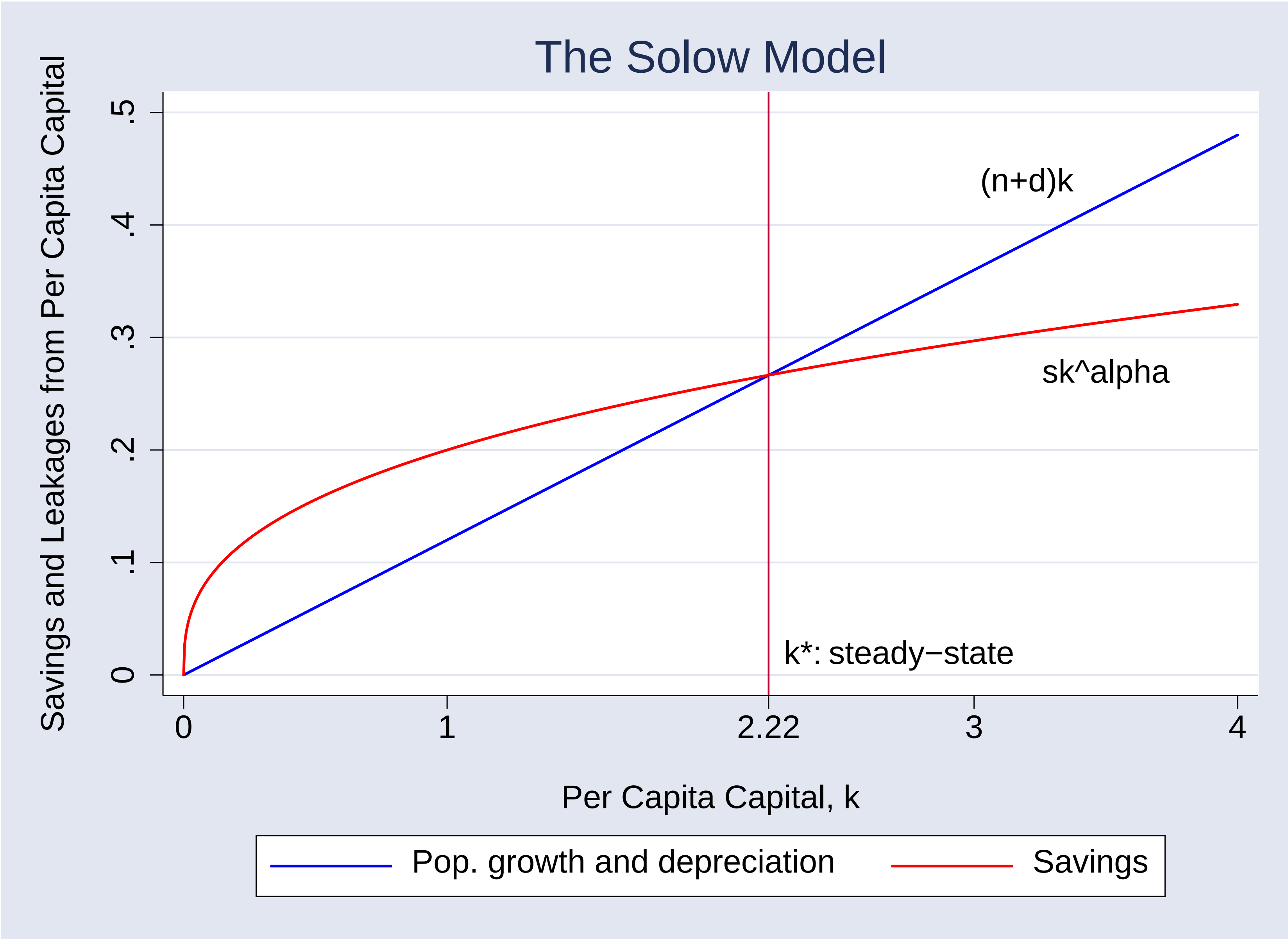

The Solow Model
Savings and Leakages from Per Capita Capital
0
.1
.2
.3
.4
.5
0
1
2.22
3
4
Per Capita Capital, k
(n+d)k
sk^alpha
k*: steady−state
Pop. growth and depreciation
Savings

In the diagram...

