

**THE DISTRIBUTION OF EARNINGS IN AN EQUILIBRIUM SEARCH
MODEL WITH STATE-DEPENDENT OFFERS AND
COUNTEROFFERS***

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We construct an equilibrium job search model with on-the-job search in which firms implement optimal-wage strategies under full information in the sense that they leave no rent to their employees and counter the offers received by their employees from competing firms. Productivity dispersion across firms results in wage mobility both within and across firms. Workers may accept wage cuts to move to firms offering higher future wage prospects. Equilibrium productivity dispersion across ex ante homogeneous firms can be endogenously generated. Productivity dispersion then generates a nontrivial wage distribution which is generically thin-tailed, as typically observed in the data.

1. INTRODUCTION

In spite of a rather successful debut, job search theory seemed to receive Diamond's (1971) critique as a death sentence. Diamond showed that no wage dispersion could persist in an equilibrium job search model with endogenous firm behavior, because it is never in the interest of any firm to offer a wage above the unemployed workers' common reservation wage. The distribution of wage offers thus degenerates to the so-called "monopsony wage," and the original job search model collapses to a simple labor market monopsony model. From then on, the revival of job search theory became somewhat conditional on the demonstration of the ability of equilibrium search models to generate equilibrium wage dispersion.

This demonstration came in two parts. First, Diamond's argument was easily circumvented by introducing heterogeneity in the reservation wages of workers, which causes wage dispersion in equilibrium (see, e.g., Albrecht and Axell,

* Manuscript received March 2000; revised and accepted March 2001.

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1984), for the theory, and Eckstein and Wolpin, 1990, for an application. Second, Burdett and Mortensen (1998) pointed out that allowing employed workers to look for a better position while employed forces firms to compete with one another in their search for new employees, which leads them to implement differentiated wage policies. The latter approach, in particular, has proved extremely successful and has received considerable theoretical and empirical attention in the past decade. Its implications are not limited to equilibrium wage dispersion. It also brings forceful insights as to why large firms pay better than small firms, why senior workers are better paid and less mobile than junior workers, or what determines the average duration of unemployment spells.

This article builds on Burdett's and Mortensen's idea that labor market frictions and on-the-job search matter in wage determination. The difference with Burdett's and Mortensen's original model, however, resides in the wage-setting mechanism. This difference has a twofold aspect.

First, it is implicitly assumed in Burdett and Mortensen (1998) that firms have incomplete information on the reservation wages of job applicants. The optimal wage-setting mechanism in this case is a take-it-or-leave-it, unconditional wage offer. This assumption of one-sided incomplete information thus imposes the stringent restriction that each firm is bound to offer the *same* wage to all the workers it comes in contact with, regardless of their previous situation. As a result, firms from the right tail of the wage offer distribution meeting workers from the left tail of the distribution of reservation wages (e.g., unemployed workers), yield an important share of their rent to these workers. In this article we examine the alternative hypothesis of optimizing firms in a complete-information environment. Firms still speak first (i.e., they have all the "bargaining power") and offer each worker the minimum wage needed to attract them given their type.

Second, in Burdett's and Mortensen's world, firms are assumed to be entirely passive when attacked by their competitors. If one of their employees receives an attractive offer from another firm, they just let him go. What if the incumbent firm knows where the offer comes from and tries to retain its worker by making a counteroffer, if able to? The counteroffer might then in turn be countered by the other firm, the succession of offers and counteroffers resulting in a Bertrand competition in wages between the two employers.² There are of course obvious incentive problems that may rule out outside offer matching as a dominant strategy. For instance, firms may face a concern for "fairness" on the part of workers,

² Note that duopolistic competition in search models is not a new idea. In Burdett and Judd (1983), for example, customers/workers sample restaurants/employers in search for the best price/wage. Marginal productivity payments occur if workers (searching for a job) can simultaneously apply to at least two would-be employers. If search does not always generate two simultaneous offers, then equilibrium wages are equal to neither marginal productivity nor reservation wages but are necessarily dispersed even if both workers and firms are identical. This setup was recently used by Acemoglu and Shimer (1999) in a model of endogenous productivity determination. One could also refer to a more recent trend in the literature which explores the consequences of workers applying to multiple vacancies and firms receiving multiple applications in a market structured by an auction mechanism. See, for example, Julien et al. (1999). Finally, Dey and Flinn (2000) independently examined a wage formation process somewhat similar to ours, yet with an additional Nash bargaining component which makes the wage-setting mechanism they consider differ from Bertrand competition.

making it very costly in terms of work effort to pay differentiated wages to identical workers employed at similar jobs. Also, rewarding on-the-job search by matching outside offers encourages on-the-job search, hence increasing wage costs and labor turnover. Firms (large firms in particular) might therefore find it optimal to commit themselves not to match outside offers. Now, there are means other than the wage for a firm to make a job more attractive to a worker, the use of which may not be as conspicuous as an increase in the wage. Among those means, one can think of working conditions that can be improved, extra hours that can be offered, and promises that can be made to the worker regarding future promotions, with reputation effects warranting the employer's commitment. What we refer to as the "wage" in the upcoming theory may therefore be viewed more generally as the flow valuation that workers assign to the corresponding job, which may include some nonmonetary amenities.

Yet, it might well be that real-world firms do have incomplete information about the reservation wages of job applicants (maybe more so in the case of unemployed workers than in the case of employees). It might also be that employers do not (always) match outside offers made to their employees. Surely, reality lies somewhere in between our complete information story and Burdett's and Mortensen's incomplete information assumption. We believe that, before turning our attention to the more realistic situation where point observations imperfectly signal true types and patient observation permits learning, the complete-information case is well worth exploring, especially as it leads to rather interesting results, of which we now give a brief account.

1.1. *Summary of the Main Results.* In an economy where all workers have the same reservation wage when unemployed (because they have the same opportunity cost of employment) and all firms extract the same marginal productivity from all workers, it is straightforward to see that the equilibrium wage distribution that would arise under those alternative two assumptions should degenerate to a mixture of two mass points: One at the unemployed workers' common reservation wage and the other at the common marginal productivity, because allowing firms to counter alternative wage propositions made to their employees triggers a Bertrand competition between these firms, the outcome of which is that the lucky contacted employee is paid the highest bid firms can make, which equals the marginal productivity of labor and yields zero profits to the firms. The equilibrium wage distribution is therefore a mixture of the monopsony wage (the reservation wage of unemployed workers) and the Walrasian wage.

One is thus tempted to conclude that perfect information and Bertrand competition annihilate the effect of on-the-job search on equilibrium wage dispersion. Clearly, additional heterogeneity is required to generate an equilibrium wage distribution showing some resemblance to the actual one. Now, this was also true of the imperfect information, wage-posting model of Burdett and Mortensen. On-the-job search indeed generates a continuous earnings distribution in the latter setting, but one that has an upward sloping density, clearly at odds with observation. The empirical applications of this model definitely demonstrated the

necessity of allowing for exogenous sources of heterogeneity.³ We extensively analyze the case of firms differing in productivities (assuming linear technologies) and heterogeneous workers with respect to their valuations of leisure.

To further investigate the differences between the basic mechanism governing equilibrium wage dispersion in our model relative to Burdett's and Mortensen's, we adapt the idea of Acemoglu and Shimer (1999) of endogenous productivity dispersion to our model economy. We consider an extension in which *ex ante* identical firms optimally choose different productivities through different capital investment choices. We show that a result of continuous productivity dispersion arises, which is similar to Burdett's and Mortensen's prediction of continuous wage dispersion. Firms compete with one another in the activities of recruiting and retaining workers. Since they are *ex ante* identical and all follow the same (optimal) wage-setting rule, they have to use their investment policies to differ from one another. As a result, the shape of the equilibrium distribution of earnings is related to that of the distribution of productivities, which is shown to be typically downward sloping at its right end, as observed in the data.

In the perfect-information Bertrand-competition economy we consider in this article, wages are dispersed within firms as well as between them. Wages rise with seniority within a given firm as well as with experience. In addition, workers may experience wage cuts when passing from a firm with a low productivity to a firm with a (sufficiently) higher productivity. The idea here is that more productive firms yield better career prospects because they have the capacity to grant higher wage hikes when engaged in harsh competition. Hence, workers employed at the top of the wage scale in a low-productivity firm may be prepared to undergo a wage contraction to work in a more productive firm.

1.2. Outline of the Article. The next section exposes the basic environment in which we are going to work. Section 3 gives a formal exposition and solution of the model under the assumption of an exogenous continuous distribution of firm productivities. Section 4 endogenizes productivity dispersion. Section 5 offers some numerical results and further comments. Section 6 concludes the article. The Appendix contains some of the less essential algebraic steps.

2. THE BASIC THEORETICAL SETUP

Things start exactly as in the original Burdett and Mortensen (1998) framework. We consider a search-theoretic model featuring a market with a constant number M of workers, and an endogenous number N of firms operating constant-returns-to-labor technologies. Workers are equally skilled and perfectly substitutable. Yet, the marginal productivity of labor (hereafter denoted p) may differ across firms because the machines operated by the workers may be different. The assumption of constant returns to labor is essential. A firm is willing to employ any worker so long as it gets paid less than its marginal productivity. As a result, no firm is ever

³ See Bowlus et al. (1995, 1998) and Bontemps et al. (1999, 2000).

induced to fire a given worker to replace him/her by a less costly unemployed in the context of this model.⁴

Workers can either be employed or unemployed, and the aggregate unemployment rate is denoted by u . The unemployed are contacted by firms at a rate λ_0 . All firms have the same probability of being sampled (random matching technology).⁵ We also allow workers to search for a better job while employed, so firms make offers to employed workers as well. The arrival rate of offers to on-the-job searchers is λ_1 . The pool of unemployed workers is regularly fueled by two mechanisms: First, as is customary, we assume that layoffs occur at the exogenous rate δ . Second, a constant flow μM of newborn workers begin their working life as unemployed. Finally, to keep population constant over time, we assume that every living worker, employed or not, faces a constant mortality rate μ . Although not essential, the birth–death process will be shown to be useful in that it allows workers to apply a zero discount rate to the future; the death risk alone is enough to make the sum of future payoffs finite. However, for the sake of generality, we will primarily consider the nonzero discounting case, and accordingly allow agents to discount the future at a rate $\rho \geq 0$.

