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# EQUILIBRIUM WAGE DISPERSION AND INTERINDUSTRY WAGE DIFFERENTIALS\*

JAMES D. MONTGOMERY

This paper develops a search-theoretic explanation of interindustry wage differentials. Given coordination problems in the labor market, the probability of filling a vacancy is an increasing function of the wage offered; in equilibrium, firms that find vacancies more costly will offer higher wages. The model thus explains the persistence of interindustry wage differentials and their correlation with industry-average capital-labor ratio and profitability. Additionally, the model predicts that high-wage firms will receive more applications per job opening and that wages in the labor market will behave as strategic complements.

According to the standard competitive model of the labor market, homogeneous workers should receive equal compensation across firms. But if coordination problems in the labor market make it difficult for firms and workers to form matches, a single-wage equilibrium seems unlikely. If the probability of filling a job opening is an increasing function of the wage offered, firms that find vacancies more costly will offer higher wages. In this paper I formally model this intuition, providing a theoretical explanation for a number of stylized facts: interindustry wage differentials are persistent (i.e., equilibrium phenomena) and are correlated with such industry attributes as the average capital-labor ratio and profitability. In addition to predicting that firms with higher valuations of output will pay higher wages, the model also suggests that these firms will receive more applications per job opening and that wages in the labor market will behave as "strategic complements."

The paper is organized as follows. In Section I, I offer a search-theoretic explanation for interindustry wage differentials, discussing the applicability of previous search models and presenting a verbal description of my model. In Section II, I analyze the simplest version of this model (the case of two job seekers and two job openings), deriving the equilibrium vector of wages and discussing the model's empirical predictions. In Section III, I examine the

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more realistic case of many applicants and many job openings, demonstrating that the basic results derived in Section II go through. In Section IV, I examine the efficiency of the model's equilibrium. I discuss relaxing several of the model's assumptions in Section V and conclude in Section VI.

### I. TOWARD A SEARCH-THEORETIC EXPLANATION

A large number of studies have demonstrated the existence of persistent interindustry wage differentials for observationally equivalent workers.<sup>1</sup> Moreover, these differentials have been shown to be positively correlated with such industry attributes as profitability and the capital-labor ratio. To account for these findings, both market-clearing and nonmarket-clearing explanations have been offered. Market-clearing explanations include unobserved ability differences and the presence of compensating differentials.<sup>2</sup> Nonmarket-clearing explanations include a variety of efficiency wage models (in which above-market wages are paid to reduce turnover or absenteeism, to eliminate shirking, or for "sociological" reasons) and rent sharing (driven by insider bargaining power).<sup>3</sup>

While the generic "efficiency-wage hypothesis" is often given as an explanation of interindustry wage differentials, different stylized facts seem to be better explained by different versions of this hypothesis. For instance, the correlation between industry average profitability and wage differentials seems to fit well with the "sociological" version in which workers' notions of a "fair" wage are influenced by firm profitability. The correlation between the industry average capital-labor ratio and wage differentials, on the other hand, fits well with the monitoring hypothesis: shirking becomes a bigger problem in firms utilizing expensive machinery.

In a recent paper Lang [1991] has suggested that search theory may provide a more complete explanation of wage differentials, generating all of the observed stylized facts without recourse to a wide variety of efficiency-wage stories. The basic argument is simple: if it is somehow difficult for firms and workers to form a match, and if a firm can increase the probability of filling a vacancy by offering a higher wage, firms for which unfilled vacancies are

1. Recent work includes Dickens and Katz [1987a,b] and Krueger and Summers [1987, 1988].

2. See Gibbons and Katz [1987] and Murphy and Topel [1987] for analysis of the unobserved ability hypothesis.

3. See Katz [1986] for a survey of the efficiency wage literature.

relatively more expensive will pay higher wages. Thus, highly profitable firms (in which valuable orders would go unfilled if a vacancy persisted) and firms with high capital-labor ratios (in which expensive machinery would sit idle if a vacancy persisted) will pay higher wages. Search-theoretic models may thus represent an important contribution to the interindustry wage differential literature.

