School of Graduate Studies and Research

Department of Economics

Economics 816

Advanced Macroeconomic Theory I

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Final Exam

December 16, 2010

The examination period is three hours. There are five questions and you are to answer all of them. Please write all answers in the exam booklets. The point totals allocated to each question (or part of a question) are included. Use these to budget your time.

Answer Questions 1 and 2 "True", "False", or "Uncertain" and explain.
(9 points each)

1. When agents face idiosyncratic risk against which they cannot fully insure because of borrowing constraints, inflation has particularly negative effects on welfare as it erodes agents' savings and makes them more likely to be liquidity constrained.

2. "Demand-side" shocks (*e.g.* shifts in preferences or fluctuations in government spending) have relatively minor effects in standard real business cycle models because they do not directly affect labour supply.

Answer Questions 3, 4, and 5 as succinctly as possible without leaving out anything important.

3. Consider the basic Pissarides matching model. Let the labour force be normalized to 1. Let u denote the measure of *unemployed workers* and v the measure of *vacant jobs* in a stationary equilibrium. Jobs are filled according to the aggregate matching function:

$$M = u^{\alpha} v^{1-\alpha} \qquad \alpha \in (0,1)$$

Workers maximize their consumption. They may be either employed or unemployed, and the values of being in these two states are denoted W and U, respectively. Employed workers receive flow *wage* w for the duration of their employment. When unemployed, workers receive flow consumption z.

There are a large number of potential firms (jobs) whose main decision is whether to enter by posting a vacancy. A job is *active* if it is matched with a worker and producing. In this case it realizes flow production p and pays the worker wage w. A job is *vacant* while it is waiting to match with a worker. Vacant jobs incur flow costs of $c > 0$. Let the values of vacant and active jobs be denoted V and J, respectively. There is free entry of firms/jobs.

Matches between firms and workers are dissolved exogenously at Poisson rate s.

a. Let the interest rate be denoted r. Write down expressions for the values of vacancies, V, filled jobs, J, and workers, both unemployed, U, and employed, W. Use these to construct a single expression for the joint value (to both the worker and the firm) of a match in a stationary equilibrium. *(15 points)*

b. Suppose now that both productivity, p, and the separation rate, s, are random variables. Explain how the *net* productivity of a match, $p - z$ affects the responsiveness of u and v to fluctuations in both p and s in equilibrium.
(8 points)

4. Consider an economy in which a large number of identical households choose consumption and savings to maximize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \ln c_t + \eta \ln(1 - h_t) \qquad \beta \in (0,1). \quad \eta > 0$$

Output is produced by a large number of identical firms, each of which operates a technology of the following form:

$$y_{it} = z_t k_{it}^{\theta} h_{it}^{1-\theta} Y_t^{\phi} \qquad \theta \in (0,1), \quad \phi \in (0, 1-\theta),$$

where y_i, k_i, and h_i are output, use of capital, and employment for firm i, and Y is *aggregate* output, which is taken as given by all individual firms.

Households rent capital and labour to firms in competitive factor markets, taking the capital rental rate, r_t, and wage, w_t, as given. z_t is a stochastic productivity parameter. For all t, $z_t \in \{z_L, z_H\}$ with $z_L < z_H$ and $\pi_{ij} = \mathrm{Prob}\{z_{t+1} = z_j | z_t = z_i\}$ for $i, j \in \{L, H\}$.

One unit of period t+1 capital can be produced using one unit of period t output. Capital depreciates fully after one period.

a. Write down the optimization problem for a household in this economy which takes all prices as given maximizes utility by choosing consumption and leisure and may save by accumulating capital and/or buying both one and two-period risk-less bonds. *(6 points)*

b. In the competitive equilibrium, derive expressions for the prices of both one and two-period risk-less bonds and for the term premium, defined as the ratio of the expected return to holding a two-period bond for one period to the return on a one-period bond. *(15 points)*

c. Suppose that households can also trade shares in firms. These shares pay dividends equal to the firms' profits per shareholder each period. What do you expect the price of such share to equal in equilibrium? *(5 points)*

5. Consider an economy populated by a unit measure of *ex ante* identical households who have limited opportunities for insurance. Households preferences are represented given by:

$$U(c_t) = \mathrm{E} \sum_{t=1}^{\infty} \beta^t \ln c_t.$$

Households may borrow or lend at the risk-free rate q_t and face a exogenous limit of $-b$ on their borrowing. Output is produced by a large number of identical firms that operate the following technology:

$$Y_t = AK_t^{\theta} L_t^{1-\theta}.$$

Capital is accumulated through households' savings, depreciates at rate δ, and is rented to firms at rental rate, r_t, in a competitive market.

I. Suppose that in each period households receive a random labour endowment $\ell_t \in \{\ell_{\min}, \ell_{\max}\}$, with

$$\pi_{ij} = \mathrm{Prob}\left\{\ell_{t+1} = \ell_j | \ell_t = \ell_i\right\}.$$

They supply this labour inelastically to firms in a competitive market, where w_t is the wage. The measure of households receiving each specific labour endowment is constant over time.

a. Define a stationary competitive equilibrium (SCE) in which both the interest rate and distribution of households over states is time invariant. *(7 points)*

b. In equilibrium, what is the relationship between the interest rate on borrowing, q_t, and the rental rate for capital, r_t? *(5 points)*

c. Is the SCE allocation efficient? If not, describe the nature of the inefficiency, and compare the equilibrium allocation to an efficient one. *(8 points)*

II. Suppose now households are endowed with one unit of labour each period and that the labour market is characterized by search and matching along the lines of Pissarides (2000). Firms create vacancies at cost ξ per period, and matches take place according to a matching function, $M(u, v) = Bu^{\alpha} v^{1-\alpha}$, with the usual properties. Each period unemployed workers receive unemployment benefit $z > 0$ and matches end with probability s. Wages are determined by Nash bargaining, with workers' "bargaining power" given by α.

d. Modify your definition of equilibrium from part **a.** to account for this change in the environment. *(5 points)*

e. Will the equilibrium allocation for this economy be efficient? Compare the efficiency properties of this equilibrium to that which you considered in part **c.** above. *(8 points)*