As announced in the introductory discussion, we depart from the Burdett and Mortensen hypotheses by assuming that firms have perfect information about the characteristics (reservation wage) of the workers who apply for a job. It then follows that:

- (i) Firms vary their wage offers according to the characteristics of the particular worker they meet instead of being bound to offer the same wage to all workers;
- (ii) Firms counter the offers received by their employees from competing firms instead of being completely passive in the face of such offers.

Under these assumptions, if two firms with identical productivities get into competition for a single worker, then the wage offer should increase until it reaches the maximal bid firms can make, i.e., p , yielding zero marginal profit. Now assuming that the two firms have different productivities $p' > p$, then the most productive firm, p' will keep the worker, because the upper bound of all wage offers a type p firm can make is precisely its productivity p , the most productive firm will be able to offer more attractive wages and still make positive profits on the worker. Moreover, the most productive firm will obviously not offer the worker a wage strictly above the productivity p of the less productive firm. Hence, the productivity of the firm he is currently working in is the supremum of all possible wages a particular worker can get from his next offer. More productive firms are therefore

⁴ Of course, in reality, firms may face capacity constraints, demand rationing, and/or diminishing returns to scale, which makes this last idea relevant again. We leave the analysis of this important extension to future research.

⁵ This is also a disputable assumption. One could oppose this assumption with the alternative assumption of “balanced matching” as in Burdett and Vishwanath (1988) (the probability that a worker samples a given firm is proportional to the firm’s size), or the endogenous matching process of Robin and Roux (1998) and Mortensen (1998). Since we are mostly interested in the wage-setting mechanism in this article, we leave the analysis of more involved matching processes for future research.

more attractive to workers. We also see that the minimum wage hike making a worker willing to move from a firm of type p to a firm of type p' thus depends on both productivities. It should increase with p and decrease with p' , because if p imposes an upper bound on the mobility wage in the current competition, p' will be an upper bound for the mobility wage the worker will be able to obtain from a potential competition between firm p' and a potential challenger $p'' > p'$ after the worker has left firm p to move to firm p' .

Note also that an employee of a firm p can benefit from an offer made by a firm $p' < p$ if p' is large enough to outbid the worker's current wage: The current type p employer has to match the best possible offer the type p' raiding firm can make to the worker, which may imply a wage hike.

Finally, we assume that there exist legal restrictions on the wage contracts: They are long-term contracts which can be renegotiated only by mutual agreement. The only way for an employer to break the contract, if the employee does not want to, is to fire that employee. So a firm cannot cancel a promotion a worker has obtained from his/her employer after he/she received an alternative offer, once the worker has eventually turned down the offer. It follows that wage cuts within the firm are not permitted. For such wage cuts to appear, something must change in the environment that makes the firing threat credible; productivity shocks, for instance, as in Mortensen and Pissarides (1994). In this article, we shall assume that opportunity costs of employment b , firms' productivities p , arrival rates λ_0 , λ_1 , δ , and μ , and the discount rate ρ do not experience any such shocks.

3. THE MODEL WITH EXOGENOUS PRODUCTIVITY DIFFERENCES

In this section, we turn to a formal description of the model under the assumption that differences in firm productivities (p) are exogenously given. We relax this assumption and endogenize productivity in Section 4.

3.1. Worker Behavior. Let us first introduce some notation. Firms have heterogeneous productivities, which are distributed over $[p, \bar{p}]$ according to the continuous distribution Γ (cdf).⁶ To save on space, we also define $\bar{\Gamma}(\cdot) = 1 - \Gamma(\cdot)$. Workers are ex ante heterogeneous with respect to their opportunity cost of employment, which we denote by $b > 0$ and assume to be distributed over some interval $[\underline{b}, \bar{b}]$ according to the continuous cdf H_0 . New workers are assumed to draw their value of b randomly in the distribution H_0 .⁷ To avoid the substantial complication that would arise from asymmetric information, we assume that the type b of any jobless worker is fully observed by firms, as is the current wage of any

⁶ The more general case of possible atoms in Γ was considered in a working paper version of this article, which is available from the authors upon request.

⁷ This heterogeneity is not essential here. Firm heterogeneity alone will be shown to be sufficient for generating continuous wage dispersion. Yet, contrary to the Burdett and Mortensen (1998) wage-posting model, we shall see that allowing workers to differ in their opportunity cost of employment does not make the economic model considerably more complicated, whereas it increases the fit of the equilibrium wage distribution with actual data.

employed worker.⁸ Workers are risk-neutral and maximize the present discounted sum of expected future income flows.

The lifetime utility of an unemployed worker with employment opportunity cost equal to b (a worker of type b , for short) is denoted by $V_0(b)$, and that of the same worker paid w in a firm of type p is $V(b, w, p)$. The optimal wage offer of a firm of type p willing to hire an unemployed worker of type b is the minimum wage $\phi_0(b, p)$ that compensates this worker for the opportunity cost of employment, which is defined by

$$(1) \quad V[b, \phi_0(b, p), p] = V_0(b)$$

The novelty here is that, since more productive firms are more attractive to workers, the minimum wage at which an unemployed worker is willing to work in a given type p firm now depends on p , as shown by Equation (1).

Finally, denoted by $\phi(p, p')$ is the optimal wage offer that a firm with productivity p' wants to make to a worker (of type b) employed in a firm with productivity $p < p'$, and that the worker is willing to accept.⁹ The best offer that the firm of type p can make to the worker is to set his wage exactly equal to p . The highest level of utility the worker can attain by staying in the firm of type p is therefore $V(b, p, p)$. Accordingly, he accepts to move to a firm of type $p' > p$ if the latter offers at least the wage $\phi(p, p')$ defined by

$$(2) \quad V[b, \phi(p, p'), p'] = V(b, p, p)$$

Any less generous offer on the part of the type p' firm is successfully countered by the less productive, type p firm.

Although a strictly rigorous mathematical derivation of the optimal worker behavior should proceed more carefully, to save on space and on the reader's patience, we shall assume from the beginning that $\phi(p, p')$ increases with p and $\phi_0(b, p)$ decreases with p , exactly as we have implicitly assumed in the above two definitions of reservation wages that the value function $V(b, w, p)$ was increasing in the wage. Again, these assumptions will be confirmed later on.

The next step is to define the value functions $V_0(\cdot)$ and $V(\cdot)$. Since offers accrue to unemployed workers at rate λ_0 , and since only firms with a productivity at least equal to b can make an offer to unemployed workers with type b , $V_0(b)$ solves the following Bellman equation:

$$(\rho + \mu + \lambda_0)V_0(b) = b + \lambda_0\bar{\Gamma}(b) \cdot E_p\{V[b, \phi_0(b, p), p] \mid p > b\} + \lambda_0\Gamma(b) \cdot V_0(b)$$

⁸ The introduction of an information asymmetry at this stage would complicate the wage-setting process considerably (see, e.g., Kennan and Wilson, 1993, for a review of the bargaining problem under asymmetric information).

⁹ Here we assume a priori that ϕ does not depend on the worker's type, b , a conjecture that will be confirmed later on. This independence is intuitive, since ϕ stems from the worker's comparison of the values of two different jobs, with no reference to the state of unemployment.

Using definition (1) to substitute $V[b, \phi_0(b, p), p]$ by $V_0(b)$ in the latter equation then yields

$$(3) \quad V_0(b) = \frac{b}{\rho + \mu}$$

As to employees, things are a little bit more involved. Consider a worker employed in a type p firm and receiving a wage $w \leq p$. This worker is hit by offers from other firms at rate λ_1 . If the offer stems from a firm with productivity p' such that $\phi(p', p) \leq w$, then the challenging firm is obviously less productive—hence less attractive—than his current employer, and it cannot even offer the worker his current wage. The worker thus rejects the offer and continues his job at an unchanged wage rate. Now, if the offer stems from a type p' firm with $w < \phi(p', p) \leq p$, then obviously the less productive challenging firm will not be able to attract the worker, but the more productive current employer will have to grant the worker a raise—up to $\phi(p', p)$ —to retain him from accepting the other firm’s offer. Finally, if the offer is made by a firm with productivity greater than p , i.e., $p' > p$, then the worker eventually accepts the offer and goes working in the type p' firm for exactly $\phi(p, p')$.

Define the threshold productivity $q(w, p)$ by $\phi[q(w, p), p] = w$, so that $\phi(p', p) \leq w$ if $p' \leq q(w, p)$. Contacts with firms with productivity less than $q(w, p)$ end up not causing any raise because the incumbent employer (with productivity p) can outbid its challenger by offering a wage *lower* than w . Since in addition, layoffs and deaths still occur at respective rates δ and μ , we may now write the Bellman equation solved by the value function $V(b, w, p)$:

$$(4) \quad (\rho + \delta + \mu + \lambda_1 \bar{\Gamma}[q(w, p)])V(b, w, p) \\ = w + \lambda_1 [\Gamma(p) - \Gamma[q(w, p)]] E_{p'}\{V(b, p', p') \mid q(w, p) < p' \leq p\} \\ + \lambda_1 \bar{\Gamma}(p)V(b, p, p) + \delta V_0(b)$$

If the challenging firm’s productivity p' is such that $q(w, p) < p' \leq p$, then the firm-type p keeps its employee but must promote him to the wage $\phi(p', p)$ such that $V[b, \phi(p', p), p] = V(b, p', p')$. If $p' > p$, then the firm-type p' wins the competition and hires the worker at the wage $\phi(p, p')$ defined by (2): $V[b, \phi(p, p'), p'] = V(b, p, p)$.

