“Residual” Wage Disparity

in Directed Search Equilibrium

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Abstract

We examine how much of the observed wage dispersion among similar workers can be explained as a consequence of a lack of coordination among employers. To do this, we construct a directed search model with homogenous workers but where firms can create either good or bad jobs, aimed at either employed or unemployed workers. Workers in our model can also sell their labor to the highest bidder. The stationary equilibrium has both *technology dispersion* – different wages due to different job qualities, and *contract dispersion* – different wages due to different market experiences for workers. The equilibrium is also constrained-efficient – in stark contrast to undirected search models with technology dispersion. We then calibrate the model to the US economy and show that the implied dispersion measures are quite close to those in the data.

JEL codes: E24, E25, J31, J24, J64

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INTRODUCTION

It has long been established that a large proportion of wage disparity cannot be explained by differences in the observed characteristics of workers. In fact, in the empirical labor literature, it is generally agreed that approximately two thirds of wage dispersion is "residual" – it occurs within narrowly defined groups of workers. (See, for example, Katz and Autor (1999).) This has always posed a challenge to theory – particularly in the light of Diamond's (1971) critique of wage dispersion, in equilibrium, with homogeneous workers. For this reason, several researchers have attributed this dispersion to "unobserved heterogeneity" among workers, with the implication that finer observations could ultimately resolve the issue.

Search theorists, on the other hand, have sought to explain this phenomenon as an equilibrium outcome with workers who are, in fact, homogeneous. Burdett and Judd (1983), for example, explore two variants of search that allow for equilibrium dispersion: non-sequential search and "noisy sequential search". Both variants, however, rely on *ex post* worker heterogeneity in order to support the result. More recently, Burdett and Mortensen (1998) argue that, in the presence of on-the-job search and Poisson arrival rates, dispersion *must* occur in equilibrium. Their model has a continuous distribution of wage offers in equilibrium, for homogenous workers. This result is sensitive to some of the underlying assumptions, however. For example, it is important that they assume that incumbent firms cannot respond by adjusting wages when being raided by other firms.[1] It is also not clear how this result would change if arrival rates were not parametric but, instead, determined by the choices of agents in the model.

Another strand of search theory has emerged recently, which focuses precisely on this issue of where buyers would choose to search, when guided by some information about sellers. This has come to be known as "directed search" theory. Following Montgomery (1991), in most directed search models, the search friction is motivated by a simple coordination problem in the presence of capacity constraints.[2] Sellers are capacity-constrained, in any period, by the fact that they have a fixed

[1] Coles (2001) considers cases where their result is robust to changes in this assumption.
[2] Not all directed search papers model this as a coordination problem. See, for example, Moen (1997).

number of objects to sell. Buyers, even when aware of the locations and prices of all the sellers, face a friction if they all move simultaneously: too many buyers may arrive at any one seller. If this seller has fewer units of the good to sell than demanded by the buyers, some buyers will be unable to purchase the good. At the same time, there may be other sellers that have too few buyers approach them, so some of the good may be left unsold. Thus, in the face of this coordination problem, some buyers and some sellers may end up frustrated even if the number of units for sale (in the aggregate) is the same as the number of units that buyers would like to purchase. In these models, the only symmetric equilibrium is one in which all buyers randomize when choosing which seller to approach. This randomization implies an endogenous matching function that resembles, in several important ways, the function used in the matching literature (for example, Pissarides (2000)).

This basic structure has been explored recently in several papers. Within it, three different sources of equilibrium wage dispersion among homogenous workers have been identified. Julien, Kennes, and King (2000) show that, when workers auction their labor, since some workers will receive more bidders than others, some workers will enjoy higher wages than others. Thus, wages can differ simply due to the randomization inherent in the coordination problem. We will refer to this type of dispersion here as "*contract dispersion*". Secondly, as shown in Acemoglu and Shimer (2000), if different jobs have different productivities, this can lead to homogeneous workers being paid differently in different jobs. We will refer to this as "*technology dispersion*".[3] The third source of wage dispersion, explored in Burdett, Shi and Wright (2001) and Shi (2001a) comes from the fact that prices charged will, in general, be a function of the severity of the capacity constraint. This draws on Peters' (1984) insight that, in capacity-constrained settings, buyers face a trade-off between prices and probability of sale. We can think of this as "*capacity dispersion*".

The concept of capacity dispersion forces us to think about which types of agents are on which side of the market and what, exactly, is being sold in the labor market. Acemoglu and Shimer (2000), Burdett, Shi, and Wright (2001) and Shi (2001a,b) follow the tradition in search theory where *firms* act as *sellers* – selling jobs

[3] Acemoglu and Shimer's (2000) model also has the added friction of non-sequential search: workers cannot see posted wages unless they pay a cost to receive a sample of them.

to workers. In Julien, Kennes, and King (2000), we model *workers* as being in the more traditional role as sellers in this market. While it seems reasonable to consider that capacity dispersion may play a major role when different sizes of firms sell jobs, it seems clear that this role would be significantly diminished when individual workers are sellers.[4]

In this paper we argue that a large proportion of the observed "residual" wage dispersion can be explained as a consequence of the basic coordination problem that underlies these directed search models. To do this, we construct the simplest possible model of this type, in which endogenous contract and technology dispersion are obtained in equilibrium. We model workers as sellers of labor, and allow firms to create vacancies of different types: high and low productivity (with different associated costs). The setup is significantly simpler than in Acemoglu and Shimer's (2000) paper, largely because we do not have the added complication of non-sequential search.[5] This allows us to derive explicit solutions for the endogenous variables. It also allows us to isolate the effects of the coordination problem alone.

We start by first examining the properties of a static model, and derive necessary and sufficient conditions for technology dispersion to exist in equilibrium, when firms are free to enter and choose their technologies. We then extend the model to a dynamic (infinite horizon) environment which allows for search, both on and off the job, and separations. We solve for values of the endogenous variables in the stationary equilibrium, and show that this equilibrium is constrained-efficient. Parameter values are then chosen so that the model matches the mean weekly wage and unemployment rate of the US economy in 1995. Key statistics of the numerical wage distribution generated by the model are then compared with those from empirical studies. Among the results, we find that the standard deviation of the log of these wages is approximately 54% of the figure given, in the Katz and Autor (1999) study, for the entire wage distribution in 1995. Perhaps more strikingly, when considering the 90-10 percentiles of the log wage distribution, the model predicts a figure of 1.08, which is quite close to the approximate 1.15 figure reported, by Katz and Autor, for "residual" wage dispersion in that year.

[4] In Julien, Kennes, and King (2001), we provide a more detailed comparison of these frameworks.
[5] Another key difference is that we allow for firm entry here, rather than fixing the number of firms.

The constrained-efficiency result is consistent with similar results in the directed search literature with homogeneity (for example: Moen (1997) and Julien, Kennes and King (2000)). However, it stands in stark contrast with those in the "undirected search" literature. For example, Sargent and Ljungquist (2000) conclude:[6]

> "In the case of heterogeneous jobs in the same labor market with a single matching function we establish the impossibility of efficiency without government intervention."

This is clearly a case where the implications of direct and undirected search theory differ substantially. The assumption that matching probabilities are unaffected by behaviour, inherent in undirected search, leads to a congestion that distorts the welfare properties of the equilibrium. When agents can choose matching probabilities, this distortion is removed.

The paper is organized as follows. Section 1 presents and analyses the static model. Section 2 then presents the structure of the dynamic model. Section 3 presents analytical results concerning the stationary equilibrium. The quantitative analysis of the model is presented in Section 4. The conclusions of the study are given in Section 5, along with a general discussion. The proofs of all the propositions in the paper are contained in the Appendix.

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[6] Acemoglu (2001) and Davis (2001) reach similar conclusions.

1. THE STATIC MODEL

Consider a simple economy with a large number N of identical, risk neutral, job candidates where each candidate has one indivisible unit of labor to sell. There are $M_i = \phi_i N$ vacancies of two types: $i \in \{1,2\}$, where $\phi_i \geq 0$, and are determined by free entry. The productivity of a worker is $y_0 = 0$ if unemployed and $y_i > 0$ if employed in a job of type *i*, where $y_2 > y_1$. It costs k_i to create a vacancy, where $k_2 > k_1$ and $y_i \geq k_i \geq 0 \ \forall i$. Each vacancy can approach only one candidate. The order of play is as follows. Given *N*, M_i vacancies of each type *i* enter the market. Once the number of entrants has been established, vacancies choose which candidate to approach. Once vacancies have been assigned to candidates, wages are determined through an ascending-bid (English) auction.[7] We solve the model using backwards induction.

