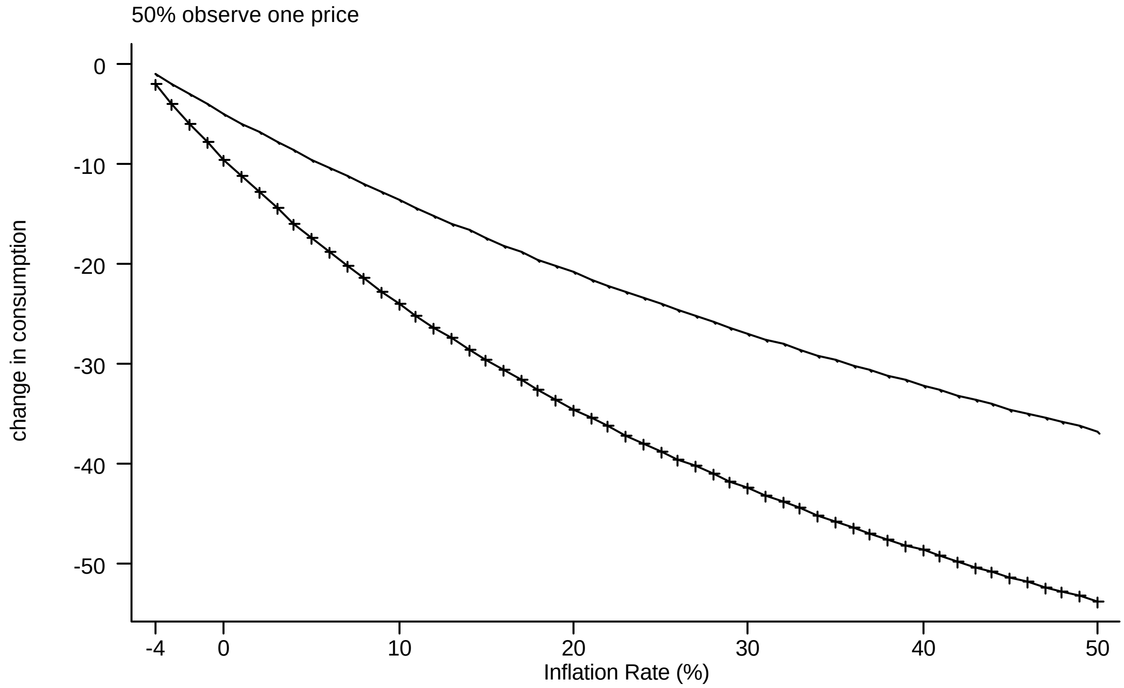


Figure 4

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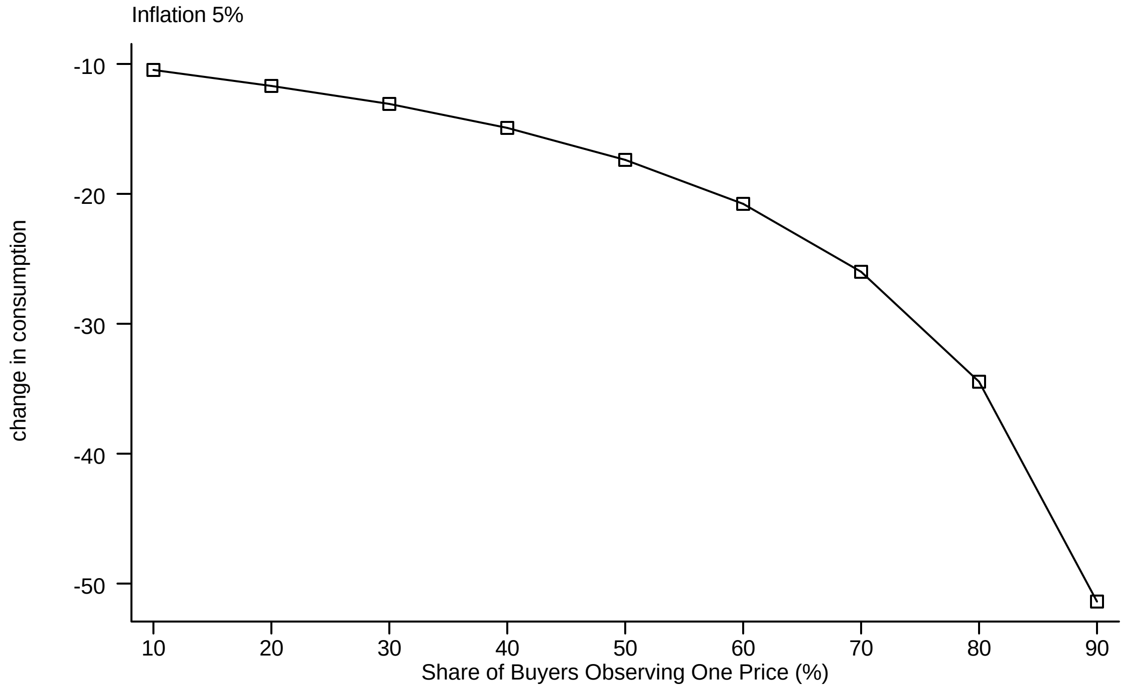
Inflation 5%
change in consumption
-10
-20
-30
-40
-50
10
20
30
40
50
60
70
80
90
Share of Buyers Observing One Price (%)

Figure 5