$sy = sk^{\alpha}$**:** represents saving (and investment) *per capita*

$(n + d)k$**:** represents the amount of investment needed to keep *per capita* capital constant given:

n**:** population growth

d**:** depreciation

The level of *per capita* capital at which

$$\dot{k} = sk^{\alpha} - (n + d)k = 0 \tag{25}$$

is called the "steady-state capital stock *per capita*", and denoted k^*.

Note: In the steady-state, aggregate capital, $K(t)$, is *not* constant:

$$\frac{\dot{K}}{K} = \frac{\dot{k}}{k} + \frac{\dot{L}}{L} = n > 0. \qquad (26)$$

Similarly, total (not *per capita*) output, $Y(t)$ grows as well:

$$\begin{aligned} Y &= K^{\alpha} L^{1-\alpha} \\ \ln Y &= \alpha \ln K + (1-\alpha) \ln L \end{aligned}$$

So,

$$\begin{aligned} \frac{\dot{Y}}{Y} &= \alpha \frac{\dot{K}}{K} + (1-\alpha) \frac{\dot{L}}{L} \\ &= \alpha n + (1-\alpha) n \\ &= n. \qquad (27) \end{aligned}$$

The economy grows at the rate of population growth.

This means that in the Solow model, growth of *per capita* income is *not* sustained.

In the steady-state, *per capita* income is constant

Outside of the steady-state, there will be growth, positive or negative:

1. Suppose $k(t) < k^*$: $sk^\alpha - (n+d)k = \dot{k} > 0$

Capita *per capita* grows over time.

2. Suppose $k(t) > k^*$: $sk^\alpha - (n+d)k = \dot{k} < 0$

Capital *per capita* falls over time.

These forces cause the economy to tend toward the steady-state over time.

What does the Solow model say about differences across countries with regard to *levels* and *growth rates* of per capita income?

1. Levels:

In the steady-state $\dot{k} = 0$, so:

$$
\begin{aligned}
sk^{*\alpha} &= (n+d)k^* \\
k^{*\alpha-1} &= \frac{n+d}{s} \\
k^* &= \left[\frac{s}{n+d}\right]^{\frac{1}{1-\alpha}} \qquad (28)
\end{aligned}
$$

Per capita income then satisfies:

$$y^* = k^{*\alpha} = \left[\frac{s}{n+d}\right]^{\frac{\alpha}{1-\alpha}} \tag{29}$$