Wage Determination

Each worker conducts an ascending-bid auction, where his reserve wage is simply his outside option $y_0 = 0$. In equilibrium, the wage w_i^j of a worker who is employed in a job of productivity *i*, and who had a second best offer from a job of productivity *j* is given by:

$$w_i^j = y_j \tag{1.1}$$

for all $i \in \{1,2\}$ and $j \in \{0,1,2\}$.

The Assignment of Vacancies to Workers

As is standard in directed search environments,[8] when considering the location choice of buyers, attention is restricted to the unique symmetric mixed strategy equilibrium in which each buyer of each type randomizes over sellers. Consequently,

[7] We justify the usage of an auction in this type of environment in Julien, Kennes, and King (2001). The form of auction is irrelevant, since revenue equivalence holds here. See, for example, McAfee and McMillan (1987).
[8] See, for example, Burdett, Shi and Wright (2001) and Shi (2001a,b).

in a large market, the probability p_i that a worker is approached by a vacancy of maximum productivity y_i is given by:

$$p_i = \begin{cases} p_0 = e^{-\phi_1} e^{-\phi_2} \\ p_1 = e^{-\phi_2}(1 - e^{-\phi_1}) \\ p_2 = (1 - e^{-\phi_2}) \end{cases} \tag{1.2}$$

It also follows that, in a large market, from the pool of vacant jobs of productivity y_i, a candidate obtains either (i) no offer, (ii) one offer, and (iii) multiple offers with probabilities $e^{-\phi_i}$, $\phi_i e^{-\phi_i}$ and $1 - \phi_i e^{-\phi_i} - e^{-\phi_i}$, respectively. Therefore, the probability distribution of wages is given by:

$$w_i^j, p_i^j = \begin{cases} w_0^0 = 0, & p_0^0 = & e^{-\phi_1} e^{-\phi_2} \\ w_1^0 = 0, & p_1^0 = & e^{-\phi_2} \phi_1 e^{-\phi_1} \\ w_1^1 = y_1, & p_1^1 = & e^{-\phi_2}(1 - \phi_1 e^{-\phi_1} - e^{-\phi_1}) \\ w_2^0 = 0, & p_2^0 = & \phi_2 e^{-\phi_2} e^{-\phi_1} \\ w_2^1 = y_1, & p_2^1 = & \phi_2 e^{-\phi_2}(1 - e^{-\phi_1}) \\ w_2^2 = y_2, & p_2^2 = & 1 - \phi_2 e^{-\phi_2} - e^{-\phi_2} \end{cases} \tag{1.3}$$

where p_i^j denotes the probability that worker obtains a wage w_i^j.

If the numbers of vacancies were given exogenously (i.e., ϕ_1 and ϕ_2 were parameters) then (1.3) would represent the final solution of the model. Examining (1.3), it is clear that wage dispersion has two sources: contract dispersion and productivity dispersion. For example, the difference in the wages $w_1^1 = y_1$ and $w_1^0 = 0$ is due entirely to contract dispersion: in both cases, the productivity of the job is low, but workers who earn w_1^1 had an outside offer from another low productivity job whereas workers who earn w_1^0 did not. In order to receive the highest wage $w_2^2 = y_2$, workers need to be on the right end of both contract and productivity dispersion: the presence of at least one high productivity vacancy is required to make this wage technically feasible, and the presence of at least one other high productivity vacancy, as an outside offer is required to make this wage an equilibrium outcome. It is also clear that contract dispersion can be at least as important to workers as productivity dispersion. For example, a worker in a high productivity job earns a wage equal to $w_2^0 = 0$ with probability p_2^0 while a worker in low productivity job earns a higher wage of $w_1^1 = y_1$ with probability p_1^1. Both of these probabilities are positive if $\phi_1, \phi_2 > 0$. We now turn to the determination of ϕ_1 and ϕ_2.

Vacancy Entry

The profit of a firm is equal to its output minus its vacancy creation cost and the wage it pays to the worker. Therefore, the profit π_i^j of a vacant job of productivity y_i that makes an offer to a worker who has a best rival offer of productivity y_j is given by:

$$\pi_i^j = \max\{y_i - y_j, 0\} - k_i \tag{1.4}$$

The expected profit π_i of a vacant job of productivity y_i is given by:

$$\pi_1 = \max\{q_1^0 y_1 - k_1, 0\} \tag{1.5}$$

$$\pi_2 = \max\{q_2^0 y_2 + q_2^1 (y_2 - y_1) - k_2, 0\} \tag{1.6}$$

where q_i^j is the probability that a firm earns a profit equal to π_i^j. The probability that a vacant job does not face offer competition from a rival job of productivity y_i is given by $e^{-\phi_i}$. Therefore $q_1^0 = q_2^0 = e^{-\phi_1} e^{-\phi_2}$ is the probability that the vacant job does not face a rival vacant job of either productivity, and $q_2^1 = (1 - e^{-\phi_1}) e^{-\phi_2}$ is the probability that a vacant job faces a low productivity rival but not a high productivity rival. The supply of vacant jobs of productivity y_i is determined by free entry, so the expected profit π_i of a vacant job of productivity y_i is equal to zero in equilibrium:

$$\pi_1 = \pi_2 = 0 \tag{1.7}$$

The assumption that the output of a particular type of job is greater than the cost of the job vacancy does *not* guarantee that the supply of jobs of that type is positive. (For example, it is easy to see that $q_1^0 y_1 - k_1$ can be negative if ϕ_2 is sufficiently large – making q_1^0 sufficiently small.) Therefore we do not know, based on our present assumptions, whether or not the two different jobs will exist in equilibrium. The following proposition presents necessary and sufficient conditions for this type of productivity dispersion.

Proposition 1:Both types of jobs exist in equilibrium ($\phi_i > 0 \quad \forall i$)if and only if the following conditions hold:

$$y_2 - k_2 > y_1 - k_1 \qquad \text{and} \qquad y_1 / k_1 > y_2 / k_2 .$$

Moreover, when these conditions hold, then the equilibrium values of ϕ_1 and ϕ_2 are given by:

$$\phi_1 = \ln(y_1 / k_1) - \ln((y_2 - y_1)/(k_2 - k_1)) \tag{1.8}$$

$$\phi_2 = \ln((y_2 - y_1)/(k_2 - k_1)) \tag{1.9}$$

The first condition in Proposition 1 ensures that the supply of high productivity jobs is always positive if the output of a good job net of its capital cost exceeds the output of a bad job net of its capital cost. The second condition implies that the supply of low productivity jobs is always positive if the output of a bad job per unit of capital is greater than the output of a good job per unit of capital. These two conditions are satisfied by the simple assumption of a diminishing marginal product of capital.

Under these conditions, equations (1.3), (1.7), (1.8), and (1.9) completely solve for the equilibrium payoff structure in the static model.

Constrained Efficiency

We now consider the problem of a social planner that is able to control entry, but still faces the same coordination friction as private agents. The planner chooses $\phi_1 \geq 0$ and $\phi_2 \geq 0$ to maximize total expected surplus *S*:

$$S = \max_{\phi_1, \phi_2} N\{(1-e^{-\phi_2})y_2 + e^{-\phi_2}(1-e^{-\phi_1})y_1 - \phi_1 k_1 - \phi_2 k_2\}$$

Proposition 2: The decentralized equilibrium is constrained efficient.

The reasoning behind the efficiency result is as follows. Consider the choice of whether or not to add one more low quality vacancy. With some probability, the employer with this new vacancy will approach a candidate that is also approached by some other vacancy, (of either of high or low quality). In this case, if this other vacancy is also low quality, then with some probability, the entering vacancy will hire the worker, so the gains to the match with the other employer will be lost. This is an external cost associated with the new vacancy. However, there is also a benefit created: the match of the entering vacancy and the worker. Clearly, this cost and this benefit exactly cancel each other. Thus, the social return from such a new vacancy is zero. Due to the auction mechanism, this is precisely the private return that a new low quality vacancy gets in this case.