But in applying existing search models to the issue of wage differentials, a number of problems emerge. First, as argued by Rothschild [1973], many search models are of a "partial partial-equilibrium" nature, examining behavior on only one side of the market. Most job-search models focus upon the decision made by job seekers facing an exogenously given distribution of wage offers, ignoring the role played by firms in determining these wages.<sup>4</sup> While the industrial organization literature on "equilibrium price dispersion" models both sides of the market, equilibrium in these models is based upon a zero-expected-profit condition for firms.<sup>5</sup> As I am concerned with the effect of profitability on wages, such a condition is unacceptable. Finally, both the standard labor market search models and the "equilibrium price dispersion" literature (in its labor market form) assume that jobs are "hidden" from workers—job seekers are unaware of the location of any particular firm. Such an assumption seems difficult to reconcile with the existence of a nearly costless and fairly extensive listing of job openings (i.e., newspaper help-wanted advertising) found in real-world labor markets.<sup>6</sup>

In view of these problems I offer below an alternative model of equilibrium wage dispersion to explain interindustry wage differentials. To motivate this model (which will be developed formally in following sections), I offer the following story.

The labor market consists of  $M$  job seekers and  $N$  firms (each having one job opening). While workers are identical, the value of output varies across firms. (One might imagine that some firms have greater product market power than others, or that some utilize more capital per worker and thus produce more output per worker.) Firms attract applicants by (costlessly) placing a help-wanted advertisement in the local newspaper. Thus, the night before the newspaper is printed, each firm (simultaneously) calls the newspaper and places an ad stating the wage it is offering. In the morning the job seekers (costlessly) receive the newspaper and are

4. See Mortensen [1986] for a survey of labor-market search literature.

5. See, for example, Butters [1977] and Burdett and Judd [1983].

6. Lang [1991] makes a similar argument; see footnote 19 for a comparison of his analysis with my own.

thus informed of all job openings and the wage offered at each. Each job seeker then selects a firm at which he will apply and makes his way to that firm's personnel office. If a job seeker finds that he is the only one who has applied for a given job, he is hired and is paid the offered wage. If more than one person has applied for a job, however, the firm randomly selects a worker from all those who applied.

Given this institutional arrangement, it seems likely that some applicants will receive no offers and that some firms will fail to fill their vacancies.<sup>7</sup> To decrease the probability that their vacancies go unfilled, firms with higher valuations (those with greater product market power or higher capital-labor ratios) will offer higher wages. The model thus predicts wage dispersion in equilibrium. (In the above story two assumptions have been made for analytical convenience: firms have only one job opening; and workers may apply to only one firm. As discussed in Section V, either (or both) of these assumptions may be relaxed without altering the basic results of the model.)

The story told above, rather than merely assuming that firms receive a stochastic number of applications, incorporates maximizing behavior on the part of both firms and workers; it thus satisfies the Rothschild critique. But in contrast to models in the "equilibrium price dispersion" literature, equilibrium is not based upon a zero-profit condition. Instead, equilibrium is derived from the requirement that the expected value of applying to a firm will be equal across firms. Intuitively, a high-wage firm is likely to receive many applications. Though the worker hired at that firm receives a high wage *ex post*, the *ex ante* probability of receiving the job is relatively small. A low-wage firm at which an applicant is relatively likely to be hired offers the same *ex ante* expected payoff. Finally, the present model demonstrates that a "hidden jobs assumption" is not crucial in a search model; wage dispersion may arise even in cases where job seekers are fully informed of the location and the wage offered at each job opening in the labor market.

## II. THE $2 \times 2$ CASE

To develop the model's basic framework, I first examine the simplest case of two applicants and two firms. Obviously, real-world labor markets consist of many applicants and many firms.

7. Mortensen [1976] presents a similar model in which the labor market comprises numerous "labor exchanges." Each worker must select an exchange and, in equilibrium, must randomize. Wages are determined by supply and demand within each exchange, however, rather than by firms as in the present analysis.

But (as will be demonstrated in the next section) the major results of the  $2 \times 2$  case go through in the case of a large labor market. Moreover, in this simple case one may obtain closed-form solutions for many of the parameters of interest, derive each firm's reaction function (its wage as a function of the other firm's wage), and plot these reaction curves to determine the equilibrium wage vector.