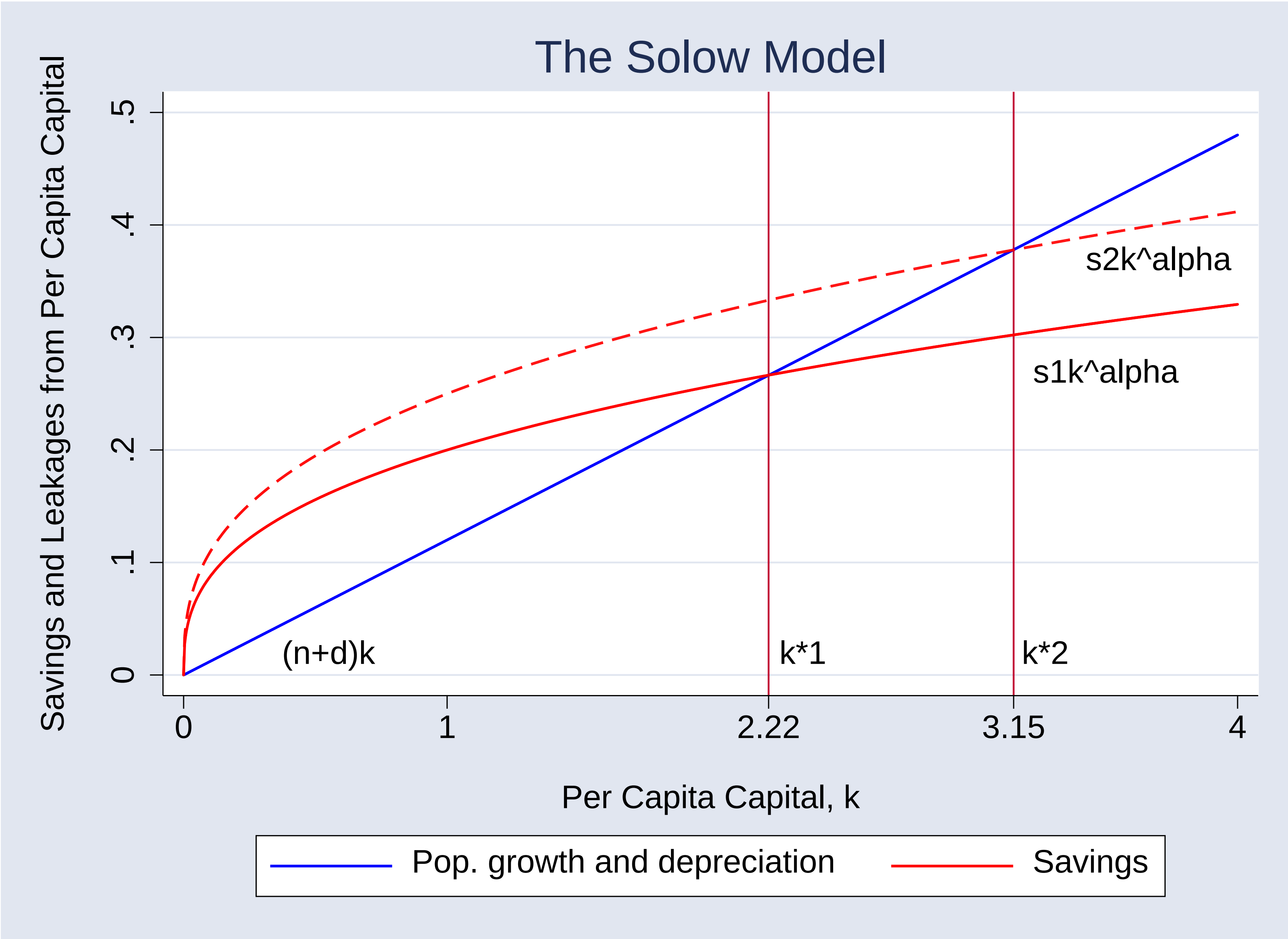
Income *per capita* is:

1. Increasing in the savings rate, s

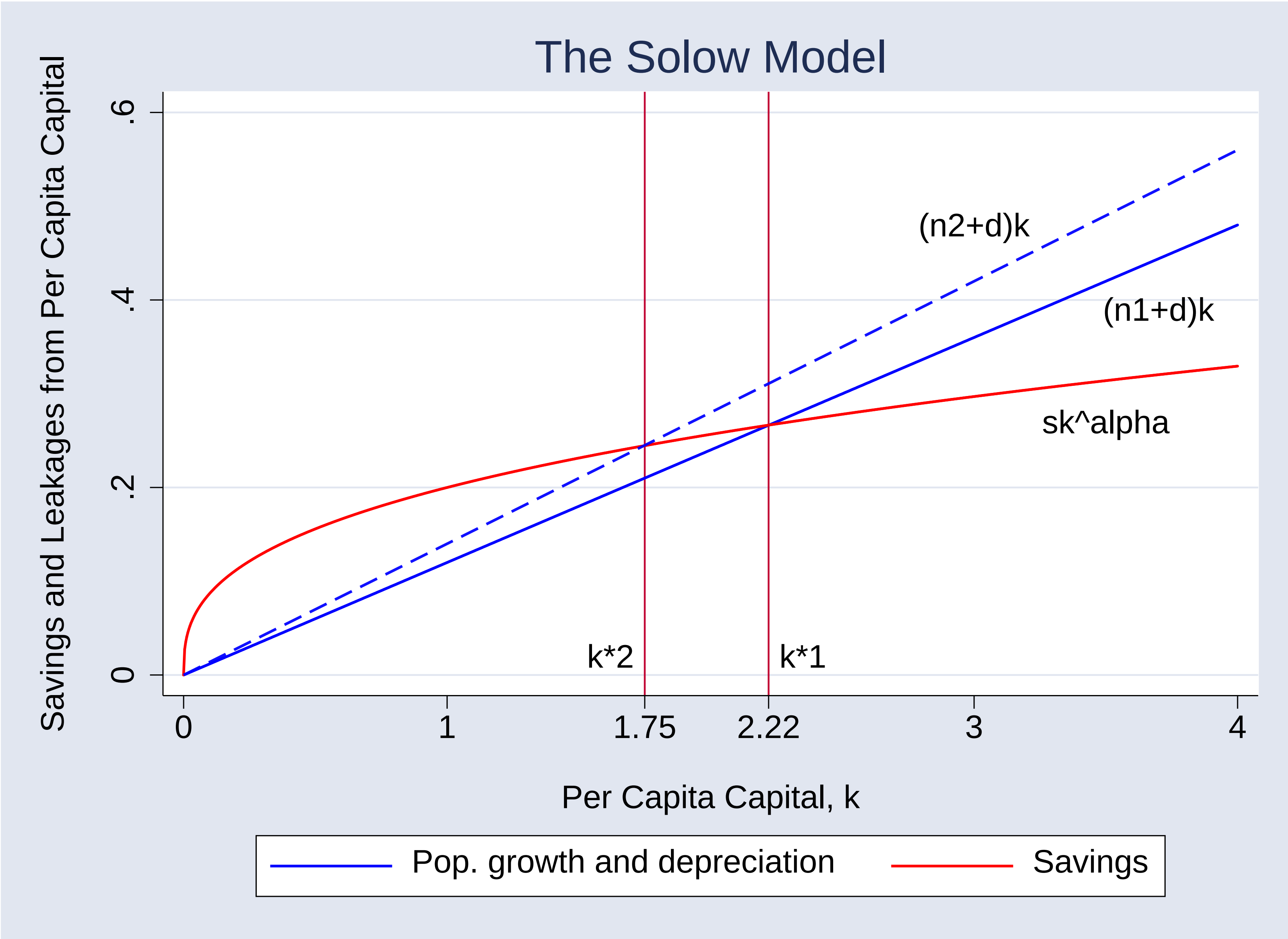
2. Decreasing in the population growth rate, n

3. Decreasing in the deprecation rate, d

Diagrams...



The Solow Model
Savings and Leakages from Per Capita Capital
0
.1
.2
.3
.4
.5
0
1
2.22
3.15
4
Per Capita Capital, k
s2k^alpha
s1k^alpha
(n+d)k
k*1
k*2
Pop. growth and depreciation
Savings


The Solow Model
Savings and Leakages from Per Capita Capital
0
.2
.4
.6
0
1
1.75
2.22
3
4
Per Capita Capital, k
(n2+d)k
(n1+d)k
sk^alpha
k*2
k*1
Pop. growth and depreciation
Savings

These predictions of the Solow model can be taken to the data:

1. Investment rates and GDP per worker:

Jones's Figure 2.6:

Over the period 1960-90, there is a *positive* relationship as suggested by the Solow model.

There are, however, many exceptions, especially among poor countries.

2. Population and GDP per worker:

Jones's Figure 2.7:

Over the same period, there is a *negative* relationship, again as suggested by the model. Still, there are many exceptions. and a lot of variation.