If, however, the other vacancy is of high quality, then, again, the social value of the entering low quality vacancy is zero and the payoff will be zero, though the auction mechanism. If the entering low quality vacancy approaches a worker whom otherwise would *not* be matched, then a social benefit is generated: the value of the match y_1. The expected marginal social benefit of the new vacancy is therefore the probability that the new vacancy will be alone when it approaches a worker, multiplied by y_1. The marginal social cost of generating a new vacancy is simply the cost of creating the vacancy k_1. A social planner equates these two, and so does a private entrant.

A similar line of reasoning holds for the creation of a new high quality vacancy. In this case, however, if the other vacancy is of low quality, then the new high quality vacancy will hire the worker with probability one. Here, the gains to the match with the other vacancy y_1 will be lost, but the gains to the new match will be y_2, so the net social gains are $(y_2 - y_1)$. Once again, through the auction mechanism, this is precisely the private return that a new high quality vacancy receives. In all cases the private and social returns are equated.

It is also worthwhile to note that the role of the worker as *seller* is crucial here. In a similar model, but where firms play the role of seller of jobs, Jansen (1999) shows that only one type of job can exist in equilibrium.

2. THE DYNAMIC MODEL

There is large number, *N*, of identical risk neutral workers facing an infinite horizon, perfect capital markets, and a common discount factor $\beta > 0$. In each time period, each worker has one indivisible unit of labor to sell. At the start of each period $t = 0,1,2,3,...$, there exist E_{0t} unemployed workers, of productivity $y_0 = 0$, and E_{it} workers in jobs of productivity $y_i > 0$ where $i \in \{1,2\}$. Also, at the beginning of each period, there exist $M_{it} = \phi_{it}(N - E_{1t} - E_{2t})$ vacant jobs of each productivity type directed at unemployed workers and $\hat{M}_{2t} = \hat{\phi}_{2t} E_{1t}$ high productivity vacant jobs directed at employed workers in jobs of productivity y_1.[9] In each period a vacant job has a capital cost of k_i such that $y_i \geq y_j$ and $k_i \geq k_j$ $\forall$ $i \geq j$. Also, any match in any period may dissolve in the subsequent period with fixed probability $\rho \in (0,1)$. In each period, any vacant job can enter negotiations with at most one worker.

Within each period, the order of play is as follows. At the beginning of the period, given the state, new vacancies enter. Once the number of entrants has been established, vacancies choose which workers to approach. Once new vacancies have been assigned to candidates, wages are determined through the auction mechanism.

[9] Note that no low productivity vacant job are directed at employed workers in high productivity jobs.

Wage Determination

Let Λ_{it} denote the expected discounted value of a match between an unemployed worker and a job of productivity y_i at the start of any period. Through the auction, the workers share W_{it}^{j} of the expected discounted value Λ_{it} is equal to the expected discounted value Λ_{jt} of a match between the worker and the worker's second best available job offer:

$$W_{it}^{j} = \Lambda_{jt} \tag{2.1}$$

The Assignment of Vacancies to Workers

Unemployed workers advertise auctions with a reserve price of Λ_{0t} while workers in low productivity jobs advertise auctions with a reserve price of Λ_{1t}. The workers are distinguishable only by their employment state. As in the static model, we restrict attention to the unique symmetric mixed strategy equilibrium in which each vacancy randomises over each relevant group of workers. Consequently, the new hires of H_{2t} high productivity workers and H_{1t} low productivity workers are given respectively by:

$$H_{2t} = (N - E_{1t} - E_{2t})p_{2t} + E_{1t}\hat{p}_{2t} \tag{2.2}$$

$$H_{1t} = (N - E_{1t} - E_{2t})p_{1t} - E_{1t}\hat{p}_{2t} \tag{2.3}$$

where $p_{2t} = (1 - e^{-\phi_{2t}})$, $p_{1t} = (1 - e^{-\phi_{1t}})e^{-\phi_{2t}}$ and $\hat{p}_{2t} = (1 - e^{-\hat{\phi}_t})$. The fraction ρ of all jobs dissolve in the next period, therefore, the supply of worker of each type evolves according to the following transition equations:

$$E_{it+1} = (1 - \rho)(E_{it} + H_{it}) \qquad i \in \{1,2\} \tag{2.4}$$

The randomness of job offers implies that a worker can obtain either one, multiple or no job offers from vacancies of either type. Therefore, it follows that the expected present value of an unmatched worker satisfies:

$$V_t = (p_{0t}^0 + p_{1t}^0 + p_{2t}^0)\Lambda_{0t} + (p_{1t}^1 + p_{2t}^1)\Lambda_{1t} + p_{2t}^2\Lambda_{2t} \qquad (2.5)$$

where $p_{0t}^0 + p_{1t}^0 + p_{2t}^0 = (1+\phi_{1t}+\phi_{2t})e^{-\phi_{1t}}e^{-\phi_{2t}}$ is the probability that a worker has one or fewer offers, $p_{2t}^1 + p_{1t}^1 = e^{-\phi_{2t}}(1-\phi_{1t}e^{-\phi_{1t}} - e^{-\phi_{1t}}) + \phi_{2t}e^{-\phi_{2t}}(1-e^{-\phi_{1t}})$ is the probability of multiple offers only one of which is possibly good, and $p_{2t}^2 = 1-\phi_{2t}e^{-\phi_{2t}} - e^{-\phi_{2t}}$ is the probability of multiple good offers.

Vacancy Entry

The expected profit Π_{it} of a job of productivity y_i making an offer to an unemployed worker satisfies:

$$\Pi_{1t} = \max\{(\Lambda_{1t} - \Lambda_{0t})e^{-\phi_{1t}}e^{-\phi_{2t}} - k_1, 0\} \qquad (2.6)$$

$$\Pi_{2t} = \max\{(\Lambda_{2t} - \Lambda_{0t})e^{-\phi_{1t}}e^{-\phi_{2t}} + (\Lambda_{2t} - \Lambda_{1t})(1-e^{-\phi_{tt}})e^{-\phi_{2t}} - k_2, 0\} \qquad (2.7)$$

where $e^{-\phi_{tt}}e^{-\phi_{2t}}$ is the probability that a low or high productivity job does not face a rival, and $(1-e^{-\phi_{1t}})e^{-\phi_{2t}}$ is the probability that a high productivity job faces only a low productivity rival. The expected profit of an offer by a high productivity to a worker in a low productivity job is given by:

$$\hat{\Pi}_{2t} = \max\{(\Lambda_{2t} - \Lambda_{1t})e^{-\hat{\phi}_{2t}} - k_2, 0\} \qquad (2.8)$$

where $e^{-\hat{\phi}_{2t}}$ is the probability that high productivity job does not face a competing offer from a rival high productivity job. The supply of vacant jobs of productivity y_i is determined by free entry. Thus

$$\Pi_{1t} = \Pi_{2t} = \hat{\Pi}_{2t} = 0 \tag{2.9}$$

The value of an unmatched worker in the next period determines the outside option of an unmatched worker in the current period, so

$$\Lambda_{0t} = \beta V_{t+1} \tag{2.10}$$

The total surplus of a high productivity job is equal to the output of a high productivity job plus the discounted future flow of income from such a job weighted by the probability of an exogenous job separation into unemployment:

$$\Lambda_{2t} = y_2 + \beta[\rho V_{t+1} + (1-\rho)y_2] + \beta^2(1-\rho)[\rho V_{t+2} + (1-\rho)y_2] + ... \tag{2.11}$$

Wages in low productivity jobs are bargained with the understanding that the worker will get the increase of surplus associated with any potential favourable future bargain between the worker and a high productivity job during the worker's tenure at a low productivity job. Therefore, the expected present value of being a worker in a low productivity job must incorporate the probability of moving into a higher paying (high productivity) job in a subsequent period. Hence

$$\Lambda_{1t} = y_1 + \beta(\rho V_{t+1} + (1-\rho)X_{t+1}) + \hat{p}^1_{1t+1}\beta^2(1-\rho)(\rho V_{t+1} + (1-\rho)X_{t+2}) + ... \tag{2.12}$$

where $X_t = (\hat{p}^1_{1t}y_1 + \hat{p}^1_2\Lambda_{1t} + \hat{p}^2_{2t}\Lambda_{2t}))$ summarizes three possible outcomes: $\hat{p}^1_{1t} = e^{-\hat{\phi}_{2t}}$ is the probability that the employed worker is not recruited, $\hat{p}^1_{2t} = \hat{\phi}_{2T}e^{-\hat{\phi}_{2t}}$, is the probability that the employed worker is recruited by one good job, and $\hat{p}^2_{2t} = 1 - \hat{\phi}_{2t}e^{-\hat{\phi}_{2t}} - e^{-\hat{\phi}_{2t}}$ is the probability that the worker is recruited by one or more high productivity jobs.