It is easiest to solve this model backwards. Assuming that the firms have already chosen the wages they will offer, the problem facing the applicants may be described as a noncooperative game. As discussed in the last section, each applicant may apply to only one firm. If the applicants choose different firms, both are hired, and each receives the wage offered by his firm. (For example, if applicant 1 applies to firm 1 and applicant 2 applies to firm 2, then both are hired. The first receives  $w_1$ , and the second receives  $w_2$ .) If both applicants apply to the same firm, however, the firm "flips a coin" to decide which applicant receives the job. (Thus, if both applicant 1 and applicant 2 apply to firm 1, each receives  $w_1$  with probability one half.)

Assuming that  $(1/2)w_2 < w_1 < 2w_2$ , the game just described has three Nash equilibria: two asymmetric pure-strategy equilibria and one symmetric mixed-strategy equilibrium.<sup>8</sup> The strategy pairs (firm 1, firm 2) and (firm 2, firm 1) constitute the pure-strategy equilibria. In the mixed-strategy equilibrium each worker applies to firm 1 with probability  $p$  (and to firm 2 with probability  $(1 - p)$ ). Intuitively,  $p$  will vary directly with  $w_1$ ; firm 1 increases the probability each worker applies to it by increasing its wage relative to firm 2's wage.

In the present analysis I shall focus upon the mixed-strategy equilibrium. While in the simple  $2 \times 2$  case presented above a pure-strategy equilibrium may seem more likely, this implies coordination on the part of the applicants. (One story might be that the applicants contact each other after reading the want ads to make sure they do not apply to the same firm.) But in a large labor market with many openings and many applicants, such coordination becomes nearly impossible. Given a listing of job openings and the wage offered at each, a participant in a large economy (in which coordination is infeasible) would likely expect a greater number of applications to be made at positions offering higher wages. In

8. The assumption that  $w_1$  and  $w_2$  are not "too far apart" is made without loss of generality; the higher wage is never more than double the lower wage regardless of the difference in valuations. See the Appendix for the proof.

equilibrium, job seekers will then be indifferent (*ex ante*) between applying for a low-wage position (with a high probability of acceptance) and a high-wage position (with a low probability of acceptance).<sup>9</sup>

To solve for the mixed-strategy equilibrium, recall the equilibrium condition that the expected value of applying to a job is constant across jobs:

$$w_1 \cdot \text{pr}(\text{getting job at firm 1}) = w_2 \cdot \text{pr}(\text{getting job at firm 2}).$$

From applicant 1's point of view,  $\text{pr}(\text{getting job at firm 1})$  is equal to  $\text{pr}(\text{applicant 2 applies to firm 2}) + (1/2)\text{pr}(\text{applicant 2 applies to firm 1})$ . (If applicant 2 applies to firm 2, applicant 1 will receive the job at firm 1 for sure; if applicant 2 instead applies to firm 1, the employer flips a coin to determine which applicant receives the job.) An analogous equation holds for the probability of getting a job at firm 2.

As applicant 2 (in equilibrium) applies to firm 1 with probability  $p$  and to firm 2 with probability  $(1 - p)$ , the equilibrium condition becomes

$$w_1 \cdot [(1/2)p + (1 - p)] = w_2 \cdot [p + (1/2)(1 - p)].$$

Solving for  $p$ ,

$$p = (2w_1 - w_2)/(w_1 + w_2).$$

As predicted above, both  $\partial p/\partial w_1$  and  $\partial(1 - p)/\partial w_2$  are positive. A firm increases the probability that each applicant applies to it (its "application probability") by increasing its own wage.