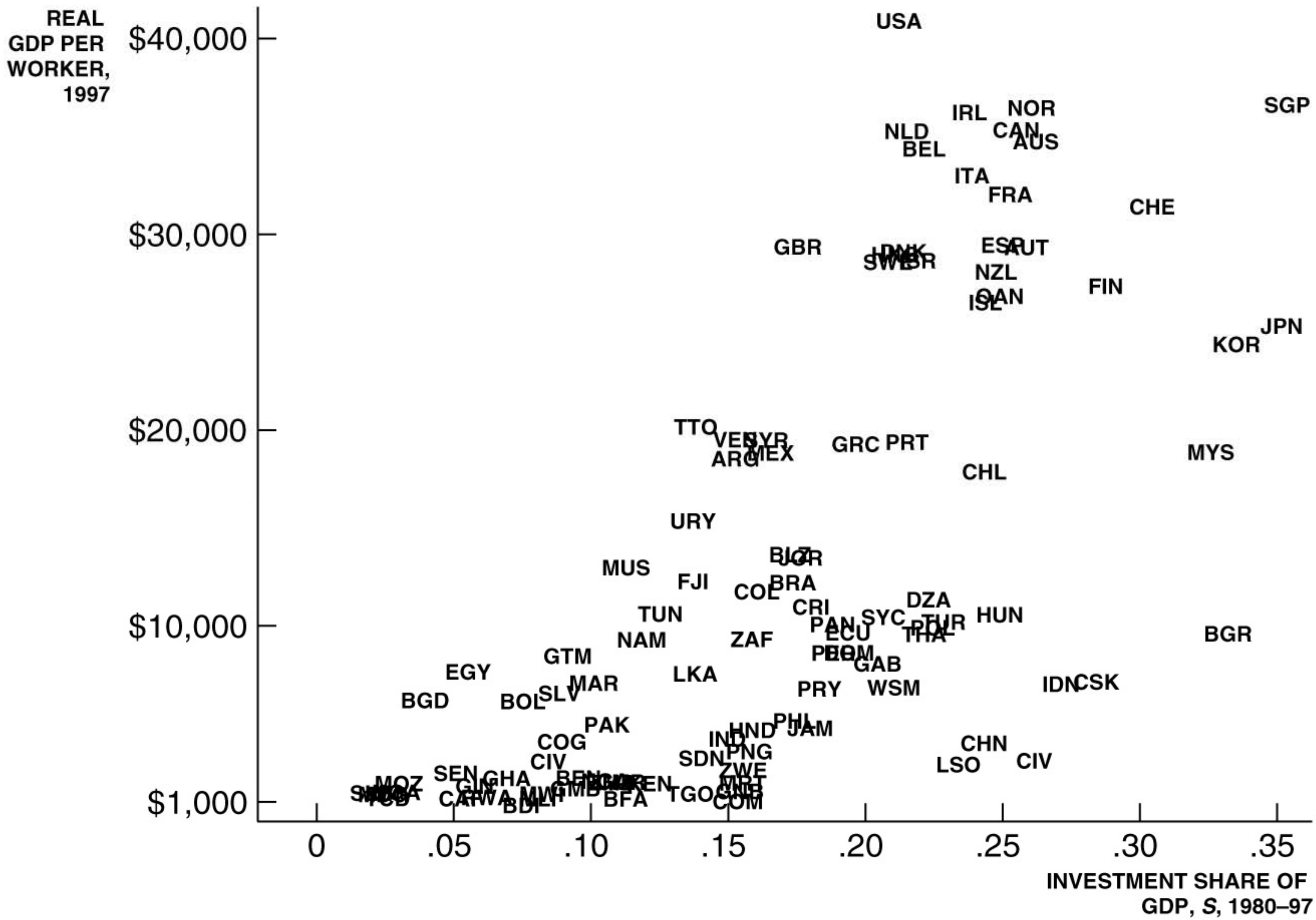


FIGURE 2.6 **GDP PER WORKER VERSUS THE INVESTMENT RATE**

Economic Growth, 2nd Edition

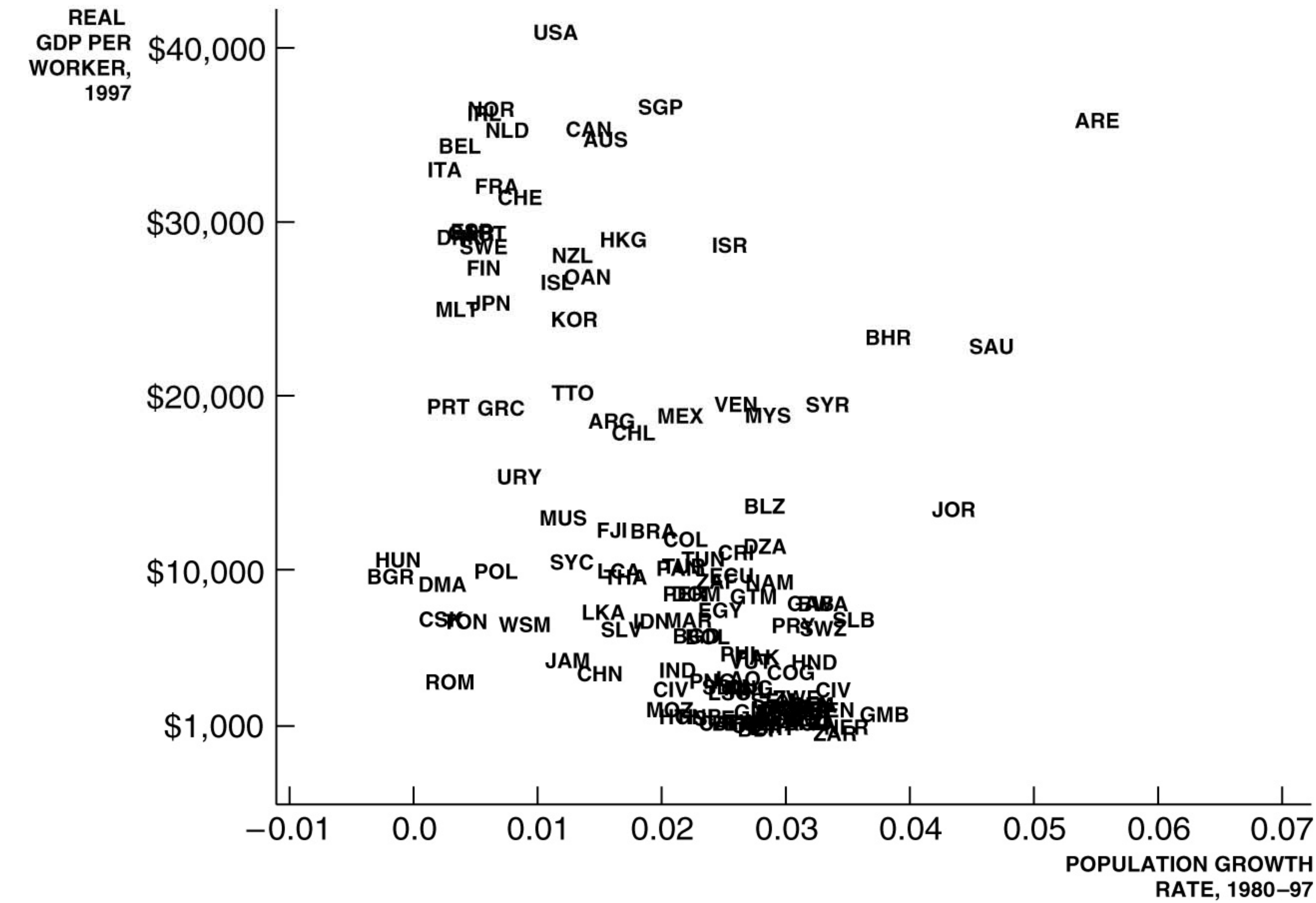