In this paper we will, for the most part, restrict our attention to the stationary equilibrium. However, the following proposition establishes that certain values are stationary in any equilibrium of this model.

Proposition 3: The equilibrium values of $\{\phi_{1t}, \phi_{2t}, \hat{\phi}_{2t}, \Pi_{1t}, \Pi_{2t}, \hat{\Pi}_{2t}, V_t, \Lambda_{0t}\ \Lambda_{1t}\ \Lambda_{2t}\}$, denoted by $\{\phi_1, \phi_2, \hat{\phi}_2,\ \Pi_1,\ \Pi_2\ , \hat{\Pi}_2, V\ ,\ \Lambda_0, \Lambda_1, \Lambda_2\}$, are stationary.

For the remainder of the paper, we restrict our attention to the stationary equilibrium.

3. THE STATIONARY EQUILIBRIUM

The following propositions characterize some of the important features of the stationary equilibrium. The first concerns the fractions of the workforce that are assigned, at the end of every period, to the different types of jobs.

Proposition 4: In the stationary equilibrium, the fraction n_i of workers in each productivity state y_i is given by:

$$n_0 = \frac{\rho p_0}{1-(1-\rho)p_0} \tag{3.1}$$

$$n_1 = \frac{(1-(1-n_0)(1-\rho))p_1}{\rho+(1-\rho)\hat{p}_2} \tag{3.2}$$

$$n_2 = 1 - n_1 - n_0 \tag{3.3}$$

where the p_i 's are given by equation (1.2) and $\hat{p}_2 = (1-e^{-\hat{\phi}_2})$.

Notice that the stationary structure allows us to use some of the results developed in Section 1, which considers the static model. The next proposition establishes a sufficient condition for on-the-job search to exist in equilibrium.

Proposition 5: Vacant good jobs are directed at workers employed in bad jobs if $(y_2 - y_1) > (1-\beta(1-\rho))k_2$, in which case the supply of these jobs is determined by:

$$k_2 = (y_2 - y_1)e^{-\hat{\phi}_2} + \beta(1-\rho)(\hat{\phi}_2 e^{-\hat{\phi}_2} + e^{-\hat{\phi}_2})k_2 \tag{3.4}$$

This condition ensures that good jobs will open up in response to the existence of bad jobs. In particular, it ensures that firms will recruit workers in bad jobs. However, it does not ensure that good job vacancies will be opened up in head to head competition with bad jobs in the recruitment of unemployed workers. In other words, we still have to determine whether ϕ_2 is strictly positive. It also does not address the existence of bad jobs in equilibrium. These two concerns are considered in the following two propositions.

Proposition 6: Unemployed workers receive more good offers on average than workers in bad jobs. The supply of good jobs aimed at unemployed workers is determined by:

$$(k_2 - k_1)e^{-\hat{\phi}_2} = k_2 e^{-\phi_2} \tag{3.5}$$

where $(k_2 - k_1) < k_2$ implies $\phi_2 > \hat{\phi}_2$.

The next proposition establishes the existence of an equilibrium with on-the-job search.

Proposition 7: An equilibrium with $\phi_1, \phi_2, \hat{\phi}_2$>0 exists. The supply of bad jobs in this equilibrium is determined by:

$$k_1 = \left(y_1 + \beta(1-\rho)(k_1\phi_1 + k_2\phi_2 - k_2\hat{\phi}_2)\right)e^{-\phi_1}e^{-\phi_2} \tag{3.6}$$

and the supply of good jobs is determined by equations (3.4) and (3.5).

Equations (3.4), (3.5) and (3.6) determine the stationary equilibrium values of ϕ_1, ϕ_2, and $\hat{\phi}_2$. (That is, they determine the numbers of vacancies of the different types in equilibrium.) Computationally, the system is recursive: (3.4) determines $\hat{\phi}_2$, then (3.5) determines ϕ_2, then (3.6) solves for ϕ_1. While simple analytical solutions are not available, it is straightforward to compute these values numerically, for any given vector of parameters ($y_1, y_2, k_1, k_2, \beta, \rho$) that satisfies the restriction in Proposition 5. Before proceeding to the numerical analysis, however, it is useful to draw out some more analytical results.

Proposition 8: The expected values of workers in the different states are given by:

$$V = \frac{y_2 - \left(k_1(1+\phi_1+\phi_2) + k_2(1+\phi_2)e^{-\phi_2}e^{\hat{\phi}_2})(1-\beta(1-\rho))\right)}{1-\beta} \tag{3.7}$$

$$\Lambda_0 = \beta V \tag{3.8}$$

$$\Lambda_2 = \frac{y_2 + \beta\rho V}{1-\beta(1-\rho)} \tag{3.9}$$

$$\Lambda_1 = \frac{y_1 + \beta(\rho V + (1-\rho)(1-\hat{\phi}_2 e^{-\hat{\phi}_2} - e^{-\hat{\phi}_2})\Lambda_2))}{1-\beta(1-\rho)(\hat{\phi}_2 e^{-\hat{\phi}_2} + e^{-\hat{\phi}_2})} \tag{3.10}$$

With ϕ_1, ϕ_2, and $\hat{\phi}_2$ determined in equations (3.4), (3.5) and (3.6), the values of $V, \Lambda_0, \Lambda_1,$ and Λ_2 can now be determined by the equations in Proposition 8. Once again, this is a recursive system , with V determined in (3.7), then Λ_0 and Λ_2 determined in equations (3.8) and (3.9). With V and Λ_2 determined, (3.10) determines Λ_1.

We can now solve for the period wages in the stationary equilibrium. These are determined by:

$$w_0^0 = 0 \tag{3.11}$$

$$\frac{w_2^j + \beta\rho V}{(1-\beta(1-\rho))} = \Lambda_j \qquad j \in \{0,1,2\} \tag{3.12}$$

$$\frac{w_1^j + \beta(\rho V + (1-\rho)(1-\hat{\phi}_2 e^{-\hat{\phi}_2} - e^{-\hat{\phi}_2})\Lambda_2))}{1-\beta(1-\rho)(\hat{\phi}_2 e^{-\hat{\phi}_2} + e^{-\hat{\phi}_2})} = \Lambda_j \qquad j \in \{0,1\}$$

(3.13)

where w_i^j denotes the wage per period of a worker in state W_i^j. The following proposition now presents the entire wage distribution in the stationary equilibrium.

Proposition 9: The wage distribution in the stationary equilibrium is as given in the following tables.