Having examined the game played by the applicants, I now return to the firms' problems. Taking the other firm's wage as given, each firm sets its own wage to maximize expected profit, which is equal to the product of the job opening's "markup" (the difference between the opening's valuation and the wage offered) and the probability that the firm fills the vacancy. Mathematically, each firm solves

$$\max_{w_i} (v_i - w_i) \cdot \text{pr}(\text{firm } i \text{ receives at least one applicant} | w)$$

9. While I have ruled out the pure-strategy equilibria on the (intuitively plausible) grounds that coordination is difficult in a large labor market, the mixed-strategy equilibrium has been justified mainly by the process of elimination. A stronger argument supporting mixed-strategy Nash equilibria is that they are "reduced-form" versions of pure-strategy Bayesian equilibria where each of the players has "a little" private information. See Tirole [1986] for further explanation.

for  $i \in \{1, 2\}$ , where  $w$  is the vector  $(w_1, w_2)$ . As discussed in the previous section, job openings may have different valuations across firms. As wage differentials are correlated with a variety of factors, I wish to remain flexible in my interpretation of the source of these differences. Intuitively,  $v_i$  might be high because firm  $i$  possesses product market power or because it maintains a high capital-labor ratio. In either case high-valuation firms find vacancies costly.<sup>10</sup>

Defining  $p_i$  as the probability that each worker applies to firm  $i$  (i.e., firm  $i$ 's "application probability"), the firms' objective functions may be rewritten as

$$(v_i - w_i) \cdot [1 - (1 - p_i(w))^2].$$

Substituting for  $p_1$ , firm 1 solves

$$\max_{w_1} (v_1 - w_1) \cdot 3w_2(2w_1 - w_2)/(w_1 + w_2)^2.$$

The first-order condition (after much algebra) yields firm 1's reaction function (i.e.,  $w_1$  as a function of  $w_2$  and  $v_1$ ):

$$R_1(w_2) = w_2(w_2 + 4v_1)/(5w_2 + 2v_1).$$

Similar calculations yield firm 2's reaction function:

$$R_2(w_1) = w_1(w_1 + 4v_2)/(5w_1 + 2v_2).$$

Unfortunately, one cannot obtain a closed-form solution for the equilibrium wage vector  $(w_1^*, w_2^*)$ . Several results should, however, be noted. First, given the convexity and continuity of the reaction functions, the equilibrium will be unique. Moreover, given that each firm's wage offer is increasing in its own valuation, the firm with the higher valuation will offer a higher wage:  $v_1 > v_2$  implies that  $w_1^* > w_2^*$  and  $p_1 > 1/2 > p_2$ . Given the symmetry of the reaction functions, firms with equal valuations will offer the same wage:  $v_1 = v_2$  implies that  $w_1^* = w_2^*$  and  $p_1 = p_2 = 1/2$ .

Three major predictions emerge from the simple  $2 \times 2$  case examined above. First, the firm with the higher valuation will offer a higher wage. If the difference in the valuations were the result of differences in capital-labor ratios, for example, the more capital-

10. One might object to this formation, arguing that a firm's vacancy represents the absence of the "marginal" worker, whose productivity should be equal to his wage. (The firm should thus always be (almost) indifferent to filling this job as the vacancy constitutes only a second-order profit loss.) In response, I would argue that real-world technologies are not perfectly differentiable. In a plant with a Leontief production technology such that each worker operates a single machine, for example, machines will sit idle if vacancies are unfilled.



intensive firm would pay a higher wage than would the labor-intensive firm. Intuitively, an unfilled vacancy is more costly for the capital-intensive firm, and it thus offers a higher wage in order to increase the probability it receives at least one application. (A similar story may be told about firms possessing different degrees of market power.) The model presented above thus generates the stylized facts cited earlier: interindustry wage differentials are persistent (i.e., equilibrium phenomena) and are correlated with industry average profitability and the capital-labor ratio.

Second, the model predicts varying "queue lengths" across job openings. As demonstrated in the examples above, the higher-valuation firm pays a higher wage and thereby increases the probability that each applicant applies to that firm. Defining "queue length" at firm  $i$  as the expected number of applications received ( $2p_i$ ), the higher-valuation (and therefore higher-wage) firm expects a longer queue. Empirically, one would thus expect industries paying higher differentials to receive more applications per job opening. (Note that this prediction does not come out of a standard model of unobserved ability or compensating differentials). While little direct evidence is available on application rates across industries, this prediction seems to be supported by the preliminary findings of Holzer, Katz, and Krueger [1987] who analyze Employment Opportunity Pilot Project (EOPP) data on application rates.<sup>11</sup>

Third, wages in the present model are "strategic complements"; an increase in one firm's wage provokes wage increases at other firms.<sup>12</sup> While I shall not formally develop the empirical implications of this result, it seems potentially important in understanding the effect of minimum-wage laws, product-market shocks, and union power on the general level of wages in the labor market. (The standard textbook analysis of minimum-wage laws, for example, maintains that an increase in the minimum wage will cause wages to fall in the uncovered sector. The present model suggests that wages in both the covered and uncovered sectors may rise in

11. Similarly, Krueger [1988] finds that the queue length for federal government job openings is positively correlated with the ratio of government wages to private-sector wages.