Imposing $w = p$ in the latter relationship, we easily get

$$(5) \quad V(b, p, p) = \frac{p + \delta V_0(b)}{\rho + \delta + \mu}$$

Plugging this back into (4), and replacing the expectation term by its expression, we finally get a definition for $V(\cdot)$:

$$(6) \quad (\rho + \delta + \mu + \lambda_1 \bar{\Gamma}[q(w, p)])V(b, w, p) = w + \lambda_1 \int_{q(w, p)}^p \frac{x + \delta V_0(b)}{\rho + \delta + \mu} d\Gamma(x) \\ + \lambda_1 \bar{\Gamma}(p) \frac{p + \delta V_0(b)}{\rho + \delta + \mu} + \delta V_0(b)$$

3.2. *Wages.* By definition, the function $q(w, p)$ is the type of the incumbent employer from which a type p firm can poach a worker with a wage offer of w . It is therefore true that

$$\begin{aligned} V(b, w, p) &= V(b, q(w, p), q(w, p)) \\ &= \frac{q(w, p) + \delta V_0(b)}{\rho + \delta + \mu} \end{aligned}$$

Plugging this expression into (6), we finally obtain that

$$(7) \quad q(w, p) = w + \frac{\lambda_1}{\rho + \delta + \mu} \int_{q(w, p)}^p \bar{\Gamma}(x) dx$$

We can now derive an expression and some properties of the reservation wages $\phi_0(\cdot)$ and $\phi(\cdot)$. We begin with the threshold wage $\phi(p, p')$ for a pair of productivities such that $p' > p$. Substituting $\phi(p, p')$ for w in (7), using the fact that $q[\phi(p, p'), p'] = p$, and rearranging terms, we obtain

$$(8) \quad \phi(p, p') = p - \frac{\lambda_1}{\rho + \delta + \mu} \int_p^{p'} \bar{\Gamma}(x) dx$$

This expression requires some comments. First, we see that, as conjectured, $\phi(p, p')$ does not depend on the worker's type b . Second, a worker employed at a firm with productivity p and who receives an alternative offer from a firm with productivity p' , not necessarily greater than p , will be promoted but never to a wage greater than p . The productivity of the current employer therefore imposes an upper bound on the next promotion and the challenger's productivity imposes an upper bound on the second next promotion. Therefore, the employee's reservation wage $\phi(p, p')$ increases with p and falls with p' , because to some extent they are willing to trade a lower share of the total rent today for a larger share tomorrow. It is thus more difficult to draw a worker out of a more productive type p firm, and workers are more easily willing to work at more productive type p' firms.

This in turn has two crucial implications. The first is that workers may be willing to accept wage cuts, even though they are not threatened with losing their job. Consider, for instance, the top rank worker in a type p firm, who earns exactly $w = p$, and assume that this worker gets an offer from a firm of type $p' > p$. Then he is going to be willing to work for the type p' firm for any wage above $\phi(p, p')$, which is *strictly less than his current wage* p , according to Equation (8). The second key implication of ϕ 's properties is that senior workers are predicted by the model to be on average less mobile than junior workers. To see this, note that a worker making w in a type p firm is "upgraded" (i.e., either promoted or hired by a better firm) when he receives an offer from a type p' firm such that either $p' \leq p$ and $w \leq \phi(p', p)$, in which case he gets a raise, or $p' > p$, in which case he goes to the firm of type p' . This makes workers with long tenures, who on average have

received more offers and therefore get higher wages in better firms, less likely to receive an attractive offer that would result in an upgrade.¹⁰ The latter feature is preserved from the Burdett and Mortensen (1998) model, whereas the former is a novelty.

We now turn to the unemployed workers' reservation wages $\phi_0(\cdot)$. First note that, by definition,

$$V(b, q[\phi_0(b, p)], q[\phi_0(b, p)]) = V[b, \phi_0(b, p), p] = V_0(b)$$

From (3) and (5), this implies that $q[\phi_0(b, p)] = b$. Finally, remarking that $\phi_0(b, p) = \phi(q[\phi_0(b, p)], p)$, and substituting these two equalities into (7), we obtain

$$(9) \quad \phi_0(b, p) = b - \frac{\lambda_1}{\rho + \delta + \mu} \int_b^p \bar{\Gamma}(x) dx$$

Again, this calls for some comments. First, we see that unemployed workers are prepared to work for a wage $\phi_0(\cdot)$ that is *less* than the opportunity cost of employment b . This is because being employed means not only earning a wage, but also getting better employment prospects. Second, as we naturally expected, $\phi_0(\cdot)$ turns out to be a *decreasing* function of p . Since more productive firms yield better future job opportunities, they are more attractive to workers and take advantage of this feature by offering lower wages. Third, $\phi_0(b, p) = p$ for $p = b$, confirming that only these firms with $p \geq b$ hire unemployed workers with type b . Fourth, the reservation wage does not depend on the arrival rate of offers λ_0 . In standard search theory, reservations wages do depend on λ_0 , because the wage offers are not necessarily equal to the reservation wage. A longer search duration may thus increase the value of the eventually accepted job. Here, this does not happen. Firms always pay the reservation wage to workers; therefore, there is no gain to expect from rejecting an offer and waiting for the following one. Finally, we see by comparing (8) and (9) that $\phi_0(\cdot)$ and $\phi(\cdot)$ have identical analytical expressions. Nevertheless, for expositional clarity, we will keep both notations in the remainder of this article.

3.3. *Firm-Level Worker Flows and Wage Distributions.* Let $L(w | p)$ denote the number of employees paid a wage $\leq w$ in a type $p \geq w$ firm. We now proceed to derive the value of $L(w | p)$ as a function of the distribution of productivities. The $L(w | p) \times N d\Gamma(p)$ workers paid less than w by firms with type p leave this category either because they are laid off—which occurs at rate δ —or because they die—which occurs at rate μ —or finally because they receive an attractive offer, which grants them a wage increase. From previous paragraphs, we see that only those workers who receive an offer from a firm with productivity

¹⁰Formally, the probability that a particular offer is able to upgrade a worker with characteristics (w, p) is $\bar{\Gamma}[q(w, p)]$. $\bar{\Gamma}$ is a decreasing function, whereas it is easy to see that $q(w, p)$ is a nondecreasing function of tenure.

not less than $q(w, p) \leq p$ will either see their wage raised above w , or leave their type p employer to a more productive firm $p' > p$. Such offers accrue at rate $\lambda_1 \bar{\Gamma}[q(w, p)]$. On the side of inflows, workers entering the category (w, p) come from two sources. Either they are hired from a firm with productivity less than $q(w, p)$, or they come out of unemployment. Let $L(p) = L(p | p)$ denote total employment in a firm of type p (the highest paid worker in such a firm earns exactly $w = p$). The number of workers hired by such a firm from firms with productivities less than $q(w, p)$ is equal to $\lambda_1 \times d\Gamma(p) \times \int_p^{q(w,p)} L(x) N d\Gamma(x)$ (offer arrival rate \times probability of picking a firm of type p \times total employment of all firms with productivity less than $q(w, p)$). Finally, the unemployed workers willing to work in a type p firm for less than w are those whose reservation wage does not exceed w , i.e., those with b such that $\phi_0(b, p) \leq w \Leftrightarrow b \leq q(w, p)$. If we denote the cdf of opportunity costs of employment among unemployed workers (which is different from the basic distribution H_0 , from which we shall derive it; see Equation (14)) by H , the inflow of unemployed workers into the category (w, p) is thus equal to $\lambda_0 \times d\Gamma(p) \times uMH[q(w, p)]$, where u designates the rate of unemployment.

The stationarity of $L(w | p)$ thus implies

$$(10) \quad (\delta + \mu + \lambda_1 \bar{\Gamma}[q(w, p)]) L(w | p) N \\ = \begin{cases} \lambda_0 u M H[q(w, p)] & \text{if } w \leq \phi(p, p) \\ \lambda_0 u M H[q(w, p)] + \lambda_1 N \int_p^{q(w,p)} L(x) d\Gamma(x) & \text{if } w > \phi(p, p) \end{cases}$$

For small values of the wage w —specifically, for $w \leq \phi(p, p)$ —the last integral term vanishes. For those values of w , the latter equation thus directly yields a closed-form characterization of $L(w | p)$. Note that only workers just coming out of unemployment will accept wages lower than w in this case. Indeed, workers having already experienced at least one period in employment were employed in a firm with productivity above p , and therefore have a reservation wage greater than $\phi(p, p)$.

To obtain an expression of $L(w | p)$ in the case $w \geq \phi(p, p)$, let us first determine the total workforce employed by a type p firm. This is most easily done by considering the stock of workers employed at all firms with types less than p , which equals $N \int_p^p L(x) d\Gamma(x)$. This stock is depleted at rate $[\delta + \mu + \lambda_1 \bar{\Gamma}(p)]$ (layoffs + deaths + offers from more attractive firms), whereas it is fueled by hiring of unemployed workers. The flow of unemployed workers hired into firms with productivities less than p is given by $\lambda_0 u M \int_p^p H(x) d\Gamma(x)$. Once again equating inflows and outflows for the stock of workers at hand leads to

$$(11) \quad N \int_p^p L(x) d\Gamma(x) = \frac{\lambda_0 u M \int_p^p H(x) d\Gamma(x)}{\delta + \mu + \lambda_1 \bar{\Gamma}(p)}$$

An expression for $L(p)$ is obtained by differentiation with respect to p :

$$\begin{aligned}
 (12) \quad L(p) &= \frac{\lambda_0 u M}{N} \cdot \frac{[\delta + \mu + \lambda_1 \bar{\Gamma}(p)] \cdot H(p) + \lambda_1 \int_p^p H(x) d\Gamma(x)}{[\delta + \mu + \lambda_1 \bar{\Gamma}(p)]^2} \\
 &= \frac{\lambda_0 u M}{N} \cdot \frac{\int_b^p [\delta + \mu + \lambda_1 \bar{\Gamma}(x)] dH(x)}{[\delta + \mu + \lambda_1 \bar{\Gamma}(p)]^2}
 \end{aligned}$$

where the last equation is obtained by an integration by parts. Note that $L(p)$ obviously increases with p .