Wages
$w_0^0 = 0$
$w_1^0 = \Lambda_0 - \beta\rho V - \beta(1-\rho)(\Lambda_2 - (\Lambda_2 - \Lambda_0)(1+\hat{\phi}_2)e^{-\hat{\phi}_2})$
$w_1^1 = y_1$
$w_2^0 = (1-\beta(1-\rho))\Lambda_0 - \beta\rho V$
$w_2^1 = (1-\beta(1-\rho))\Lambda_1 - \beta\rho V$
$w_2^2 = y_2$

Fraction of workforce earning each wage
$n_0^0 = n_0$
$n_1^0 = n_1(\phi_1 e^{-\phi_1})/(1-e^{-\phi_1})$
$n_1^1 = n_1 - n_1^0$
$n_2^0 = [1+n_0(1-\rho)/\rho]e^{-\phi_1}e^{-\phi_2}$
$n_2^1 = [1+n_0(1-\rho)/\rho]\phi_2 e^{-\phi_2}(1-e^{-\phi_1}) + n_1\hat{\phi}_2 e^{-\hat{\phi}_2}(1-\rho)/\rho$
$n_2^2 = n_2 - n_2^0 - n_2^1$

Given the parameters $(y_1, y_2, k_1, k_2, \beta, \rho)$ and equations (3.1)-(3.10), the equations in Proposition 9 determine the wage structure in the stationary equilibrium. At this point, it is useful to compare this structure with that of the static model (given in equation (1.3)). Clearly, w_0^0, w_1^1, and w_2^2 are the same in the two models. While the reasoning why $w_0^0 = 0$ is straightforward in both models, w_1^1 and w_2^2 may need some explanation. The key is that, in each period, the expected value of profits for the each firm is driven down to zero. If two (or more) vacancies of the same type (but none of the other type) land at the doorstep of the same worker, any chance of a positive *ex post* profit for these firms disappears. The cost they paid to generate the vacancy, is already sunk. They are, in effect, just like firms in the static game. The value of holding the job open into the next period is zero. As in the static game, Bertrand competition between the two identical firms drives the current payoff to zero. The value of w_2^1 is also determined, as in the static model, by the surplus associated with a low quality job.

Unlike the value of firms, the value of workers is not driven to zero in the dynamic model. Whereas, in the static model, each worker's outside option is zero; in the dynamic model, an unemployed worker's outside option is $\Lambda_0 > 0$. If a worker receives only one low quality vacancy, the auction mechanism determines that this worker will receive exactly his outside option. The value of w_1^0 in Proposition 9 is simply the period wage consistent with that. The determination of w_2^0 is entirely analogous.

Before turning to the numerical analysis of this model, we first consider, once again, the question of constrained efficiency, where the social planner chooses to maximize the total expected surplus

$$S = \max_{\{E_t^2, E_t^1, H_t^2, H_t^1, M_t^1, M_t^2\}} \sum_{t=0}^{\infty} \beta^t \{y_2(E_{2t} + H_{2t}) + y_1(E_{1t} + H_{1t}) - k_1 M_{1t} - k_2 M_{2t} - k_2 \hat{M}_{2t}\}$$

subject to equations (2.2), (2.3) and (2.4).

Proposition 10: The stationary equilibrium is constrained-efficient.

4. QUANTITATIVE ANALYSIS

There are six parameters in this model: ($y_1, y_2, k_1, k_2, \beta, \rho$). To assess the quantitative significance of the dispersion in this theory, as our baseline, we picked parameter values to approximate the US economy in 1995. We chose this year for two reasons. First, this theory abstracts from any cyclical features, and is essentially a theory of an economy that is performing well – the only friction being the basic coordination problem. Arguably, this was the case in the US at that time. Second, 1995 is the last year considered in Katz and Autor's (1999) study, which presents many statistics that are relevant for this theory.

Parameter Values

The Katz and Autor (1999) study analyses weekly data. With an annual discount rate of 5%, this implies a weekly discount factor of $\beta = 0.999$. Using Kuhn and Sweetman's (1998) estimate of a 4% monthly separation rate, we set the weekly $\rho = 0.01$. To focus on an equilibrium with on-the-job search, given the values of β and ρ, we restricted our choices of y_1, y_2, k_1 and k_2 to satisfy the condition stated in Proposition 5. We set $y_1 = 150$, which is at the lower end of the observed distribution. We chose the values of y_2, k_1 and k_2 to match the average weekly wage in 1982 dollars ($255), the "natural" rate of unemployment (3.9%) and the vacancy rate 2.6%[10]. These values were $y_2 = 1131.3$, $k_1 = 1500$, and $k_2 = 76000$. [11]

Results

[10] The actual unemployment rate in 1995 was 5.6%. We chose 3.9% as our approximate target for the unemployment rate because the unemployment rate settled down to that number in subsequent years, and this theory is really a theory of the natural rate. The 2.6% figure for the vacancy rate was extrapolated from Blanchard and Diamond (1989), using labor force figures from the BLS and the vacancy index from the Conference Board.

[11] The values of k_1 and k_2 may seem quite high, when considering weekly costs. However, we have modelled this so that these costs terminate once a vacancy is filled – and vacancies are filled quite quickly in equilibrium. In reality, there fixed costs when creating jobs, and these can be quite large when considering the capital that is used to match with a worker. Following Pissarides (2000), to keep the state vector as small as possible, we model these costs as flow costs.

Table 4.1, below, presents the equilibrium wage distribution, for this set of parameters.

Equilibrium Wage Distribution	
Wages	Fraction of Workforce
$w_0^0 = 0$	$n_0^0 = 0.0393$
$w_1^0 = 127.11$	$n_1^0 = 0.0967$
$w_1^1 = 150$	$n_1^1 = 0.0075$
$w_2^0 = 231.31$	$n_2^0 = 0.2812$
$w_2^1 = 251.83$	$n_2^1 = 0.5501$
$w_2^2 = 1131.3$	$n_2^2 = 0.0252$
Table 4.1	

It is quite clear from this table that both productivity dispersion and contract dispersion play important roles in wage determination. For example, among workers that receive only one job offer, those that receive this offer from a high productivity vacancy receive a wage of $w_2^0 = 231.31$, while those that receive the offer from a low productivity vacancy receive only $w_1^0 = 127.11$. This difference is due entirely to productivity dispersion. However, among those workers that take jobs with high productivity vacancies, those that had no other offer receive $w_2^0 = 231.31$, those whose second-best offer came from a low-productivity vacancy receive $w_2^1 = 251.83$, while those whose second-best offer came from another high productivity vacancy receive $w_2^2 = 1131.3$. The difference of these three wages is driven purely by contract dispersion.

Table 4.1 also shows that, in the stationary equilibrium, most workers are in good jobs. Adding n_1^0 and n_1^1, we can see that only 10.42% of workers are in bad jobs. Altogether, 85.65% of workers are in good jobs. However, very few (2.52%) are paid the top wage of $w_2^2 = 1131.3$. Due to contract dispersion, 28.12% earn only $w_2^0 = 231.31$, while 55.01% earn $w_2^1 = 251.83$. This leaves 3.93% unemployed.

Table 4.2 shows the stationary equilibrium values of some of the other key variables.

Other Key Variables in Equilibrium	
Good Vacancies Aimed at Workers in Bad Jobs	$\hat{\phi}_2 = 0.0516$
Good Vacancies Aimed at Unemployed Workers	$\phi_2 = 0.0715$
Bad Vacancies Aimed at Unemployed Workers	$\phi_1 = 0.1471$
Value of Unemployed Worker	$\Lambda_0 = 233{,}649$
Value of a Bad Job Match	$\Lambda_1 = 235{,}515$
Value of a Good Job Match	$\Lambda_2 = 315{,}541$
Table 4.2	

From this table, it can be seen that the probability of a worker receiving a good job offer, when unemployed ($1-e^{-\phi_2}$ = 0.069) is higher than the receiving one when already employed in a bad job ($1-e^{-\hat{\phi}_2}$ = 0.0503). This occurs because of the extra bargaining power a worker in a bad job has: if successfully recruited, he must be paid $w_2^1 = 251.83$, rather the wage $w_2^0 = 231.31$ paid to a worker that was previously unemployed. Overall, the probability of a worker leaving a current job to take another (0.0503) one is approximately one quarter the probability of a currently unemployed worker finding a job ($1-e^{-\phi_1}e^{-\phi_2}$ = 0.1964). Rephrasing this, in equilibrium, the "offer arrival rate" for unemployed workers is significantly higher than the "offer arrival rate" of employed workers. This is something that has been observed empirically, and is typically assumed in "undirected search" models with on-the-job search.[12]

[12] See, for example, Pissarides (1994).

From Tables 4.1 and 4.2, another feature of the equilibrium can be seen. Although the vacancy/unemployment ratios for good and bad jobs are quite similar in magnitude, in the stationary equilibrium, the vast majority of workers are in good jobs. On-the-job-search is significant enough to drive this result. Workers in bad jobs know that they will not stay there for very long. This is also reflected in the fact that the ratio $\Lambda_2 / \Lambda_1 = 1.34$ is significantly smaller than the value of $y_2 / y_1 = 7.54$. The values of the matches include all expected returns to both the firm and the worker. Thus, as can be seen from equation (3.10), the value of Λ_1 takes into account the fact that the worker will, most likely, move on to a good job in the future.

