Economics 816 **Fall 2010**
Advanced Macroeconomic Theory I **Allen Head**

Midterm Exam

November 4, 2010

Answer all questions. The number of points assigned to each question or part is indicated. Answer question 1 *True*, *False*, **or** *Uncertain* **and justify your answer. For questions 2, 3 and 4, answer concisely, without leaving out important information.**

1. *(12 points)* If money enters a real business cycle model via a cash-in-advance constraint, then the recursive competitive equilibrium allocation cannot be computed as the solution to a social planning problem.

2. *(12 points)* Consider the following operator, T, on a space of bounded and continuous functions:

$$p_1(d) = Tp_0(d) = \beta \mathrm{E}_t \left[\frac{d}{d'} \left[p_0(d') + d' \right] \right] \tag{1}$$

Here $d, d' \in \{d_1, d_2\}$ and $\pi_{ij} = \mathrm{Prob}[d' = d_j \,|\, d = d_i]$, $i, j = 1, 2$. Show that T is a contraction.

3. Consider a version of the Mehra-Prescott model in which preferences have the "external habit" form. A representative household maximizes:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{c_t}{\nu_t}\right)^{1-\alpha} - 1}{1-\alpha} \qquad \alpha > 0, \tag{2}$$

where $\nu_t = d_t^{\delta} d_{t-1}^{1-\delta}$ is a benchmark level of aggregate consumption *taken as given* by the individual household. Each household owns a unit share in a "tree" which yields fruit d_t each period, with

$$d_{t+1} = \gamma_{t+1} d_t, \qquad \forall t. \tag{3}$$

The gross growth rate of income, γ_{t+1}, is a random variable which takes on a value of either γ_1 or γ_2, with $\gamma_2 < \gamma_1$. In each state, the probability that the state will remain the same next period is 1/3.

a. *(13 points)* Formulate the optimization problem for a household that may trade tree shares and risk-less bonds that pay one unit of consumption good in the next period as a dynamic programming problem.

b. *(13 points)* Derive expressions for the state-contingent prices, $\bar{q}_i^1$, and returns, $\bar{R}_i^1$, for a risk-less one-period bonds. In which state is the return on such a bond highest? Explain.

4. Suppose that the economy has many productive units (say, firms) each of which can produce a homogeneous consumption/investment good using the following technology:

$$y_{it} = z_t k_{it}^{\alpha} h_{it}^{1-\alpha} K_t^{\psi} \qquad \alpha, \psi \in (0,1) \tag{4}$$

where y_{it}, k_{it}, and h_{it} are output and employment of capital and labour by productive unit i in period t. K_t refers to the *aggregate* capital stock of the economy. In most other respects the economy is like that of a standard real business cycle model: There are a large number of identical households with preferences given by:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \eta \ln(1 - h_t) \right] \qquad \beta \in (0,1) \quad \eta > 0; \tag{5}$$

where c_t and h_t are the households consumption and hours of work in period t respectively. Capital is accumulated also in the usual way:

$$K_{t+1} = (1-\delta)K_t + X_t \qquad \delta \in (0,1) \tag{6}$$

where X_t is aggregate investment in the current period. Each period the economy experiences two shocks. First, aggregate total factor productivity is random, and second the *government* consumes a stochastic quantity of goods, g_t, which it finances with lump-sum taxes on households:

$$\ln z_{t+1} = \rho \ln z_t + \epsilon_{t+1} \qquad \rho \in (0,1) \tag{7}$$

$$\ln g_{t+1} = \omega \ln g + \xi_{t+1} \qquad \omega \in (0,1). \tag{8}$$

a. *(14 points)* Set up a dynamic programming problem of which the solution is a recursive competitive equilibrium. Is there a restriction on the parameter ψ which is needed to ensure that there is a solution to the functional equation? Explain.

b. *(9 points)* Will the recursive competitive equilibrium allocation solve the "social planner's" problem? If not, in general terms how will the two allocations differ?

c. *(15 points)* Derive an equation that could be used to measure technology shocks in the data under the assumption that this model is correct. Compare your measure of technology shocks to the "Solow residual":

$$SR_t = \ln Y_t - \alpha \ln K_t - (1-\alpha) \ln H_t \tag{9}$$

d. *(12 points)* How do you expect an increase in government spending to affect both labour supply and the wage in a recursive competitive equilibrium? What role does the parameter ψ play in this?