12. This terminology was introduced by Bulow, Geanakoplos, and Klemperer [1985]. In oligopoly models with differentiated products where firms engage in price competition, prices are typically "strategic complements." In oligopoly models with Cournot competition, however, an increase in one firm's output prompts other firms to decrease output; quantities in these games are thus "strategic substitutes."

response to such an increase.<sup>13</sup>) Additionally, the finding that wages are strategic complements may provide an economic rationale for the widespread use of community wage surveys by corporate personnel departments. In the present model each firm will be (justly) concerned with its position in the community wage hierarchy, as relative wages determine its probability of attracting applicants.

### III. EXTENSION TO A LARGE LABOR MARKET

Having examined the simplest case of two applicants and two firms in order to develop intuition, I now examine the more realistic case in which the labor market comprises many applicants and job openings. In the following analysis I assume  $M$  applicants and  $N$  job openings. As before, each firm solves

$$\max_w (v_i - w_i) \cdot \text{pr} \{ \text{firm } i \text{ receives at least one applicant} | w \}.$$

This objective function may be rewritten as

$$(1) \quad (v_i - w_i) \cdot [1 - (1 - p_i(w))^M],$$

where  $w$  is the vector of wages  $(w_1, \dots, w_N)$  and  $p_i$  is again the probability that each worker applies to firm  $i$  given  $w$ .

The mixed-strategy application probabilities (i.e., the  $p_i$ 's) are determined by two conditions: (1) the expected value of applying to each job opening is constant across openings; and (2) the summation of these application probabilities over all job openings must equal one. Mathematically,

$$(2) \quad w_i \cdot \text{pr}(\text{getting a job at firm } i) \\ = w_j \cdot \text{pr}(\text{getting a job at firm } j) \quad \forall i, j$$

and

$$(3) \quad \sum_{i=1}^N p_i = 1.$$

As  $\text{pr}(\text{getting a job at firm } i) = [1 - (1 - p_i)^M]/p_i M$ , condition (2)

13. Gramlich [1976] provides some empirical support for this hypothesis. In a related study Grossman [1983] finds that an increase in the minimum wage compresses the overall wage distribution in the short run; this result could also be generated from the present model.

may be rewritten as<sup>14</sup>

$$(2') \quad w_i \cdot [1 - (1 - p_i)^M] / p_i M = w_j \cdot [1 - (1 - p_j)^M] / p_j M \quad \forall i, j.$$

As condition (2') represents a system of  $N - 1$  equations and condition (3) provides the  $N$ th equation, one could (at least conceptually) solve for each of the application probabilities as a function of the vector of wages, substitute into the firm's objective function, and ultimately derive reaction functions ( $w_i$  as a function of all other wages  $w_{-i}$ ).

But while a large labor market could be modeled in this fashion, this formalization implicitly places an enormous informational burden upon firms. To determine the reactions of other firms and ultimately derive its own wage offer, each firm must know the entire vector of valuations. More realistically, one would expect that firms in a large labor market base their wage offers upon some (simpler) measure of labor market "tightness." In this setting, each firm takes labor market tightness as given, considering only the direct effect of its wage upon its own application probability; equilibrium is achieved through fluctuations in labor market tightness.<sup>15</sup>

Formally, each firm  $i$  maximizes its objective function (1) subject to

$$(2'') \quad w_i \cdot [1 - (1 - p_i)^M] / p_i M = K,$$

where  $K$  is a (direct) indicator of labor market tightness.<sup>16</sup> This constraint states the (direct) relationship between the firm's wage offer and its application probability. Substituting (2'') into the firm's objective function, firm  $i$ 's problem becomes

$$\max_{p_i} v_i [1 - (1 - p_i)^M] - p_i MK,$$

and the first-order condition is

$$(4) \quad v_i M (1 - p_i)^{M-1} = MK,$$

14. Intuitively, the probability of being hired at firm  $i$  is equal to the expected number of applicants at firm  $i$  ( $= 1 - (1 - p_i)^M$ ) divided by the expected number of applicants at firm  $i$  ( $= p_i M$ ).