The next table, Table 4.3, compares some of the statistics from this example with those from US data.

Comparing Statistics		
Statistic	*Model*	*US Data*
Mean Wage	255.55	255.00
Standard Deviation Log Wage	0.327	0.616
90%-10% Log Wage	1.08	1.54 (1.15)
Unemployment Rate	3.93	5.6 (3.9)
Vacancy Rate	2.6	2.6

Table 4.3

The values of the parameters were chosen so that the mean wage, the unemployment rate, and the vacancy rate were close to those in the data. The mean weekly wage for males in the US was approximately \$255 in 1995. The unemployment rate 5.6% overall, with an estimated natural rate of 3.9%. The corresponding figures from the model are \$255.55 and 3.93%. Katz and Autor report that the standard deviation of the log wage in the US overall in 1995 was 0.616. In the model, the corresponding figure is 0.327 – approximately 53% of the figure in the data. Thus, one could argue that 53% of this observed dispersion was due to the coordination problem, which results in both productivity dispersion and contract dispersion among workers that are effectively homogeneous. This result is reinforced by another statistic reported by Katz and Autor. They report the differences of the 90th and 10th percentiles of the log wage distribution, both overall and for the "residual" wage distribution. In the US, overall, in 1995, this figure was approximately 1.54 overall and 1.15 for the residual distribution. In the model, this figure is 1.08. Thus, by this measure, this simple model can explain a large proportion of the residual wage dispersion.

We can also use this model for local comparative static exercises – comparing the equilibrium outcomes across stationary equilibria with different parameter values. The following table presents the results from this exercise, for small perturbations around the parameters in the above base case.

	y_1	k_1	y_2	k_2	β	ρ
$\hat{\phi}_2$	-	0	+	-	+	-
ϕ_2	-	+	+	-	+	-
ϕ_1	+	-	-	+	-	+
V	+	-	+	-	+	-
Λ_0	+	-	+	-	+	-
Λ_1	+	-	+	-	+	-
Λ_2	+	-	+	-	+	-
w_1^0	+	-	-	+	+	-
w_1^1	+	0	0	0	0	0
w_2^0	+	+	+	-	+	-
w_2^1	+	0	+	-	+	-
w_2^2	0	0	+	0	0	0
n_0^0	-	+	+	-	+	+
n_1^0	+	+	-	+	-	+
n_1^1	+	-	-	+	-	+
n_2^0	-	+	+	-	+	-
n_2^1	+	-	-	-	-	-
n_2^2	-	+	+	-	+	-
$\overline{w}$	+	+	+	-	+	-
$\sigma_{\log w}$	-	+	+	-	+	-
$\log 90 - \log 10$	-	+	+	-	+	-

Table 4.4: Comparative Statics

Most of the signs in this table are quite intuitive. Two that are not immediately obvious are $\partial n_0^0 / \partial y_2 > 0$ and $\partial n_0^0 / \partial k_2 < 0$. That is, the unemployment rate is a decreasing function of the productivity, and an increasing function of the cost, of a good job. This is understandable, however, when observing that it is also the case that $\partial n_1^0 / \partial y_2 < 0$, $\partial n_1^1 / \partial y_2 < 0$ and $\partial n_1^0 / \partial k_2 > 0$, $\partial n_1^1 / \partial k_2 > 0$. In this case higher values of y_2, and lower values of the cost k_2, while increasing the number of good jobs, generate a decrease in bad jobs and lower the overall employment rate.

Another interesting feature that comes out in this table is that higher values of the separation rate ρ lead to higher unemployment rates, but less dispersion. This leads to a reduction in the expected present value of the stream of future payoffs, which affects the expected return from good jobs disproportionately since they have higher costs to be paid up-front. This reduces the number of good jobs, and the wage in good jobs, while encouraging the entry of bad jobs. Overall, unemployment goes up, due to the large direct effect of separations on unemployment. However, dispersion is reduced by the diminished relative value of good jobs. This offers an alternative explanation for the negative correlation observed between these variables, and analysed, in an undirected search model by Delacroix (2001).

5. CONCLUSIONS

From this analysis, it appears that a large proportion of the observed wage disparity among similar workers can be seen as a direct consequence of the lack of coordination among employers. When each employer chooses, independently, the quality of a job and the candidate to offer it to, then the theory predicts that we will observe both contract dispersion and technology dispersion. In the absence of this coordination problem, all employers would choose the same type of job, and would pay the same wage to identical workers. Quantitatively, when calibrating the model to match observed mean wages and unemployment rates, we found that, despite its simplicity, it can come remarkably close replicating the dispersion statistics that have been calculated, in independent studies, for US data.

We also found that the equilibrium allocations are constrained-efficient in the sense that a planner could do no better unless able to eliminate the coordination problem, (and hence, the matching friction). In particular, the policies advocated in (for example) Acemoglu (2001), which influence the relative composition of good and bad jobs without reducing the matching frictions, would only hurt here. This is an example of how conclusions can be quite different in models with directed and undirected search.

One appealing feature of this model is that the measures of dispersion are unaffected by simple scaling up of the productivities and costs. Future work, therefore, could imbed this model into a framework with asset accumulation and innovative activity, to examine the joint determination of dispersion, growth, and unemployment.

APPENDIX

Proof of Proposition 1:

Using (1.5)-(1.7), the expected profits of the two types of vacancies are: $\pi_1 = e^{-\phi_1} e^{-\phi_2} y_1 - k_1 = 0$ and $\pi_2 = e^{-\phi_1} e^{-\phi_2} y_2 + e^{-\phi_2}(1-e^{-\phi_1})(y_2 - y_1) - k_2 = 0$. Solving these simultaneously yields (1.8) and (1.9). It is easily shown that $\phi_1, \phi_2 > 0$ iff $y_2 - k_2 > y_1 - k_1$ and $y_1 / k_1 > y_2 / k_2$. ■

Proof of Proposition 2:

An interior maximum of the social planning problem satisfies $k_1 = e^{-\phi_1} e^{-\phi_2} y_1$ and $k_2 = e^{-\phi_1} e^{-\phi_2} y_2 + e^{-\phi_2}(1-e^{-\phi_1})(y_2 - y_1)$ which is the same as the decentralised economy. It follows from the proof of Proposition 1 that $y_2 - k_2 \geq y_1 - k_1$ and $y_2 / k_2 \geq y_1 / k_1$ imply $\phi_1, \phi_2 \geq 0$. ■

Proof of Proposition 3:

In a stationary equilibrium the values of $\{\phi_{1t}, \phi_{2t}, \hat{\phi}_{2t}, \Pi_{1t}, \Pi_{2t}, \hat{\Pi}_{2t}, V_t, \Lambda_{0t}\ \Lambda_{1t}\ \Lambda_{2t}\}$ are given by

(A.1) $$V = \Lambda_2 - (\Lambda_1 - \Lambda_0)(1 + \phi_1 + \phi_2) e^{-\phi_1} e^{-\phi_2} - (\Lambda_2 - \Lambda_1)(1 + \phi_2) e^{-\phi_2}$$

(A.2) $$\Pi_1 = \max\{(\Lambda_1 - \Lambda_0) e^{-\phi_1} e^{-\phi_2} - k_1, 0\}$$

(A.3) $$\Pi_2 = \max\{(\Lambda_2 - \Lambda_0) e^{-\phi_1} e^{-\phi_2} + (\Lambda_2 - \Lambda_1)(1 - e^{-\phi_1}) e^{-\phi_2} - k_2, 0\}$$

(A.4) $$\hat{\Pi}_2 = \max\{(\Lambda_2 - \Lambda_1) e^{-\hat{\phi}_2} - k_2, 0\}$$

(A,5) $$\Pi_1 = 0$$

(A.6) $$\Pi_2 = 0$$

(A.7) $$\hat{\Pi}_2 = 0$$

(A.8) $$\Lambda_0 = \beta V$$

(A.9) $$\Lambda_2 = \frac{y_2 + \beta \rho V}{1 - \beta(1-\rho)}$$

(A.10) $$\Lambda_1 = \frac{y_1 + \beta(\rho V + (1-\rho)(1 - \hat{\phi}_2 e^{-\hat{\phi}_2} - e^{-\hat{\phi}_2})\Lambda_2))}{1 - \beta(1-\rho)(\hat{\phi}_2 e^{-\hat{\phi}_2} + e^{-\hat{\phi}_2})}$$