FIGURE 2.7 GDP PER WORKER VERSUS POPULATION GROWTH RATES

Economic Growth, 2nd Edition

2. Growth Rates:

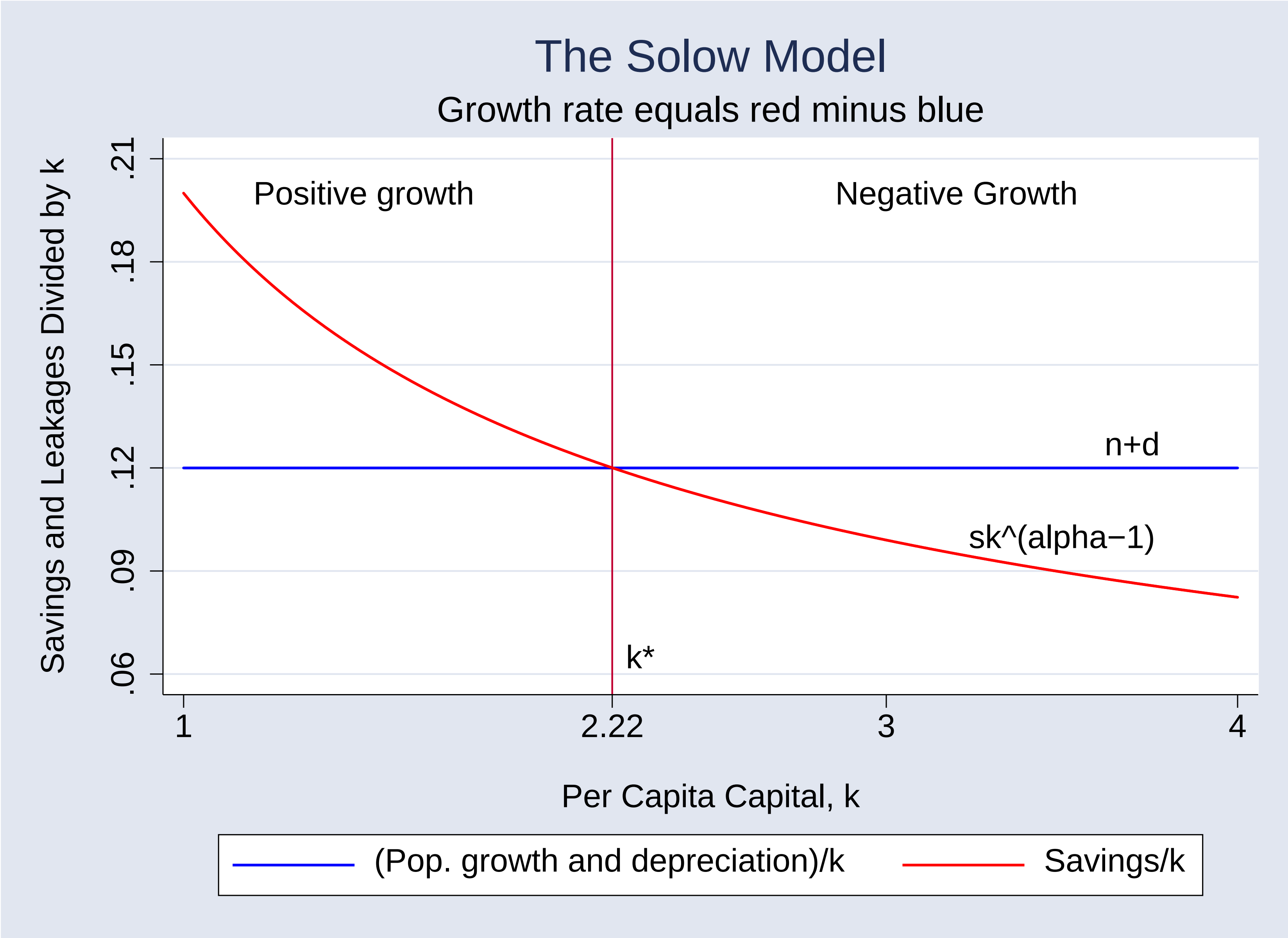
The Solow model can in principle account for vast variation across countries with regard to growth rates, *outside of the steady-state*.