15. The obvious analogy is to the pricing decision of firms in a competitive product market: each firm takes the market price as given without considering its impact on other producers; price fluctuates to clear the market.

16. From condition (1)  $K$  is the expected value of applying to any job. Thus,  $K$  represents a lower bound on the set of wages a firm could offer and expect to attract workers with positive probability. If a firm should offer a wage less than  $K$ , no applicant would apply (even though he would receive the job with probability one).

which yields

$$(5) \quad p_i = 1 - (K/v_i)^{1/(M-1)}.$$

Several results emerge from this expression for the firm's (optimal) application probability. First, the equilibrium is stable. As  $K$  rises, each firm chooses a lower application probability; an increase in labor market tightness reduces firms' "excess desire" for workers. (Recall that demand is fixed at one worker per firm.) Analogously, a reduction in labor market tightness will increase firms' "excess desire." Second, firms with higher valuations will choose higher application probabilities and must accordingly offer higher wages. The first two predictions from the  $2 \times 2$  case are thus preserved: firms with higher valuations will offer higher wages and thus generate longer queues (where "queue length" is defined as  $p_i M$ , the expected number of applicants at firm  $i$ ).

Furthermore, wages remain strategic complements in the sense that an increase in labor market tightness will prompt firms to raise wages. By substituting (5) into (2''), one obtains firm  $i$ 's wage offer:

$$w_i = M K [1 - (K/v_i)^{1/(M-1)}] / [1 - (K/v_i)^{M/(M-1)}].$$

Given  $(K/v_i) \in (0,1)$ , one can show that the partial derivative  $\partial w_i / \partial K$  is positive: the firm increases its wage as the labor market tightens. In a large labor market each individual firm has little effect on labor market tightness (and thus on other wages). But general labor market shocks—an increase in the minimum wage, for example—may substantially alter market tightness and prompt wage changes.

#### IV. EFFICIENCY

As discussed above, a coordination problem in the labor market causes some firms to receive no applications and some job seekers to receive no offers. If a social planner were able to change the structure of the labor market, perhaps assigning workers to firms, the coordination problem could be overcome, and as all firms would be operating, output would be maximized.<sup>17</sup> But assuming that the social planner must operate within the existing institu-

17. Note that the social planner's role is then essentially that of the Walrasian auctioneer discussed in Section VI.

tional structure, he will maximize expected output by setting wages, thus determining each worker's vector of application probabilities  $(p_1, \dots, p_N)$ . (One should note that wages, representing transfers between the firms and workers, will not appear in the social objective function. All that matters from a welfare perspective is the (weighted) sum of the probabilities that each firm will operate.)

More formally, the social planner solves

$$\max_p \sum_{i=1}^N v_i [1 - (1 - p_i)^M],$$

subject to

$$\sum_{i=1}^N p_i = 1,$$

where  $p = (p_1, p_2, \dots, p_N)$  is the vector of application probabilities and (as before)  $v_i$  is the value of firm  $i$ 's output. Defining  $\lambda$  as the Lagrange multiplier on the planner's constraint, the first-order conditions yield

$$v_i M(1 - p_i)^{M-1} = \lambda \quad \forall i.$$

The planner thus equates marginal productivities across firms;  $\lambda$  is determined by the requirement that the application probabilities sum to one.

But as shown in the previous section, this is identical to the outcome obtained through noncooperative wage formation. From equation (4) the first-order condition of firm  $i$ 's problem is

$$v_i M(1 - p_i)^{M-1} = MK \quad \forall i.$$

In equilibrium, marginal productivities are equated across firms;  $K$  (like  $\lambda$  above) is determined by the requirement that the application probabilities sum to one. Equilibrium in the large labor market is thus (constrained) efficient.

