Economics 816 **Fall 2011**
Advanced Macroeconomic Theory I **Allen Head**

Assignment 5

(Due Friday 12/2)

1. Consider the basic Pissarides matching model. Let the labour force be normalized to 1. Let u denote the measure of *unemployed workers* and v the measure of *vacant jobs* in a steady-state equilibrium. Jobs are filled according to the aggregate matching function:

$$M = u^{\alpha} v^{1-\alpha} \qquad \alpha \in (0,1)$$

Workers maximize their consumption. They may be either employed or unemployed, and the values of being in these two states are denoted W and U, respectively. Employed workers receive flow *wage* w for the duration of their employment. When unemployed, workers receive flow consumption z.

There are a large number of potential firms (jobs) whose main decision is whether to enter by posting a vacancy. A job is *active* if it is matched with a worker and producing. In this case it realizes flow production p and pays the worker wage w. A job is *vacant* while it is waiting to match with a worker. Vacant jobs incur flow costs of $pc > 0$. Let the values of vacant and active jobs be denoted V and J, respectively. There is free entry of firms/jobs.

Matches between firms and workers are dissolved exogenously at Poisson rate λ.

a. Derive and interpret an expression for the level of unemployment in a steady-state equilibrium as a function of the *labour market tightness*, $\theta \equiv v/u$.

b. Let the interest rate be denoted r. Derive expressions for V, J, U, and W as functions of w, r, and economy parameters in a steady-state equilibrium.

c. What happens to your expressions as $r \longrightarrow 0$? Give an intuitive explanation for this.

2. Consider a situation in which production requires a match between two agents of two different types. (You may think of them as a worker and a firm). Assume that all agents, regardless of type, have identical preferences. In particular, all agents have common discount factor $\beta \in (0, 1)$. Each period two agents match, and then as a pair receive n draws of a match value, π, from a cumulative distribution function, $F(\pi)$. If the two agents jointly decide to proceed with any of these matches, then they remain matched forever and each enjoy utility π per period. The number of draws, n, that a matched pair receives is given by

$$n = g(k_1, k_2)$$

where k_i is the disutility from generating match draws incurred by the type i agent *previous* to being matched with the type $\ell \neq i$ match partner. The function g is assumed to be strictly increasing in both arguments.

a. Suppose that each agent of type i takes the actions of its potential type $\ell \neq i$ match partners, k_ℓ, as given. Let v_i denote the value for an unmatched type i agent *prior* to choosing k_i. Formulate Bellman equations for typical agents of the two types, and define a Nash equilibrium.

b. Consider the social planning problem of maximizing a weighted sum of the two types of agents utilities. Let ψ and $1 - \psi$ be the weights on the utilities of the type 1 and 2 agents respectively. Let $W(\psi)$ denote the social value for unmatched agents prior to choosing k_1 and k_2. Write down a Bellman equation for $W(\psi)$.

c. Will the optimal (in the sense of part **b.**) quantities of resources, k_1 and k_2, be devoted to generating match value draws in the Nash equilibrium of part **a.**? Explain.