Finally, combining Equation (11) at $p \equiv q(w, p)$ with (10) immediately shows that $L(w | p) = L[q(w, p)]$, where w takes its values between $\phi_0(b, p)$ and p .

3.4. *Aggregate Worker Flows and Wage Distributions.* The last endogenous variable we need to pin down at this point is the unemployment rate u . Since total flows in and out of unemployment are equal in steady states, we have

$$(13) \quad \left[\mu + \lambda_0 \int_b^b \bar{\Gamma}(b) dH(b) \right] u = \delta(1 - u) + \mu$$

or

$$u = \frac{\delta + \mu}{\delta + \mu + \lambda_0 \int_b^b \bar{\Gamma}(b) dH(b)}$$

Note that the unemployment rate is endogenous because of unemployed workers voluntarily rejecting uninteresting wage offers, i.e., from firms with productivity less than their opportunity cost of employment.

Moreover, as we already stressed, $H(\cdot)$ is the distribution of opportunity costs of employment among unemployed workers, which can be related to $H_0(\cdot)$, i.e., the distribution of such costs in the entire population, as follows. Denote the distribution of b 's among *employed* workers by H_e . All three distributions are related by the following two flow-balance equations:

$$\begin{aligned}
 (\mu + \delta)(1 - u) dH_e(b) &= \lambda_0 \bar{\Gamma}(b) u dH(b) \\
 [\mu + \lambda_0 \bar{\Gamma}(b)] u dH(b) &= \delta(1 - u) dH_e(b) + \mu dH_0(b)
 \end{aligned}$$

which reflect the fact that the measure of agents of type b among employed and unemployed workers remains constant. Eliminating $dH_e(b)$ from the latter two equations, we obtain

$$(14) \quad u dH(b) [\delta + \mu + \lambda_0 \bar{\Gamma}(b)] = (\delta + \mu) dH_0(b)$$

We now define $G(w)$ as the cdf of the aggregate earnings distribution. $G(w)$ hence denotes the proportion of workers earning less than w in the economy. We begin with a characterization of the support of $G(\cdot)$. The slopes of $\phi_0(\cdot)$ with respect to both its arguments imply that the lowest wage at which workers exit

unemployment is $\phi_0(\underline{b}, \bar{p})$. Similarly, the highest possible wage in the economy is clearly $\phi(\bar{p}, \bar{p}) = \bar{p}$. We know from the definitions of ϕ_0 and ϕ that $\phi_0(\underline{b}, \bar{p}) \leq \bar{p}$. The support of $G(\cdot)$ is therefore (comprises) the segment $[\phi_0(\underline{b}, \bar{p}), \bar{p}]$.

Basically, what we have to do to determine $G(w)$ is to count the number of workers earning less than w in any firm, and then sum over the relevant number of firms. This is carried out precisely in Appendix A.1. It is also shown in Appendix A.1 that $g(\cdot)$ is continuous in a neighborhood of \bar{p} and that $g(\bar{p}) = 0$, where $g(\cdot)$ is the density associated with $G(\cdot)$. In other words, this implies that the wage distribution generated by our model is always thin-tailed at its right end, whatever the distributions of firm productivities $\Gamma(\cdot)$ and worker opportunity costs of employment $H(\cdot)$.

3.5. A Comparison with the Burdett and Mortensen (1998) Wage-Posting Equilibrium. At this point, it is interesting to compare some of the predictions of our model with their counterparts in the wage-posting equilibrium analyzed by Burdett and Mortensen (1998). The version of the Burdett and Mortensen model best suited to that comparison is that with homogeneous workers (i.e., all workers having equal b 's) and dispersed firm types. Moreover, we let $\lambda_0 = \lambda_1$ so that the reservation wage of unemployed workers is then equal to b in the Burdett and Mortensen setting.¹¹ We thus consider two "twin" economies, both characterized by the same exogenous distribution of firm productivities, and only differing in their wage formation processes. A subscript "BM" refers to the "wage-posting" economy à la Burdett and Mortensen (1998), whereas the absence of subscript refers to the "Bertrand" economy described in this article.

We first look at total employment and output. Equation (12) gives employment in a type p firm in the special case of homogeneous workers as

$$L(p) = \frac{\lambda_0 u M}{N} \cdot \frac{\delta + \mu + \lambda_1}{[\delta + \mu + \lambda_1 \bar{\Gamma}(p)]^2}$$

which is exactly the same formula as in the Burdett and Mortensen model with homogeneous workers and dispersed firm productivities: $L(p) = L_{\text{BM}}(p)$ for all p . Consequently, total employment and total output are equal in both economies.

The next natural question to address is how output is split between workers and employers in the two economies. Let us first look at profits. In our economy, considering the fact that $\phi(\cdot)$ is increasing in both its arguments, the lowest-paid worker in a type p firm is one that has just been hired and thus earns $\phi_0(\underline{b}, p)$. And clearly, the highest-paid worker in a type p firm earns precisely p . Having thus defined the support of the within-firm earnings distribution for any type p

¹¹ The more general case of heterogeneous firms and workers is explored by Bontemps et al. (1999), with continuous heterogeneity and under the restriction that offer arrival rates are equal for employed and unemployed workers ($\lambda_0 = \lambda_1$). A comparison with our model is also possible in this case, at the cost of considerable additional algebraic heaviness. We choose to limit this section to the case of homogeneous workers, which gives the essential insights.

Moreover, one can show that all the results of this section are reinforced in the plausible case of $\lambda_0 > \lambda_1$.

firm, we can readily derive the value of the current operating surplus for such a firm:¹²

$$\pi(p) = \int_{\phi_0(b,p)}^P (p - w) dL(w | p)$$

Integrating by parts, we change the latter expression into

$$(15) \quad \pi(p) = \int_{\phi_0(b,p)}^P L[q(w, p)] dw$$

Then, since from (7),

$$\frac{\partial q(w, p)}{\partial w} = \left[1 + \frac{\lambda_1 \bar{\Gamma}[q(w, p)]}{\rho + \delta + \mu} \right]^{-1}$$

it follows that changing $q(w, p)$ into x in the integral in (15) yields the following equivalent expression:

$$(16) \quad \pi(p) = \int_b^P L(x) \cdot \left[1 + \frac{\lambda_1 \bar{\Gamma}(x)}{\rho + \delta + \mu} \right] dx$$

In the BM economy, profits are given by

$$(17) \quad \pi_{\text{BM}}(p) = \int_b^P L(x) dx$$

whence

$$(18) \quad \pi(p) - \pi_{\text{BM}}(p) = \int_b^P L(x) \cdot \frac{\lambda_1 \bar{\Gamma}(x)}{\rho + \delta + \mu} dx > 0$$

Firms at all productivity levels always earn greater profits in the “Bertrand” economy than in the “wage-posting” economy. This result was expected, as both economies are essentially the same, up to the difference that firms are more constrained in their set of available strategies in the “wage-posting” economy since they are not able to adapt their offers to the characteristics of the workers they meet. This constraint is obviously detrimental to firms.

We now turn to workers. Clearly, since no rent is yielded by firms to the workers they hire from the unemployment pool in the Bertrand case, unemployed workers are worse off in the Bertrand than in the wage-posting economy. Indeed, the value

¹²The following derivation of the firms’ profits is not specific to the case of homogeneous workers. The same formulae obtain in the heterogeneous case, provided that b is everywhere replaced by \underline{b} , the lower support of the distribution of employment opportunity costs.

of unemployment, $V_0(b)$, is equal to $b/(\rho + \mu)$ in the Bertrand case (see Equation (3)), while we have

$$V_0(b)|_{\text{BM}} = \frac{b}{\rho + \mu} + \frac{\lambda_0}{\rho + \mu} \int_p^{\bar{p}} \frac{\bar{\Gamma}(x)}{\rho + \delta + \mu + \lambda_1 \bar{\Gamma}(x)} dx$$

in the wage-posting case. The integral term in the last equation corresponds to the expected rent collected by unemployed workers upon being hired, coming from the fact that firms offer them wages that are, in general, strictly above their reservation wage.

It is also instructive to derive the average “quasi-rent” (i.e., how much revenue in present discounted terms in excess of what an unemployed worker gets) that a worker can expect to enjoy when employed. Taking the unconditional expectation of the rent $V(b, w, p) - V_0(b)$ with respect to the joint cdf wages and firm types over a cross section of employees yields

$$(19) \quad E(V - V_0) \equiv \int_p^{\bar{p}} \frac{NL(p)\gamma(p)}{(1-u)M} \cdot \int_{\phi_0(b,p)}^p (V(b, w, p) - V_0(b)) \cdot \frac{L(dw|p)}{L(p)} dp$$

where $(1-u)M/N$ is the average firm size and $NL(p)\gamma(p)/(1-u)M$ is the density of firm types in a cross section of workers (each firm p being weighted by the measure of workers it employs), and where $\int_A^B f(w) \cdot L(dw|p)$ denotes the Stieltjes integral of any function $f(w)$ with respect to the measure $L(dw|p)$ over the interval $(A, B]$ (note that this measure has a mass point at $w = \phi_0(b, p)$). It is shown in Appendix A.2 that this mean rent is given by

$$(20) \quad E(V - V_0) = \frac{(\delta + \mu)(\delta + \mu + \lambda_1)}{\rho + \delta + \mu} \cdot \int_p^{\bar{p}} (p - b) \cdot \frac{2\lambda_1\gamma(p)\bar{\Gamma}(p)}{[\delta + \mu + \lambda_1\bar{\Gamma}(p)]^3} dp$$

which is strictly positive as long as λ_1 is. The fact that firms are in competition for the recruitment of *employed* workers guarantees that the latter are on average strictly better off than the unemployed. It is naturally essential for that competition to be activated that workers be allowed to search on the job ($\lambda_1 > 0$, as is also evident in the wage-posting economy; see below).