To gain intuition for the preceding result, note the similarity between the large labor market case and the standard competitive labor market. The first term in the firm's (rewritten) objective function is expected (total) product; the second term is the product of the expected payment to each applicant ( $K$ ) and the expected number of applicants ( $p_i M$ ). Given that each firm takes labor market tightness as given (i.e., assumes that  $K$  is fixed), the firm's problem thus appears quite similar to the standard competitive

maximization problem

$$\max_L Q(L) - wL,$$

where  $Q$  is output (with price normalized to one),  $L$  is labor input, and  $w$  is the (fixed) market wage. Like firms in a standard competitive labor market, firms in the large labor market are essentially "price takers," as each assumes that the "supply" of applicants is perfectly elastic at price  $K$ .

But while firms in a large labor market are "price takers," firms in a small market are not. In the  $2 \times 2$  case, for example, each firm recognizes that an increase in its wage will not only attract more applicants but will also tighten the labor market. Because each firm's wage affects the other's marginal cost, an externality exists. In general, marginal costs (and thus marginal productivities) will not be equated across firms; a social planner could increase the expected value of output in the labor market.

Given that the social planner desires to maximize the expected value of output, the preceding analysis has demonstrated that wage differentials are (constrained) efficient in the large labor market case. But if the planner were instead minimizing unemployment (defined as the expected number of workers receiving no offers), he would equalize wages across the labor market, eliminating differentials entirely.<sup>18</sup> (Unemployment might enter the planner's objective function if, for instance, he was unable to make transfers between employed and unemployment workers.) Intuitively, wage differentials yield varying queue lengths across firms: high-wage firms will have long queues, while low-wage firms have short queues. As each firm hires at most one worker, many applicants who applied to high-wage firms will remain unemployed. If all firms offered the same wage, queue lengths would be equalized, and unemployment minimized.

## V. RELAXING ASSUMPTIONS

In the model above, two assumptions were made for analytical convenience; either (or both) could be loosened without altering the basic implications of the analysis. First, I assumed that each firm has only one job opening. Multiple openings could be permit-

18. Formally, the planner maximizes the *unweighted* sum of the probabilities that each firm will operate. The first-order conditions for this problem imply that application probabilities (and thus wages) should be equalized across firms.

ted at the cost of complicating the story told earlier: the newspaper help-wanted ads must now state not only the wage offered but also the number of vacancies to be filled. (*Ceteris paribus*, an applicant would apply to a firm with two openings rather than a firm with one opening, as he stands a greater chance of being hired at the first firm.) Second, I assumed that each worker could apply to only one firm. Multiple applications could also be permitted at the cost of complicating the story (and the necessary math). After each firm offers its job to someone in its queue, an applicant may now hold multiple offers (and presumably accepts the one offering the highest wage). Thus, some firms may have long queues (comprised of those applicants to whom it did not offer the job) but still have an unfilled vacancy. If this second assumption were relaxed, one might intuitively expect wage dispersion to decrease but not be eliminated entirely.<sup>19</sup>

While loosening each of these assumptions might add a measure of realism to the model, one should note that the basic equilibrium condition has not changed if this is permitted. Importantly, the expected value of applying to a firm is still constant across firms. (This condition fails to hold only when applicants are allowed to apply to *all* firms. In that case, firms can offer any positive wage and still receive applications. If applicants can apply to only  $(N - 1)$  firms, however, the equilibrium condition holds: any firm offering a wage below the labor market's "expected payoff" will receive no applications and will be forced to raise its wage.) And given this condition, one would expect the model's basic intuition to go through: firms with higher valuations will offer higher wages in order to increase the probability of filling their vacancies.

