School of Graduate Studies and Research

Department of Economics

Economics 816

Advanced Macroeconomic Theory I

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Final Exam

December 20, 2011

The examination period is three hours. There are five questions and you are to answer all of them. Please write all answers in the exam booklets. The point totals allocated to each question (or part of a question) are included. Use these to budget your time.

Answer Question 1 "True", "False", or "Uncertain" and explain

1. Government debt has no effect on either the risk-free rate or the consumption allocation in models with uninsured idiosyncratic risk and borrowing constraints. *(9 points)*

Answer Questions 2, 3, 4, and 5 as succinctly as possible without leaving out anything important.

2. Consider the asset pricing model of Lucas: There are a large number of identical households, with preferences given by:

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t u(c_t) \qquad \beta \in (0,1).$$

There is no production, and assets are traded in frictionless, Walrasian, markets. The Euler equation associated with choice of asset i may be written:

$$p_t^i u'(c_t) = \mathrm{E}_t\left[\beta u'(c_{t+1})[y_{t+1}^i + p_{t+1}^i]\right],$$

where p_t^i is the asset's price, y_t^i is its "dividend", and c_t is aggregate consumption in period t.

Consider two assets. One is a perpetual risk-less bond (*i.e.* an asset that pays one unit of consumption with certainty in every period). The other asset is risky, with stochastic payments that are on average lower than one. In equilibrium, can the risky asset have a higher price than the risk-less one? Explain. *(12 points)*

3. Recall the economy considered in question #1 of Assignment 3. In that model, the technology for production of gross output, g_t, was given by

$$g_t = z_t k_t^{\alpha_1} h_t^{\alpha_2} m_t^{1-\alpha_1-\alpha_2},$$

where z_t is a technology shock, k_t is capital, h_t is employment. and m_t is imported oil. Aggregate value added (or gross domestic product), y_t was given by

$$y_t = g_t - q_t m_t$$

where q_t denotes the exogenously given price of oil at time t. In this model it was shown that in equilibrium, value added could be written

$$y_t = A z_t^{\frac{1}{\alpha_1+\alpha_2}} k^{\frac{\alpha_1}{\alpha_1+\alpha_2}} h_t^{\frac{\alpha_2}{\alpha_1+\alpha_2}} q_t^{\frac{\alpha_1+\alpha_2-1}{\alpha_1+\alpha_2}}.$$

Suppose that we are interested in calibrating this economy and want to parameterize the technology shock process:

$$\ln z_{t+1} = \rho \ln z_t + \epsilon_{t+1}^z.$$

Is it appropriate to measure technology shocks using the "Solow residual" under the assumptions of the model? If not, characterize the differences between Solow residual-based measures of technology shocks and the "true" shocks implied by the model. *(14 points)*

4. Consider the basic Pissarides matching model. Let the labour force be normalized to 1. Let u denote the measure of *unemployed workers* and v the measure of *vacant jobs* in a stationary equilibrium. Jobs are filled according to the aggregate matching function:

$$M = u^{\alpha} v^{1-\alpha}, \quad \alpha \in (0, 1).$$

Workers maximize consumption. They may be either employed or unemployed, and the values of being in these two states are denoted W and U, respectively. Employed workers receive flow *wage*, w, for the duration of their employment. Suppose that this wage is determined by bargaining, with the worker receiving share β of the total match surplus. When unemployed, workers receive flow consumption z.

There are a large number of potential firms (jobs) whose only decision is whether to enter by posting a vacancy. A job is *active* if it is matched with a worker and producing. In this case it generates flow output p and pays the worker wage w. A job is *vacant* while it is waiting to match with a worker. Vacant jobs incur flow cost: $pc > 0$. Let the values of vacant and active jobs be denoted V and J, respectively. There is free entry of firms/jobs.

Matches between firms and workers are dissolved exogenously at Poisson rate λ.

a. Derive and interpret an expression for the level of unemployment in the steady-state equilibrium as a function of *labour market tightness*, $\theta \equiv v/u$. *(7 points)*

b. Let the interest rate be denoted r. Derive expressions for V, J, U, and W as functions of w, r, and economy parameters in a steady-state equilibrium. *(14 points)*

c. Suppose that β rises to $\beta' > \beta$ and all other parameters of the economy remain unchanged. What will happen to V, J, w, and the level of unemployment, u, in equilibrium? Explain. *(8 points)*

d. Suppose that $\beta' = \alpha$. Would the effects you described in part **c.** be associated with improvements or reductions in overall social welfare? Explain. *(6 points)*

5. Consider an economy with a large number (*i.e.* unit measure) of *ex ante* identical households. Each household has preferences ordered by:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \ln c_t, \qquad \beta \in (0,1).$$

Each period household i receives endowment $y_{it} = s_{it} w_t$, where w_t is *per capita* income and $s_{it} \in \{s_1, s_2\}$ is a weight determining household $i's$ individual income. Let the fraction of households receiving share s_i of *per capita* income be constant. A given household's *individual* endowment follows a Markov process where $\pi_{ji} = \text{Prob}\{s_{it+1} = s_i | s_{it} = s_j\}$. *Per capita* income, w_t grows at stochastic rate γ_t:

$$w_{t+1} = \gamma_t w_t, \qquad \text{where} \qquad \forall t, \quad \gamma_t \in \{\gamma_1, \gamma_2\},$$

and for all t, $\gamma_{t+1} = \gamma_t$ with probability 1/3.

a. *Perfect Insurance:* Suppose that there exists a complete set of state-contingent assets so that households can perfectly insure themselves against their individual risk. In an equilibrium of this economy, derive expressions for the state contingent prices, q_i, and returns, R_i, $i = 1, 2$ for a risk-less bond that pays one unit of consumption next period, regardless of the state. Call the average return on this asset the "risk-free rate". On what parameters does this depend? *(9 points)*

Suppose now that *per capita* income is constant with $w_t = 1$ for all t. State contingent assets are absent: Rather, households may only borrow from or lend to one another at net risk-free rate r_t, and face a constraint on their level of borrowing in each period. At each time t household i chooses consumption, c_{it}, and "assets", a_{it+1} to maximize expected utility subject to the following constraints:

$$c_{it} + a_{it+1} = (1 + r_t) a_{it} + s_{it}$$
$$a_{it} \geq -b$$

b. *Uninsurable Idiosyncratic Risk:* For this economy define a stationary competitive equilibrium in which the distributions of consumption and asset holdings are invariant and the risk-free rate is constant. Outline a procedure for computing the "risk-free rate", r, in this equilibrium. how would it differ from what you computed in part **a.**? *(13 points)*

Suppose now that uninsured idiosyncratic risk arises not from realizations of a random income process, but rather from search and matching in the labour market. In particular, imagine a setting along the lines of Pissarides (2000) (see question **4.**).

c. *Random Matching in the Labour Market:* If unemployment compensation, z, were chosen to fully insure households against labour market risk, would the equilibrium in this economy (*i.e.* the one with random matching) replicate that of the "Perfect Insurance" economy considered in part **a.**? Explain. *(8 points)*