Turning to the wage-posting economy, the mean rent in the wage-posting economy can be shown to equal¹³

$$(21) \quad E(V - V_0)|_{\text{BM}} = \frac{(\delta + \mu)(\delta + \mu + \lambda_1)}{\rho + \delta + \mu} \cdot \int_p^{\bar{p}} \frac{2\lambda_1\gamma(p)\bar{\Gamma}(p)}{\delta + \mu + \lambda_1\bar{\Gamma}(p)} \cdot \int_b^p \frac{dx}{[\delta + \mu + \lambda_1\bar{\Gamma}(x)]^2} dp$$

¹³ A precise derivation is available from the authors upon request.

Since $\bar{\Gamma}(x) > \bar{\Gamma}(p)$ for all $p > x$, one has that

$$\frac{p - b}{[\delta + \mu + \lambda_1 \bar{\Gamma}(p)]^2} > \int_b^p \frac{dx}{[\delta + \mu + \lambda_1 \bar{\Gamma}(x)]^2}$$

for all p and therefore $E_G(V - V_0) > E_G(V - V_0)|_{\text{BM}}$. On average, employed workers appropriate a larger rent for themselves relative to unemployed workers under our assumptions about wage formation than under Burdett’s and Mortensen’s. The Bertrand economy is therefore more “unequal” in a cross-group sense.

4. EQUILIBRIUM PRODUCTIVITY DISPERSION

The model is now completely solved and analyzed given a particular distribution $\Gamma(\cdot)$ of productivities. Taking $\Gamma(\cdot)$ as exogenous may be a sensible assumption in a short-run perspective. However, to the extent that a firm’s productivity follows from its investment choices, it certainly should be made endogenous in the longer run. Endogenizing $\Gamma(\cdot)$ is what this section is devoted to.

Equilibrium productivity dispersion arises from the firms’ dispersed investment choices. Assume firms are ex ante identical and endowed with a technology exhibiting constant returns to labor and decreasing returns to capital. More specifically, the output per capita of a firm with a capital stock of K is $p = f(K)$, with $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) = +\infty$, and $f'(+\infty) = 0$. Furthermore, to avoid useless analytical complication, we also assume that $f(0) < \underline{b}$, so zero investment cannot be optimal, since a firm with no capital would not even be attractive to the least picky worker in the economy. We borrow that specification from Acemoglu and Shimer (1999). Also, Robin and Roux (1998) estimate the coefficients of a Cobb–Douglas production function $K^\alpha \cdot L^\beta$ on firm data and find that β is roughly equal to 1 whereas α is much smaller, somewhere between 0 and 0.1 depending on the particular sector considered.

With this specification, and taking the user cost of capital as an exogenous constant r , the final level of profit made by a type p employer is $\Pi(p) = \pi(p) - r f^{-1}(p)$. In equilibrium, all firms must make the same (maximal) profit, say Π^* . The following thus holds in equilibrium:

$$\begin{cases} \pi(p) - r f^{-1}(p) \equiv \Pi^* & \text{for all } p \in \text{supp}(\Gamma), \\ \pi(p) - r f^{-1}(p) < \Pi^* & \text{otherwise} \end{cases}$$

Since $\pi(p) - r f^{-1}(p)$ is a constant over the support of $\Gamma(\cdot)$, and since π is differentiable, it is therefore true that

$$(22) \quad \pi'(p) = \frac{r}{f'[f^{-1}(p)]}$$

which is equivalent to (recall Equation (16))

$$\left[1 + \frac{\lambda_1 \bar{\Gamma}(p)}{\rho + \delta + \mu} \right] \cdot L(p) = \frac{r}{f'[f^{-1}(p)]}$$

that is to say

$$(23) \quad \frac{\lambda_0 u M}{N} \cdot \frac{\int_b^p [\delta + \mu + \lambda_1 \bar{\Gamma}(x)] dH(x)}{[\delta + \mu + \lambda_1 \bar{\Gamma}(p)]^2} \cdot \frac{\rho + \delta + \mu + \lambda_1 \bar{\Gamma}(p)}{\rho + \delta + \mu} = \frac{r}{f'[f^{-1}(p)]}$$

The lower bound \underline{p} of Γ 's support solves $\bar{\Gamma}(\underline{p}) = 1$ by definition. This leads to

$$(24) \quad \frac{\rho + \delta + \mu + \lambda_1}{(\rho + \delta + \mu)(\delta + \mu + \lambda_1)} \cdot \frac{\lambda_0 u M}{N} \cdot H(\underline{p}) = \frac{r}{f'[f^{-1}(\underline{p})]}$$

Note that, since both H and $r/f'[f^{-1}(p)]$ are increasing, there may a priori exist several solutions to Equation (24). Since these solutions are obviously all above \underline{b} , we can take their infimum as the value of \underline{p} .

The next proposition (proved in Appendix A.1) shows that Equations (23) and (24) indeed characterize an equilibrium solution, in the sense that no firm has an incentive to deviate.

PROPOSITION 1. *The equilibrium distribution of productivities is the unique solution to the integral Equation (23) where the minimal productivity \underline{p} is chosen as the smallest solution to Equation (24).*

The final equilibrium condition we impose is a zero-profit condition, $\Pi^* = 0$, which results in the long run from the assumption of free entry and exit of firms into competition. This last condition determines the long-run equilibrium number of competing firms, and completes the description of the equilibrium distribution of productivities.

The optimal profit is

$$\Pi^* = \frac{\rho + \delta + \mu + \lambda_1}{(\rho + \delta + \mu)(\delta + \mu + \lambda_1)} \frac{\lambda_0 u M}{N} \int_b^p H(b) db - r f^{-1}(\underline{p})$$

The envelope theorem implies that $\partial \Pi^* / \partial N < 0$. Therefore, when the number of active firms increases, profits fall. Solving $\Pi^* = 0$ for N then yields the maximal number of firms possibly operating in the market.

Unfortunately, $L(p)$ depends on the distribution of productivities in an intricate way that precludes any analytical solution for $\bar{\Gamma}(\cdot)$ to Equation (23) in the general case. A closed-form solution can nonetheless be obtained in some special cases. The case of homogeneous workers is obviously one of those. As another example, the case of a continuous distribution of opportunity costs of employment with no time discounting ($\rho = 0$) is explored in Appendix A.4.

5. SIMULATIONS AND FURTHER COMMENTS

Numerical techniques can be used to give a visual picture of the results obtained so far. The version of the model that we choose to simulate is the simple particular case of no future discounting presented in Appendix A.4.¹⁴ Note that, under the assumption of a strictly positive mortality rate μ , this assumption is consistent with the existence of the workers' value functions $V(\cdot)$ and $V_0(\cdot)$, since that constant probability flow of dying acts formally exactly as a rate of discount from their point of view. Also, the empirical literature on dynamic choice models teaches us that the discount rate is hardly identified and most studies fix/estimate it to a small value.¹⁵ The assumption adopted in this section, i.e., that of a discount rate that is only associated with the probability of death, can therefore be considered an interesting simplification yielding a very reasonable approximation of the more complete model. The question we primarily want to address with these simulations is the following: What are the distributions γ , g , and h_0 compatible with an exogenously given distribution h of employment opportunity costs among unemployed workers?

We specify the distribution h is a normal distribution $\mathcal{N}(m, \sigma^2)$ truncated below at b . The production technology is $f(K) = [K_0^{1-1/\eta} + (AK^\alpha)^{1-1/\eta}]^{\eta/(\eta-1)}$, where K stands for physical capital and K_0 is a constant that could correspond to any other sort of input like human capital for instance. Note that this specification encompasses the simple isoelastic case ($K_0 = 0, \alpha < 1$) and a more general CES form ($K_0 > 0, \eta < +\infty$). We exploit both cases in our simulations below.

Table 1 summarizes the adopted exogenous parameter values. As announced, we set the workers' discount rate ρ to 0, and the total number M of workers is normalized to 1. The basis time period being a month, the user cost of capital is consistent with an annual interest rate of 10%. In the isoelastic case, the retained value for α is borrowed from estimates by Robin and Roux (1998). In the CES case, we take $\eta > 1$ to ensure that the elasticity of f is an increasing function of K , a property which will turn out to be of some importance (see below), and we take K_0 such that $f(0) = K_0 < b$ remains consistent with our original assumptions on f . The rates of job creation (λ_0) and job destruction (δ) are roughly consistent with European standards. So is the arrival rate of offers to employed workers λ_1 (see Bontemps et al., 1999, 2000). The population renewal rate (μ), and all the parameters pertaining to the distribution h were chosen to yield reasonable values of the endogenous variables.

The resulting values of the unemployment rate u , the number of operating firms N , and the productivity range $[p, \bar{p}]$ are reported in Table 2. In both cases, the

¹⁴ We have been able to perform some simulations of the more complete model, with $\rho > 0$ (which are available upon request). Because no closed-form solution of the model is available in the general case, those exercises are far more demanding in terms of computational capacity. Moreover, they deliver results that show no qualitative difference, and hardly any quantitative difference for sensible values of ρ with the $\rho = 0$ case that we are presenting in this section.