All countries should experience ZERO growth in the steady-state.

The further a country is from its steady-state level of *per capita* capital the more rapidly it should be growing (or shrinking) to converge to that steady-state.

We can represent this in a version of our Solow model diagram:

1. Below the steady-state: positive but *declining* growth.
2. Above the steady-state: negative but *increasing* growth.


The Solow Model
Growth rate equals red minus blue
Positive growth
Negative Growth
n+d
sk^(alpha−1)
k*
Savings and Leakages Divided by k
.06
.09
.12
.15
.18
.21
1
2.22
3
4
Per Capita Capital, k
(Pop. growth and depreciation)/k
Savings/k

Technological Progress in the Solow Model

In the basic Solow model, growth occurs only as a result of factor accumulation.

There are two factors, labour and capital

1. *Labour* grows *exogenously* through population growth.
2. *Capital* is accumulated as a result of savings behaviour.

Because the technology has the neoclassical form (diminishing returns to *per capita* capital), capital accumulation cannot raise *per capita* income forever.

This does not depend on the assumption of a constant savings rate. It will happen even if $s = 1$, that is, if people save *all* of their income.

In order to generate *sustained* economic growth, some assumptions must be abandoned. Here are a couple of possibilities:

1. Constant returns to scale (diminishing returns to $k = K/L$).

2. Static technology. (*i.e.* a constant production function)

Because we are still studying the *Solow* model, we will maintain assumption #1, and allow for *technological progress*. By this we mean shifts in the production function over time.

There are many ways for the production function to “shift” over time. That is, there are many different types of technological progress. In an attempt to be consistent with two observations:

i. The return to capital is roughly constant over time.

ii. Capital's share of income is roughly constant over time.

we will assume that technological progress is *labour augmenting*...

$$Y = F(K, AL) = K^{\alpha}(AL)^{1-\alpha} \tag{30}$$

where A represents the level of technology at the current time.

Labour augmenting technological change is sometimes called "Harrod neutral" and is associated with a constant capital-output ratio (K/Y) in a "steady-state".

Other possibilities:

1. "Hicks neutral" (constant K/L): $\quad Y = AF(K, L)$
2. Capital augmenting (constant L/Y): $\quad Y = F(AK, L)$

Note: The data suggests we are experiencing *labour augmenting* technological change. Why?

The return to capital is proportional to the ration of output to capital:

$r = \alpha \frac{Y}{K}$: If Y/K is constant, so is r.

Under the assumption of the Cobb-Douglas technology, labour augmenting technological change is also Hicks neutral.

Technological change:

$$A(t) = A(0)e^{gt} \tag{31}$$

$$\ln A(t) = \ln A(0) + gt \tag{32}$$

$$\frac{\dot{A}}{A} = \frac{d \ln A(t)}{dA(t)} = g. \tag{33}$$

Here g is a *parameter* denoting the *exogenous* rate of technological progress.

Technological progress is exogenous here (like population growth) because it is determined outside the model, not as a consequence of agents' actions.

Recall the capital accumulation equation in the basic Solow model:

$$\frac{\dot{k}}{k} = s\frac{y}{k} - (d + n)$$

So, for the growth rate of *per capita* capital to be constant, it must be the case that $\frac{y}{k} = \frac{Y}{K}$ must be constant as well.

Recall that this is the distinguishing characteristic of Harrod neutral or labour augmenting technological change.