We have 10 independent equations for the 10 proposed stationary variables. The parameters $\{\beta, y_1, y_2, k_1, k_2, \rho\}$ of these equations are constant. Moreover, all of these equations are independent of the potentially non-stationary state variables E_{1t}, E_{2t}, M_{1t} etc.. Therefore, $\{\phi_{1t}, \phi_{2t}, \hat{\phi}_{2t}, \Pi_{1t}, \Pi_{2t}, \hat{\Pi}_{2t}, V_t, \Lambda_{0t}\ \Lambda_{1t}\ \Lambda_{2t}\}$ are stationary in equilibrium. ■

Proof of Proposition 4:

In a stationary equilibrium, equations (2.2), (2.3) and (2.4) imply

(i) $H_2 = (N - E_1 - E_2)p_2 + E_1\hat{p}_2$

(ii) $H_1 = (N - E_1 - E_2)p_1 - E_1\hat{p}_2$

(iii) $E_i = (E_i + H_i)(1-\rho)\ \forall i \in \{1,2\}$

Note: (a) (iii) implies $H_i = (E_i + H_i)\rho \quad \forall i \in \{1,2\}$

(b) definition: $n_i = (E_i + H_i)/N \quad \forall i \in \{1,2\}$

(c) identity: $n_0 = 1 - n_1 - n_2$

We can rewrite (i) and (ii) as follows.

(i') $\rho n_2 = (1-(n_1+n_2)(1-\rho))p_2 + (1-\rho)n_1\hat{p}_2$

(ii') $\rho n_1 = (1-(n_1+n_2)(1-\rho))p_1 - (1-\rho)n_1\hat{p}_2$

Note that (i') plus (ii') implies

(iv) $\rho(n_1+n_2) = (1-(n_1+n_2)(1-\rho))(p_1+p_2)$ or

$$\rho(1-n_0) = (1-(1-n_0)(1-\rho))(p_1+p_2)$$

This gives:

(v) $n_0 = \dfrac{\rho p_0}{1-(1-\rho)p_0}$

We can substitute (v) into (ii') to get

(vi) $n_1 = \dfrac{[1-(1-n_0)(1-\rho)]p_1}{\rho + (1-\rho)\hat{p}_2}$

Finally, by the identity

(vii) $n_2 = 1 - n_0 - n_1$ ■

Proof of Proposition 5:

Equations (A.10) and (A.9) imply that the difference between Λ_2 and Λ_1 is as follows:

(A.11) $$\Lambda_2 - \Lambda_1 = \frac{y_2 - y_1}{1-\beta(1-\rho)(\hat{\phi}_2 e^{-\hat{\phi}_2} + e^{-\hat{\phi}_2})}$$

Equation (A.4) and $\hat{\phi}_2 > 0$ imply

(A.12) $$e^{-\hat{\phi}}(\Lambda_2 - \Lambda_1) = k_2$$

Equations (A.11) and (A.12) yield equation (3.4). It is easy to see from equation (3.4) that $\hat{\phi}_2$ is always positive if $(y_2 - y_1) > (1-\beta(1-\rho))k_2$. ■

Proof of Proposition 6:

On-the-job search implies that $\phi_1, \hat{\phi}_2 > 0$. Therefore, equations (A.2) and (A.3) imply

(A.13) $$e^{-\hat{\phi}_2}(\Lambda_2 - \Lambda_1) = k_2$$

(A.14) $$(\Lambda_1 - \Lambda_0)e^{-\phi_1}e^{-\phi_2} = k_1$$

Equations (A.9), (A.13) and (A.14) can be used to eliminate Λ_2, Λ_2 - Λ_1 and Λ_1 - Λ_0 from equation (A.1). The appropriate substitutions yield

(A.15) $$V = \frac{y_2 - \left\lfloor k_1(1+\phi_1+\phi_2) + k_2(1+\phi_2)e^{-\phi_2}e^{\hat{\phi}_2})(1-\beta(1-\rho))\right\rfloor}{1-\beta}.$$

Equations (A.9) and (A.8) imply that the difference Λ_2 - Λ_0 is given by

(A.16) $$\Lambda_2 - \Lambda_0 = \frac{y_2 - (1-\beta)\beta(1-\rho)V}{1-\beta(1-\rho)}$$

In an equilibrium with good jobs aimed at unemployed workers it must be the case that

(A.17) $$(\Lambda_2 - \Lambda_0)e^{-\phi_1}e^{-\phi_2} + (\Lambda_2 - \Lambda_1)(1-e^{-\phi_1})e^{-\phi_2} = k_2$$

Substitute (A.15) and (A.16). Then substitute this expression and (A.11) into (A.17). This yields equation (3.5). ■

Proof of Proposition 7:

Equations (A.10) and (A.8) imply that the difference between Λ_1 and Λ_0 is as follows:

A.18)
$$\Lambda_1 - \Lambda_0 = \frac{1}{1-\beta(1-\rho)(\hat{\phi}_2 e^{-\hat{\phi}_2} + e^{-\hat{\phi}_2})}\left[y_1 + \frac{y_2\beta(1-\rho)(1-\hat{\phi}_2 e^{-\hat{\phi}_2} - e^{-\hat{\phi}_2})}{1-\beta(1-\rho)}\right] - \frac{(1-\beta)(1-\rho)\beta V}{1-\beta(1-\rho)}$$

If we assume that $\phi_1, \hat{\phi}_2 > 0$, we can substitute (A.15) into (A.18) to get an expression for Λ_1-Λ_0 in terms of $\phi_1, \phi_2, \hat{\phi}_2$. This expression can be substituted into equation (A.14) to yield (3.6). Therefore, an equilibrium with $\phi_1, \phi_2, \hat{\phi}_2$>0 is characterised by equations (3.4), (3.5) and (3.6). According to Propositions 1 and 2, we know that $\phi_2, \hat{\phi}_2$>0 are determined by equations (3.4) and (3.5) and that both values are positive if $(y_2 - y_1) > (1-\beta(1-\rho))k_2$. We can then substitute these values into equation (3.6) to check whether ϕ_1>0. ■

Proof of Proposition 8:

Follows directly from the equations derived in Propositions 4 through 7. ■

Proof of Proposition 9:

The values of n_i^j are obtained in a fashion similar to Proposition 4.

(i) $H_2^0 = (N - E_1 - E_2)p_2^0$

(ii) $H_2^1 = (N - E_1 - E_2)p_2^1 + E_1 \hat{p}_2^1$

(iii) $H_2^2 = (N - E_1 - E_2)p_2 + E_1 \hat{p}_2^2$

(iv) $H_2^i = (E_2^i + H_2^i)\rho \qquad \forall i \in \{0,1,2\}$

(v) $n_2^i = (E_2^i + H_2^i)/N \qquad \forall i \in \{0,1,2\}$

Note that (iv) also implies $E_2^i = (E_2^i + H_2^i)(1-\rho)\ \forall i \in \{0,1,2\}$. Recalling the proof of Proposition 4, we can rewrite (i), (ii) and (iii) as follows.