¹⁵ Among many other studies, Rust and Phelan (1997) arbitrarily choose an annual 2%, Keane and Wolpin (1997) estimate an annual 6.4%, and Gillespie (1998) chooses a daily 0.9997 which corresponds to an annual 10%. Most studies conclude from sensitivity analysis that the other parameter estimates only slightly depend on the value chosen for the discount rate.

TABLE 1
PARAMETER VALUES

Specification	Distribution h				Flows		
	m	σ	\underline{b}	δ	μ	λ_0	λ_1
Isoelastic	10300	750	10000	0.005	0.0015	0.077	0.012
CES	10300	750	10000	0.005	0.0015	0.077	0.012
	Technology				Workers		
	r	α	η	A	K_0	M	ρ
Isoelastic	0.008	0.1	–	2500	0	1	0
CES	0.008	0	1.7	10^{-6}	9950	1	0

TABLE 2
SIMULATION RESULTS

	u (percent)	\underline{p}	\bar{p}	N	$\phi(\underline{b}, \bar{p})$
Isoelastic	7.8	11867.4	13365.2	0.023	5411.1
CES	12.3	10266.8	11664.2	0.019	8438.9

generated unemployment rates lie in a reasonable range by European standards, and the predicted numbers of firms implies an average of 43 to 50 employees per firm, which is not unrealistic. All graphical results are gathered in Figures 1–4, the first two figures reporting the results obtained with an isoelastic production function, and the last two figures corresponding to the CES case. In Figures 1 and 3, panel (a) plots the chosen distribution $h(\cdot)$. The resulting distribution γ of productivities is plotted on panel (b). The corresponding $h_0(\cdot)$, which is derived from h and $\bar{\Gamma}$ according to Equation (14), is shown on panel (c). The economy-wide wage distribution, $g(w)$, can be seen on panel (d). Panel (e) displays the wage distribution within a typical firm, the analytical definition of which is $\ell(w | p) = L[q(w, p)] \cdot \partial q / \partial w$. Total employment in each firm, $L(p)$, is plotted on panel (f) as a function of productivity. In Figures 2 and 4, panel (a) displays the current operating surplus (COS), panel (b) plots the capital/output ratio, panel (c) plots the mean wage per firm, and finally panel (d) shows the output share of labor. All quantities are displayed as functions of the productivity parameter p , which is directly bound to the amount of capital K through the function f .

These graphs deserve some comment. As can be seen from panels (a) and (b) of Figures 1 and 3, there is not much qualitative difference between h and h_0 . This suggests that we would obtain very similar results by adopting a hump-shaped distribution h_0 as the exogenous primitive of our model, instead of starting with an exogenous h . The reason why we took the latter option is clearly that an analytical expression of $\bar{\Gamma}(\cdot)$ featuring $h(\cdot)$ is available (see Equation (A.10)), which greatly simplifies the computations.

As to the aggregate wage distribution $g(\cdot)$, we see that it is thin-tailed at both ends (as was theoretically predicted), which is a nice feature. Furthermore, it clearly comprises two parts: A relatively high mode on the right, and a tail on the

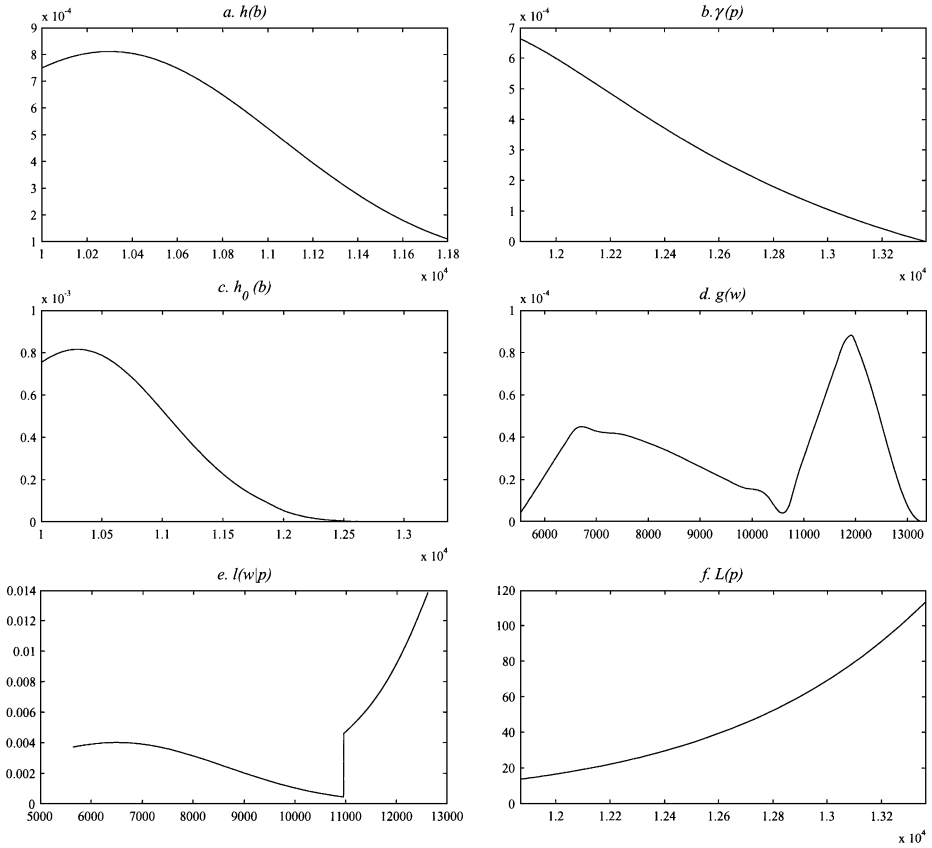


FIGURE 1

EQUILIBRIUM DISTRIBUTIONS (COBB-DOUGLAS)

left. The latter corresponds to wages in the range $[\phi_0(\underline{b}, \bar{p}), \phi(\underline{p}, \bar{p})]$, while the former covers $[\phi(\underline{p}, \bar{p}), \bar{p}]$. In other words, the left tail only contains workers in their first employment spell after a period in unemployment, whereas the right mode also contains workers having already received at least one offer while employed. Panel (d) in Figure 1 reveals that the left tail is very long in the isoelastic case. Regarding the shape of the typical empirically observed wage distribution, the left part of our artificial wage distribution is therefore admittedly not as good looking as its right part in this particular case, for this left tail is actually too long for a good fit with the data. This malformation is not as bad as it looks, however. First, as can be seen from Figure 3, panel (d), the model does much better in the CES case: The generated wage distribution is much more packed, even though it is still a bit skewed to the right. Second, the workings of the model suggest at least one way in which it could be remedied. Indeed, our model as it is written ignores any kind of institutional constraint that could distort the equilibrium wage distribution. Minimum wage constraints, in particular, are often binding (at least in Europe), and are likely to pack the wage distribution even further.

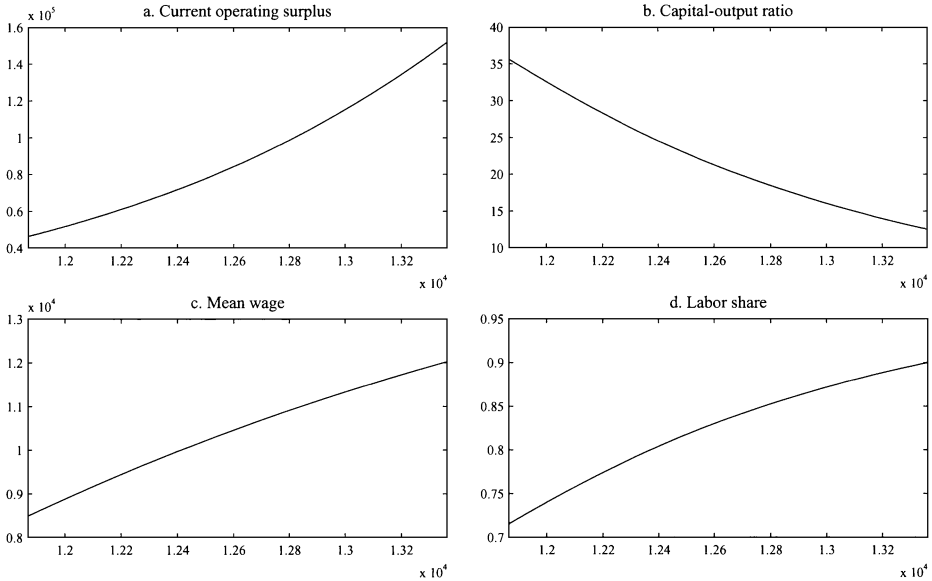


FIGURE 2

EQUILIBRIUM DISTRIBUTIONS (COBB-DOUGLAS, CONTINUED)

A similar dichotomy between first-spell employees and employees having already received an external offer is clearly reflected by the within-firm wage distribution (panel (e) in Figures 1 and 3), which turns out to be discontinuous at the wage $\phi(p, p)$.

