Economics 816 **Fall 2011**
Advanced Macroeconomic Theory I **Allen Head**

Midterm Exam

October 27, 2011

Answer all questions. The weight each question or part will receive in marking is indicated. Answer question 1 *True*, *False*, or *Uncertain* and justify your answer. For questions 2, 3 and 4, answer concisely, without leaving out important information.

1. *(12 points)* In a real business cycle model, it makes no difference whether we assume that labour is indivisible or that households have preferences that are linear in leisure.

2. *(12 points)* Consider the following operator T where $v_0(s)$ is a bounded and continuous function:

$$v_1(s) = Tv_0(s) = \max_u \{(s,u)R \begin{pmatrix} s \\ u \end{pmatrix} + \beta v_0(s')\} \tag{1}$$

Here s and u are vectors and R is a negative semi-definite matrix. Show that the operator T is a contraction mapping.

3. Consider a version of the Mehra-Prescott model in which the representative consumer maximizes

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \ln(c_t + \alpha) \qquad \alpha > 0, \tag{10}$$

and owns a unit share in a "tree" which yields fruit d_t each period, with

$$d_{t+1} = \gamma_{t+1} d_t, \qquad \forall t. \tag{11}$$

The state of the economy is the gross growth rate of income, γ_{t+1}, a random variable which takes on a value of either γ_1 or γ_2, with $\gamma_2 < \gamma_1$. Each period, the probability that the state will remain the same next period is 1/2.

Derive expressions for the state-contingent prices of

a. A risk-less bond *(13 points)*

b. An equity share in a tree. *(13 points)*

4. Consider the following two-sector real business cycle model. There are two goods, each produced using the other as an input. Preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \big[\ln c_{1t} + \ln c_{2t} + \gamma \ln(1 - n_{1t} - n_{2t})\big]. \tag{12}$$

The technology for producing each good $i = 1, 2$ is:

$$y_{it} = z_{it} k_{it}^{\alpha} n_{it}^{1-\alpha} \tag{13}$$

where z_{it}, k_{it}, and n_{it} are the technology shock, capital employment, and labor hours in sector i at time t respectively. The interdependence between sectors is embedded in the following resource constraints, which specify the investment technology:

$$k_{2t+1} = y_{1t} - c_{1t} \tag{14}$$

$$k_{1t+1} = y_{2t} - c_{2t}. \tag{15}$$

Note that (14) and (15) imply complete depreciation of capital each period. Each sector's technology shock is AR(1) in logs, $i = 1, 2$:

$$\ln z_{it+1} = \rho \ln z_{it} + \epsilon_{it+1} \qquad \rho \in (0,1), \quad \ln \bar{z}_i = 0, \tag{16}$$

where ϵ_i is iid both over time and across sectors, with mean 0 and variance σ^2.

a. Set up a Bellman equation associated with the social planner's problem for this economy. Is it appropriate to calculate a recursive competitive equilibrium allocation using this problem? *(8 points)*

b. Derive c_{it}, n_{it}, and k_{it+1}, $i = 1, 2$ as functions of the state. *(18 points)*

c. Derive and interpret equations relating sectoral outputs ($\ln y_{it}$ for $i = 1, 2$) to their lagged values and the values of the productivity levels, z_{it}, for $i = 1, 2$, and the shocks, ϵ_{it}, $i = 1, 2$, in equilibrium. Are sectoral outputs correlated over time? *(12 points)*

d. Derive and interpret an equation relating the aggregate output measure:

$$Y_t = y_{1t}^{\frac{1}{2}} y_{2t}^{\frac{1}{2}}, \tag{17}$$

to its lagged value, the z_{it}'s, and the ϵ_{it}'s in equilibrium. Is aggregate output more or less volatile in equilibrium than the sectoral outputs? *(12 points)*