(i') $\rho n_2^0 = \rho(1-n_0)p_2^0 + n_0 p_2^0$

(ii') $\rho n_2^1 = \rho(1-n_0)p_2^1 + n_0 p_2^1 + (1-\rho)n_1 \hat{p}_2^1$

(iii') $\rho n_2^2 = \rho(1-n_0)p_2^2 + n_0 p_2^2 + (1-\rho)n_1 \hat{p}_2^2$

which gives:

$$n_2^0 = [1 + n_0(1-\rho)/\rho]e^{-\phi_1}e^{-\phi_2}$$

$$n_2^1 = [1 + n_0(1-\rho)/\rho]\phi_2 e^{-\phi_2}(1-e^{-\phi_1}) + n_1\hat{\phi}_2 e^{-\hat{\phi}_2}(1-\rho)/\rho$$

$$n_2^2 = n_2 - n_2^0 - n_2^1. \quad ■$$

Proof of Proposition 10

There are two types of high productivity vacancies - M_{2t} and $\hat{M}_{2t}$. Therefore, it is actually convenient to distinguish (i) the workers that moved into good jobs from unemployment and (ii) the workers that moved into good jobs from bad jobs. Define

(B.1) $E_{2t} = \hat{E}_{2t} + \tilde{E}_{2t} \qquad i \in \{1,2\}$

Like wise $H_{2t} = \hat{H}_{2t} + \widetilde{H}_{2t}$, $i \in \{1,2\}$. In which case, the social planning problem can be stated as follows:

(B.2)

$$S = \max_{\left\{\substack{E_{1t},\hat{E}_{2t},\widetilde{E}_{2t},H_{1t},\hat{H}_{2t},\widetilde{H}_{2t},\\ M_{1t},\hat{M}_{2t},\widetilde{M}_{2t}}\right\}} \sum_{t=0}^{\infty} \beta^t \{y_2(\hat{E}_{2t} + \widetilde{E}_{2t} + \hat{H}_{2t} + \widetilde{H}_{2t}) + y_1(E_{1t} + H_{1t}) - k_1 M_{1t} - k_2(\hat{M}_{2t} + \widetilde{M}_{2t})\}$$

subject to

(B.3) $$\widetilde{H}_{2t} = (N - E_{1t} - \hat{E}_{2t} - \widetilde{E}_{2t})(1 - e^{-\phi_{2t}})$$

(B.4) $$\hat{H}_{2t} = E_{1t}(1 - e^{-\hat{\phi}_{2t}})$$

(B.5) $$H_{1t} = (N - E_{1t} - \hat{E}_{2t} - \widetilde{E}_{2t})(1 - e^{-\phi_{1t}})e^{-\phi_{2t}} - E_{1t}(1 - e^{-\hat{\phi}_t})$$

(B.6) $$E_{1t+1} = (1-\rho)(E_{1t} + H_{1t})$$

(B.7) $$\widetilde{E}_{2t+1} = (1-\rho)(\widetilde{E}_{2t} + \widetilde{H}_{2t})$$

(B.8) $$\hat{E}_{2t+1} = (1-\rho)(\hat{E}_{2t} + \hat{H}_{2t})$$

where $M_{1t} = \phi_{1t}(N - E_{1t} - \widetilde{E}_{2t} - \hat{E}_{2t})$, $M_{2t} = \phi_{2t}(N - E_{1t} - \widetilde{E}_{2t} - \hat{E}_{2t})$ and $\hat{M}_{2t} = \hat{\phi}_{2t} E_{1t}$. Note that (B.3) and (B.6) implies

(B.9) $$e^{-\phi_{2t}} = \frac{N - E_{1t} - \hat{E}_{2t} N - \widetilde{E}_{2t+1}/(1-\rho)}{N - E_{1t} - \widetilde{E}_{2t} - \hat{E}_{2t}}$$

or, alternatively,

(B.10)

$$\phi_{2t} = \frac{M_{2t}}{N - \widetilde{E}_{2t} - E_{1t} - \hat{E}_{2t}} = -(\ln(N - \hat{E}_{2t} - E_{1t} - \frac{\widetilde{E}_{2t+1}}{1-\rho}) - \ln(N - \widetilde{E}_{2t} - E_{1t} - \hat{E}_{2t}))$$

Likewise (B.5) and (B.6) imply

(B.11) $$\phi_{1t} = \frac{M_{1t}}{N - \widetilde{E}_{2t} - E_{1t} - \hat{E}_{2t}} = -(\ln(N - \frac{\widetilde{E}_{2t+1} + E_{1t+1} + \hat{E}_{2t+1}}{1-\rho}) - \ln(N - \frac{\widetilde{E}_{2t+1}}{1-\rho} - E_{1t} - \hat{E}_{2t}))$$

and (B.4) and (B.8) imply

(B.12) $$\hat{\phi}_{2t} = \frac{\hat{M}_{2t}}{E_{1t}} = -\left(\ln(E_{1t} + \hat{E}_{2t} - \frac{\hat{E}_{2t+1}}{1-\rho}) - \ln(\hat{E}_{2t})\right)$$

We can then rewrite the social planning problem.

(B.13) $$V(t) = \max_{E_t,\hat{E}_{2t}\widetilde{E}_{2t},E_{t+1},\hat{E}_{2t+1}\widetilde{E}_{2t+1}} \{\frac{y_2}{(1-\rho)}(\hat{E}_{2t+1} + \widetilde{E}_{2t+1}) + \frac{y_1}{(1-\rho)} E_{1t+1} +$$

$$k_2(N-\widetilde{E}_{2t}-E_{1t}-\hat{E}_{2t})\left(\ln(N-\frac{\widetilde{E}_{2t+1}}{1-\rho}-E_{1t}-\hat{E}_{2t})-\ln(N-\widetilde{E}_{2t}-E_{1t}-\hat{E}_{2t})\right)+$$

$$k_2E_{1t}\left(\ln(E_{1t}+\hat{E}_{2t}-\frac{\hat{E}_{2t+1}}{1-\rho})-\ln(\hat{E}_{2t})\right)+$$

$$k_1(N-\widetilde{E}_{2t}-E_{1t}-\hat{E}_{2t})\left(\ln(N-\frac{\widetilde{E}_{2t+1}+E_{1t+1}+\hat{E}_{2t+1}}{1-\rho})-\ln(N-E_{1t}-\hat{E}_{2t}-\frac{\widetilde{E}_{2t+1}}{1-\rho}\right)+\beta V(t+1)$$

The first order conditions (with a slight abuse of notation) are as follows

$$<\widetilde{E}_{2t}>\quad V'(\widetilde{E}_{2t})=k_2\phi_{2t}+k_1\phi_{1t}+k_2$$
$$<\hat{E}_{2t}>\quad V'(\hat{E}_{2t})=k_2\phi_{2t}-k_2e^{\phi_{2t}}+k_2e^{\hat{\phi}_{2t}}+k_1\phi_{1t}+k_1e^{\phi_{1t}}+k_2$$
$$<E_{1t}>\quad V'(E_{1t})=k_2\phi_{2t}-k_2e^{\phi_{2t}}-k_2\hat{\phi}_{2t}+k_2e^{\hat{\phi}_{2t}}+k_1\phi_{1t}+k_1e^{\phi_{1t}}$$
$$<\widetilde{E}_{2t+1}>\qquad 0=y_1-k_2e^{\phi_{2t}}+k_1e^{\phi_2}(1-e^{\phi_2})+\beta(1-\rho)V'(\widetilde{E}_{2t+1})$$
$$<\hat{E}_{2t+1}>\qquad 0=y_2-k_2e^{\hat{\phi}_2}-k_1e^{\phi_1}e^{\phi_2}+\beta(1-\rho)V'(\hat{E}_{2t+1})$$
$$<E_{1t+1}>\qquad 0=y_1-k_1e^{\phi_1}e^{\phi_2}+\beta(1-\rho)V'(E_{1t+1})$$

This system of equations can be solved for the steady state values of $\phi_1,\hat{\phi}_1,\phi_2$. The results are as follows

(B.14)
$$k_2=y_2e^{-\phi_1}e^{-\phi_2}+k_2(1-\rho)\phi_2\beta e^{-\phi_1}e^{-\phi_2}+k_1(1-\rho)\phi_1\beta e^{-\phi_1}e^{-\phi_2}+(1-e^{-\phi_1})(k_2-k_1)$$

(B.15) $k_2=(y_2-y_1)e^{-\hat{\phi}_2}+\beta(1-\rho)(\hat{\phi}_2e^{-\hat{\phi}_2}+e^{-\hat{\phi}_2})k_2$

(B.16) $k_1=y_1e^{-\phi_1}e^{-\phi_2}+(1-\rho)\beta(k_2\phi_2-k_2e^{-\phi_2}-k_2\hat{\phi}_2+k_2e^{\hat{\phi}_2}+k_1\phi_1+k_1e^{-\hat{\phi}_2})e^{-\phi_1}e^{-\phi_2}$

Equation (B.15) is the same as Equation (3.4). Manipulation of equations (B.14) and (B.16) yields equations (3.5) and (3.6). ■

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