Concerning the distribution of firm sizes, we see from panel (f) that the size of a type p firm, $L(p)$, is an increasing function of its productivity under both specifications. This is one of the preserved features from the original Burdett and Mortensen (1998) model. Since firms with higher productivities are able to pay higher wages, the supremum of all wages paid by a firm of type p being precisely p , this result is consistent with the widespread idea of larger firms paying higher wages. Yet, our model has another prediction: Since more productive firms are more attractive to workers—for the very reason that they *potentially* and/or *eventually* pay higher wages—low-paid workers are willing to accept even lower wages to work in a more productive firm. As a result, the within-firm earnings distribution has a wider support in larger firms: Its upper bound, p , obviously increases with p and its lower bound, $\phi_0(b, p)$ decreases with p . This raises the question of whether the *average wage* increases with firm's size, as is usually observed in the data. Letting $\bar{w}(p)$ denote the average wage in any firm of type p , we have

$$\bar{w}(p) = \frac{\int_{\phi_0(b,p)}^p w dL(w | p)}{L(p)} = \frac{pL(p) - \pi(p)}{L(p)}$$

Since $\pi(p) = r f^{-1}(p)$ from the zero-profit condition and $\pi'(p) = r(f^{-1})'(p)$ from the optimality condition for profits, we may substitute those expressions into that

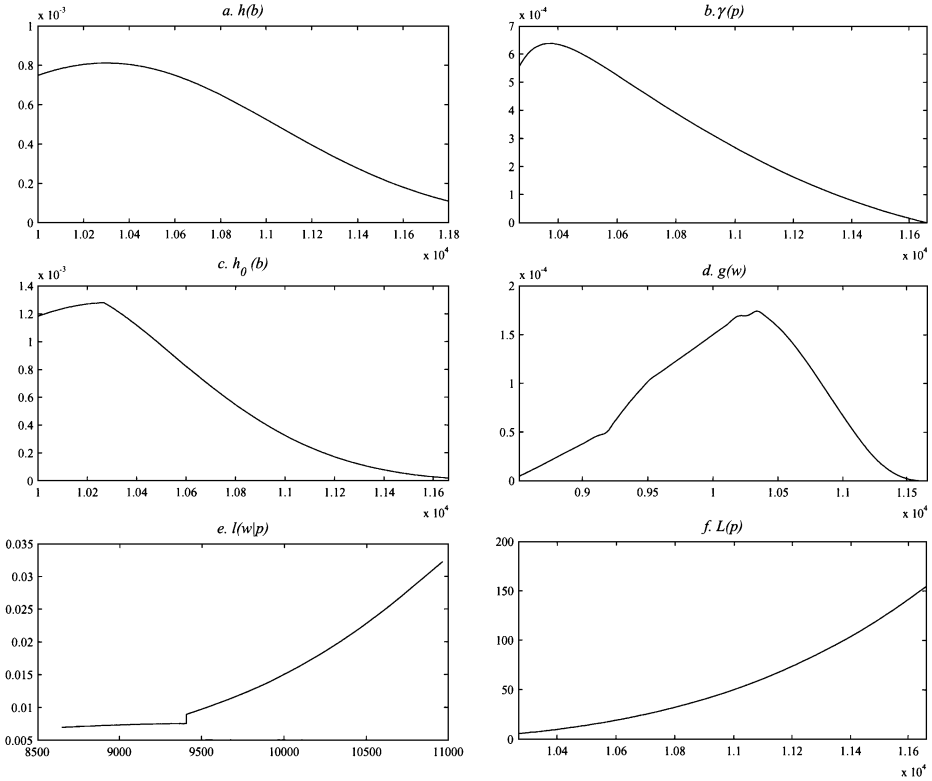


FIGURE 3

EQUILIBRIUM DISTRIBUTIONS (CES)

of $\bar{w}(p)$ to obtain

$$(25) \quad \bar{w}(p) = p - \frac{\delta + \mu + \lambda_1 \bar{\Gamma}(p)}{\delta + \mu} \cdot \frac{f^{-1}(p)}{(f^{-1})'(p)}$$

$$(26) \quad = p \cdot \left[1 - \alpha(p) \cdot \frac{\delta + \mu + \lambda_1 \bar{\Gamma}(p)}{\delta + \mu} \right]$$

where $\alpha(p)$ is the elasticity of f . This last expression shows that, provided that $\alpha(\cdot)$ does not increase too steeply with productivity, the average wage is an increasing function of p , and hence of firm size. In our isoelastic example, $\alpha(\cdot)$ is a constant and $\bar{w}(p)$ is accordingly increasing (see panel (c) of Figure 2). Furthermore, as shown on panel (c) of Figure 4, our CES example also has this property, which in practice turns out to obtain for a wide range of parameter values.

We finally turn to the labor share of value-added and to the capital/output ratio (panels (b) and (d) in Figures 2 and 4). As can easily be seen from (26), the labor share is equal to $\bar{w}(p)/p$. Moreover, since the share of capital is $rK/[pL(p)] = 1 - \bar{w}(p)/p$, the capital/output ratio is directly derived from the labor share as

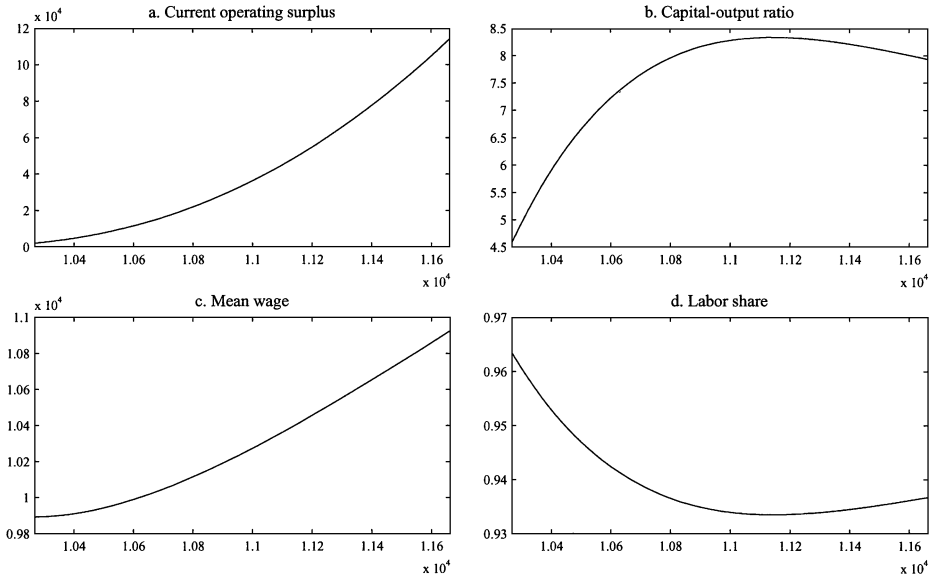


FIGURE 4

EQUILIBRIUM DISTRIBUTIONS (CES, CONTINUED)

$[1 - \bar{w}(p)/p]/r$. From (26), we thus see that

$$(27) \quad \text{labor share} = \frac{\bar{w}(p)}{p} = 1 - \alpha(p) \cdot \frac{\delta + \mu + \lambda_1 \bar{\Gamma}(p)}{\delta + \mu}$$

Its monotonicity with respect to p is therefore a priori ambiguous. In particular, in the isoelastic case, $\alpha(\cdot)$ is a constant and model thus predicts a labor share that would increase with capital, which is at variance with the facts (see, e.g., Robin and Roux, 1998, and Abowd et al., 1999). Moreover, an increasing labor share automatically implies a decreasing capital/output ratio, which also is counterfactual. But here again, the CES specification seems to deliver much more satisfactory predictions, since both indicators generally have the proper monotonicity in this case (see Figure 4).¹⁶

6. CONCLUSION

In this article we have developed a simple equilibrium model of search unemployment with on-the-job search generating equilibrium productivity and wage dispersion. The key novelty in our model is that

- (i) we allow firms to respond to the offers received by their employees from competing firms, and

¹⁶ As can be seen from Equation (27), this is where the upward slope of f 's elasticity comes into play. By offsetting the tendency to decrease $\bar{\Gamma}$, it gives the labor share its downward slope.

- (ii) we allow firms to vary their wage offers according to the reservation wage of the particular worker they meet.

These two assumptions imply that, contrary to what happens in the conventional equilibrium search model with on-the-job search, firms yield no rent to their employees, unless they are forced by their competitors. In this sense, the wage-posting strategy we allow firms to implement is optimal, which is quite natural in a world where firms are the only agents holding any market power on the labor market (in the sense that workers only have the choice to either accept or reject the offers they receive from firms), provided that full information prevails.

Our main results are the following: First, we have shown that the earnings distribution generated by our model is generically thin-tailed at both ends, as is typically observed in the data. This contrasts with the persistent, counterfactual result of the conventional search model that generates a wage distribution that is increasing over its entire support. Simple numerical exercises show that, using very standard functional forms and roughly calibrated parameter values, the model delivers fairly realistic quantitative results. Second, our model is potentially flexible enough to authorize nontrivial wage mobility both within and between firms. This makes us somewhat confident about its empirical future.

There are several obvious directions in which our model can be extended. One was mentioned in the main text and consists of introducing a binding minimum wage in our setup. Another one is to endogenize the distribution of employment opportunity costs, H , which we took as exogenously given in the simulations of this article. This can be done by introducing unemployment compensation in the form of benefits being proportional to past wages. With this (realistic) assumption, wage dispersion automatically generates dispersion in the opportunity costs of employment. Finally, another extension would be to endogenize firms' behavior even further by allowing them to choose the rate at which they meet workers through an endogenous vacancy posting decision, as in matching models of unemployment (Pissarides, 1990; Mortensen and Pissarides, 1994). A synthesis between the search and matching approaches is also available in Mortensen (1998) or in the extension of Burdett and Mortensen (1998) by Robin and Roux (1998). This would certainly yield further insights into the issue of equilibrium unemployment. All those extensions are under current investigation.

APPENDIX

A.1. *The Distribution of Earnings.* As announced in the main text, we count the number of workers earning less than w in any firm, and then sum over the relevant number of firms. The only difficulty is that some firms—the least productive ones—neither can attract workers at very low wages nor afford high-wage workers. We thus have to partition the support of Γ to get an accurate expression of G .

