1. Consider a small open economy which produces a single consumption good using only capital and experiences exogenous productivity shocks. Suppose that the economy experiences a persistent, but not permanent, productivity shock. Then the response of the current account to this shock is ambiguous. (15 points)

2. Cash-in-advance models in which the simple quantity equation always hold can easily account for the observation that monetary shocks cause real and nominal exchange rates to move in the same direction. (15 points)

3. Consider a two-period model of a small open economy. A representative agent derives utility from consumption of home and foreign goods and domestic real balances. Let the consumer’s utility function be given by:

\[ U = \ln(c_h^0 + c_f^0) + \alpha \ln \left( \frac{m_h^0}{p_h^0} \right) + \beta E_0 \left[ \ln(c_h^1 + c_f^1) + \alpha \ln \left( \frac{m_h^1}{p_h^1} \right) \right], \]

where \( c_h^t \) and \( c_f^t \) denote consumption of home and foreign goods at time \( t \) respectively and \( m_h^t / p_h^t \) is domestic real balances at time \( t \). The representative agent receives an endowment, \( y_h^t \), of home consumption good in period \( t = 0, 1 \) and can purchase nominal domestic bonds, \( B_h^0 \), nominal foreign bonds, \( B_f^0 \), and real bonds, \( b_0 \), which pay net returns, \( i_h, i_f, \) and \( \rho \) respectively. In addition, the representative agent can purchase or sell time 1 foreign currency at time 0 at the forward rate \( f_1 \). Let \( e_t \) denote the spot exchange rate at time \( t \).

a. Derive expressions for both the nominal and real exchange rates. How does each measure of the exchange rate respond to changes in money supplies and endowments? (18 points)

Suppose now that the utility function is replaced by:

\[ U = \left( c_h^0 \right)^\alpha \left( c_f^0 \right)^\gamma \left( \frac{m_h^0}{p_h^0} \right)^{1-\alpha-\gamma} + \beta E_0 \left[ \left( c_h^1 \right)^\alpha \left( c_f^1 \right)^\gamma \left( \frac{m_h^1}{p_h^1} \right)^{1-\alpha-\gamma} \right]. \]

b. Now, is the effect of a change in endowments on the nominal exchange rate unambiguous? Is it possible that changes in money supplies will affect real exchange rates? Explain. (12 points)
4. Consider a two period world economy with two countries and no uncertainty. Throughout, variables with no superscript will be “home” country variables and those with a * superscript will be foreign variables.

In each country there is a representative agent. The home country agent’s preferences are given by:

$$U(c_0, c_1) = \frac{c_0^{1-\sigma}}{1-\sigma} + \beta \frac{c_1^{1-\sigma}}{1-\sigma} \quad 0 < \beta < 1, \quad \sigma > 0.$$ 

The foreign country representative agent has similar preferences, except that \(\sigma\) is replaced with \(\sigma^*\), \(\beta\) with \(\beta^*\), etc.

Home country output at time \(t = 0, 1\) is produced using the following technology,

$$Y_t = A_t K_t^\alpha \quad 0 < \alpha < 1.$$ 

Here \(K_t\) is the home country capital stock at time \(t\) and \(A_t\) is a parameter governing productivity. Again, the foreign country has a symmetric technology. The representative agents in the home and foreign countries are endowed with \(K_0\) and \(K_0^*\) respectively.

In addition to investing in capital, an agent of either country can save (dissave) by lending to (borrowing from) the other country. Let \(B_0\) and \(B_0^*\) denote lending (borrowing) by the home and foreign countries respectively. Assume that these “bonds” work like this. The home agent buys (sells) \(B_0\) at price \(1/1+r\) at time 0 and then receives from (pays to) the foreign agent \(B_0\) units of time 1 consumption good at \(t = 1\). Here \(r\) is interpreted as the world interest rate.

a. Maximize the home agent’s utility function subject to the lifetime budget constraint and derive optimal investment and savings as functions of the world interest rate, \(r\). (15 points)

b. Describe the relationships between optimal investment and savings and the world interest rates. Are these variables unambiguously increasing or decreasing in the world interest rate? Explain. (10 points)

c. Suppose now that \(\sigma = 1\). What are the effects of an increase in \(A_1\) (home country productivity in period 1) on the world interest rate and on the current account in the home country. (15 points)