## VI. CONCLUSION

Somewhere deep in the background of the competitive labor-market paradigm is a Walrasian auctioneer equating supply and demand. In this story the auctioneer calls out a candidate market

19. Under assumptions similar to those of the Butters [1977] model, Lang [1991] analyzes the case of multiple applications, proving that in equilibrium wages will be dispersed and some applicants will remain unemployed. Lang additionally discusses the possibility that firms require job seekers to "apply in-person"; on a formal level, this case is quite similar to the model presented above. The present model may be viewed as an extension of the Lang analysis, formally developing the "in-person application" story in the context of a finite number of applicants and job openings.

wage, learns labor supply and demand at that wage, and continues this process until he finds a wage at which the number of job openings is equal to the number of applicants. Once the market wage is determined, the auctioneer might then assign each worker to a specific firm. In each setting, there is no reason why the value of output of a given job opening would influence the wage offered; any firm can obtain a worker at the market wage. But in a labor market without such an auctioneer, one would expect coordination problems to develop. If the burden of deciding which firm to apply to were placed on the applicants, a candidate single-wage equilibrium would no longer guarantee each firm one worker—some firms might receive many applications, while others might receive none. In such a case, wage dispersion becomes likely: firms that would find it relatively expensive to have unfilled vacancies will offer higher wages than firms that are nearly indifferent to filling a vacancy.

The model presented in this paper was developed to explain a number of stylized facts. Interindustry wage differentials for observationally equivalent workers are persistent over time and are positively correlated with such industry attributes as profitability and capital-labor ratios. As discussed above, each of these stylized facts falls easily out of a search model of the labor market. Additionally, the search story developed above generates other testable hypotheses. Industries paying higher wages will receive (on average) more applications, and wages should behave as strategic complements.

But while the model developed in this paper has demonstrated the potential importance of search considerations for wage determination, this intuition can hardly be billed as novel; various efficiency-wage stories of turnover and absenteeism provide similar intuition.<sup>20</sup> This model does, however, make two contributions. First, it goes beyond a “partial partial-equilibrium” story in which an ad hoc specification of application rates (or turnover or absenteeism) is assumed. Rather, the probability that each firm fills its vacancy is derived endogenously from the vector of wages offered. Second, the model demonstrates that a search model can be constructed without (perhaps unrealistically) assuming that jobs are hidden from workers and that firms must earn zero profits.

20. The nonefficiency-wage models of Lang [1991] (search considerations) and Weitzman [1989] (wage rigidity) also offer similar intuition: firms pay higher wages to increase the probability of hiring workers.



Wage dispersion in the present model is generated by (plausible) coordination problems developing in a large labor market where all job seekers can read the want ads.

#### APPENDIX

In solving the game played by the applicants, I assumed that  $(\frac{1}{2})w_2 < w_1 < 2w_2$ . If this condition were violated, this game has a unique pure-strategy equilibrium in which both applicants apply to the high-wage firm. (This firm thus becomes a monopsonist.) If  $v_1$  and  $v_2$  were far enough apart, one might expect this condition to be violated; the high-valuation firm might offer a wage sufficient to ensure that it always hires a worker. I shall now prove that this is not true. It is never optimal for the high-valuation firm to completely exclude the low-valuation firm from the labor market; the assumption that  $w_1$  and  $w_2$  are not "too far apart" is thus made without loss of generality.

Assume (without loss of generality) that  $v_1 > v_2$ . Substituting firm 1's reaction function into its objective function, one obtains firm 1's value function (exposed profit as a function of  $v_1$  and  $w_2$ ):

$$\Pi^* = \frac{(\frac{1}{4})(2v_1 - w_2)^2}{v_1 + w_2} = \frac{v_1^2 - v_1w_2 + (\frac{1}{4})w_2^2}{v_1 + w_2}.$$

This value function represents the profit earned in the "competitive" case in which both firms attract applicants with positive probability. Firm 1 may, however, guarantee it receives both applications by offering  $w_1 = 2w_2$ . Profit in this "monopsony" case can be written as

$$\Pi_1^m = v_1 - 2w_2.$$

(Profit is equal to the "markup" term alone, as  $\text{pr}(\text{firm 1 receives at least one application}) = 1$ .) Multiplying both the numerator and the denominator by  $(v_1 + w_2)$ , one obtains

$$\Pi_1^m = (v_1^2 - v_1w_2 - 2w_2^2)/(v_1 + w_2).$$

Comparing this "monopsony" profit with the "competitive" profit, it is obvious that  $\Pi_1^* > \Pi_1^m$  for all  $v_1$  and  $w_2$ ; firm 1 will never completely exclude firm 2 from the labor market.

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