Case 1 ($\phi_0(\underline{b}, \bar{p}) \leq w \leq \phi_0(\underline{b}, p)$). Here the wage is so low that the least productive firms are not even attractive to a type \underline{b} unemployed worker. Defining the function $s_0(w, b)$ by $\phi_0[b, s_0(w, b)] = w$, only firms with productivity greater than $s_0(w, b)$ can hire workers for less than w . And with w in the range $[\phi_0(\underline{b}, \bar{p}), \phi_0(\underline{b}, p)]$, it is easy to see that $s_0(w, b) \geq \underline{p}$, so not all firms will actually

be able to have employees paid less than w . Accordingly, $G(w)$ is given over that range by

$$(A.1) \quad \frac{(1-u)M}{N}G(w) = \int_{s_0(w,b)}^{\bar{p}} L(w|p) d\Gamma(p) = \int_{s_0(w,b)}^{\bar{p}} L[q(w,p)] d\Gamma(p)$$

Finally, we get the distribution function through a mere differentiation:

$$(A.2) \quad \frac{(1-u)M}{N}g(w) = \int_{s_0(w,b)}^{\bar{p}} L[q(w,p)] \frac{\partial q}{\partial w}(w,p) d\Gamma(p)$$

since $L[q(w, s_0(w, b))] = L(b) = 0$.

Note that, since by definition $s_0[\phi_0(\bar{b}, \bar{p}), \bar{b}] = \bar{p}$, the above relationship shows that $g(\cdot)$ is nil at the lower bound of its support, i.e., $g[\phi_0(\bar{b}, \bar{p})] = 0$.

Case 2 ($\phi_0(\bar{b}, \bar{p}) \leq w \leq \bar{p}$). In this case, all firms are productive enough to attract at least some workers by an offer of w , and also to employ some workers at wages higher than w . $G(w)$ is thus simply given by

$$(A.3) \quad \frac{(1-u)M}{N}G(w) = \int_p^{\bar{p}} L(w|p) d\Gamma(p) = \int_p^{\bar{p}} L[q(w,p)] d\Gamma(p)$$

and $g(w)$ is defined by

$$(A.4) \quad \frac{(1-u)M}{N}g(w) = \int_p^{\bar{p}} L[q(w,p)] \frac{\partial q}{\partial w}(w,p) d\Gamma(p)$$

Note that the continuity of $g(w)$ at $w = \phi_0(\bar{b}, \bar{p})$ is ensured by the fact that $s_0[\phi_0(\bar{b}, \bar{p}), \bar{b}] = \bar{p}$ (see Equations (A.2) and (A.4)).

Case 3 ($w \geq \bar{p}$). In this final case, all firms do have employees paid less than w , but only those more productive than w also have employees paid more than w . We thus have to distinguish between those two categories of firms to define $G(w)$:

$$(A.5) \quad \frac{(1-u)M}{N}G(w) = \int_p^w L(p) d\Gamma(x) + \int_w^{\bar{p}} L(w|p) d\Gamma(p)$$

$$(A.6) \quad = \int_p^w L(p) d\Gamma(p) + \int_w^{\bar{p}} L[q(w,p)] d\Gamma(p)$$

The corresponding value of the density $g(\cdot)$ is

$$(A.7) \quad \frac{(1-u)M}{N}g(w) = \int_w^{\bar{p}} L[q(w,p)] \frac{\partial q}{\partial w}(w,p) d\Gamma(p)$$

Since $L(w) = L[q(w, w)]$, the partial derivatives of the two integral bounds cancel each other.

This last expression has a crucial implication: It shows that $g(\bar{p}) = 0$. The implausible increasing distribution of earnings persistently obtained in Burdett and

Mortensen-like models is therefore ruled out under the current specification. Here, the $g(w)$ necessarily slopes downward in a neighborhood of the upper bound of its support.

A.2. *Derivation of the Mean Rent.* For a given pair (w, p) , the following sequence of equalities stems from Equation (5) and the definition of $q(w, p)$:

$$\begin{aligned}
 \text{(A.8)} \quad & (\rho + \delta + \mu) \cdot [V(b, w, p) - V_0(b)] \\
 & = (\rho + \delta + \mu) \cdot [V(b, q(w, p), q(w, p)) - V_0(b)] \\
 & = q(w, p) - b
 \end{aligned}$$

Substituting (A.8) in (19), we get

$$\text{(A.9)} \quad (\rho + \delta + \mu) \cdot E(V - V_0) = \int_p^{\bar{p}} \frac{N\gamma(p)}{(1-u)M} \cdot \int_{\phi_0(b,p)}^p (q(w, p) - b) \cdot L(dw | p) dp$$

The identity $L(w | p) = L[q(w, p)]$ implies that

$$L(dw | p) = L'[q(w, p)] \cdot \frac{\partial q}{\partial w}(w, p) \cdot dw$$

Using the change of variables $x = q(w, p)$ in the central integral of (A.9), we get

$$(\rho + \delta + \mu) \cdot E(V - V_0) = \int_p^{\bar{p}} \frac{N\gamma(p)}{(1-u)M} \cdot \int_b^p (x - b) \cdot L'(x) dx dp$$

Integrating by parts, first in the inner, then in the outer integral of the last line above leads to

$$(\rho + \delta + \mu) \cdot E(V - V_0) = \frac{N}{(1-u)M} \cdot \int_p^{\bar{p}} (p - b) \cdot L'(p) \cdot \bar{\Gamma}(p) dp$$

which, together with the expression (12) of $L(p)$ from which one deduces an expression for $L'(p)$, implies (20).

A.3. Γ is an Equilibrium Solution (Proposition 1). To prove that no firm has an incentive to deviate from drawing a productivity in the candidate distribution γ , we only need to prove that no investment choice resulting in a productivity outside $[\underline{p}, \bar{p}]$ yields higher profit than Π^* . And indeed, the level of profit achieved by a single firm investing to be less productive than \underline{p} can be derived from (12) and (15) as

$$\Pi(p) = \frac{\lambda_0 u M}{N} \cdot \frac{\rho + \delta + \mu + \lambda_1}{(\rho + \delta + \mu)(\delta + \mu + \lambda_1)} \cdot \int_b^p H(x) dx - r f^{-1}(p)$$

Clearly, being less productive than \underline{b} means being unable to attract any worker and therefore cannot be optimal. We thus focus on values of $p \geq \underline{b}$. $\Pi(p)$ is continuously differentiable, and such that

$$\Pi(\underline{b}) = -r f^{-1}(\underline{b}) < 0$$

(the fact that $f^{-1}(\underline{b}) > 0$ stemming from our assumption that $f(0) < \underline{b}$), and

$$\lim_{p \rightarrow +\infty} \Pi(p) \leq \lim_{p \rightarrow +\infty} \frac{\lambda_0 u M}{N} \cdot \frac{\rho + \delta + \mu + \lambda_1}{(\rho + \delta + \mu)(\delta + \mu + \lambda_1)} \cdot (p - \underline{b}) - r f^{-1}(p) = -\infty$$

from the convexity of $f^{-1}(\cdot)$. $\Pi(\cdot)$, therefore, has a (not necessarily unique) maximum reached at some point in $[\underline{b}, +\infty)$.

Since $\Pi(\underline{b}) < 0$, the maximum of Π cannot be reached at \underline{b} . This implies that the condition $\Pi'(p) = 0$ is verified at any Π -maximizing value of p . The infimum of all such points thus has to be \underline{p} , which proves that no investment below \underline{p} yields better profits than Π^* .

At the other end of the distribution, given that all firms draw their investment choices from the candidate distribution, a single firm investing to be more productive than \bar{p} , being alone at this productivity level, will not be anymore attractive to workers than a firm of type \bar{p} . From (15), the profit attained by such a firm is thus equal to

$$\Pi(p) = \int_{\phi_0(\underline{b}, \bar{p})}^{\bar{p}} L[q(x, \bar{p})] dx + (p - \bar{p})L(\bar{p}) - r f^{-1}(p)$$

It is then straightforward to see from the convexity of f^{-1} that $\Pi(\bar{p}) = \Pi^*$, $\Pi'(\bar{p}) = 0$, and $\Pi''(\bar{p}) < 0$. Thus, $\Pi(p) < \Pi(\bar{p})$ for any $p > \bar{p}$.

A.4. A Continuum of Worker Types with Zero Discounting. In this appendix, we analyze the particular situation where workers do not discount the future, which amounts to set $\rho = 0$ in the above analysis. Rewriting Equation (23) with $\rho = 0$, we get, after some rearrangements,

$$\frac{[\delta + \mu + \lambda_1 \bar{\Gamma}(p)]h(p)}{(\delta + \mu + \lambda_1)H(\underline{p}) + \int_{\underline{p}}^p [\delta + \mu + \lambda_1 \bar{\Gamma}(x)] dH(x)} = \frac{\lambda_0 u M}{N(\delta + \mu)r} h(p) \cdot f'[f^{-1}(p)]$$

which solves as

$$\begin{aligned} & (\delta + \mu + \lambda_1)H(\underline{p}) + \int_{\underline{p}}^p [\delta + \mu + \lambda_1 \bar{\Gamma}(x)] dH(x) \\ &= (\delta + \mu + \lambda_1)H(\underline{p}) \cdot \exp \left\{ \frac{\lambda_0 u M}{N(\delta + \mu)r} \int_{\underline{p}}^p f'[f^{-1}(x)] dH(x) \right\} \end{aligned}$$

Differentiating and using (24), this finally turns into

$$\begin{aligned} \text{(A.10)} \quad \delta + \mu + \lambda_1 \bar{\Gamma}(p) &= (\delta + \mu + \lambda_1) \frac{f'[f^{-1}(p)]}{f'[f^{-1}(\underline{p})]} \\ &\quad \cdot \exp \left\{ \frac{\lambda_0 u M}{N(\delta + \mu)r} \int_{\underline{p}}^p f'[f^{-1}(x)] dH(x) \right\} \end{aligned}$$

and we thus end up with an explicit expression of $\bar{\Gamma}(\cdot)$, featuring the distribution of employment opportunity costs *among unemployed workers*, $H(\cdot)$.

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