While the policy raises the *growth rate* of the stock of knowledge only in the short-run, it raises the *level* of technology (and the level of *per capita* income) permanently.

To see this, we solve for the level of *per capita* income along the balanced growth path:

1. Begin with the capital accumulation equation:

$$\dot{K} = s_K Y - dK$$

2. Now define (as usual):

$$\tilde{k} \equiv \frac{K}{AL} \quad \text{so that}$$

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$

(taking logs and differentiating with respect to time)

3. Substitute this into the capital accumulation equation to get:

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = s_K \frac{Y}{K} - (d + n + g_A) \qquad \text{where} \quad g_A \equiv \frac{\dot{A}}{A} \tag{119}$$

4. Since $\frac{Y}{K} = \frac{\tilde{y}}{\tilde{k}}$:

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = s_K \frac{\tilde{y}}{\tilde{k}} - (d + n + g_A) \qquad \text{where} \quad g_A \equiv \frac{\dot{A}}{A} \tag{120}$$

5. Now, return to the production function:

$$
\begin{aligned}
Y &= K^{\alpha}\left[AL_Y\right]^{1-\alpha} \\
\frac{Y}{L} &= \left(\frac{K}{L}\right)^{\alpha}\left(\frac{AL_Y}{L}\right)^{1-\alpha} \\
y &= k^{\alpha}[A(1-s_R)]^{1-\alpha} && (121) \\
\tilde{y} &= \frac{y}{A} = \left(\frac{k}{A}\right)^{\alpha}(1-s_R)^{1-\alpha} && (122) \\
&= \tilde{k}^{\alpha}(1-s_R)^{1-\alpha} && (123)
\end{aligned}
$$

6. Substitute the result into the equation derived in step 4.

$$\begin{aligned} \frac{\dot{\tilde{k}}}{\tilde{k}} &= \frac{s_K \tilde{k}^{\alpha}(1-s_R)^{1-\alpha}}{\tilde{k}} - (d+n+g_A) \\ &= s_K \tilde{k}^{\alpha-1}(1-s_R)^{1-\alpha} - (d+n+g_A) \end{aligned} \tag{124}$$

7. Along the balanced growth path, $\frac{\dot{\tilde{k}}}{\tilde{k}} = 0$, so solving we have

$$\tilde{k} = \left[\frac{s_K}{d+n+g_A}\right]^{\frac{1}{1-\alpha}} (1-s_R) \tag{125}$$

8. Income per unit of effective labour along the balanced growth path is:

$$\begin{aligned}\tilde{y} &= \tilde{k}^{\alpha}(1-s_R)^{1-\alpha} \\ &= \left[\frac{s_K}{d+n+g_A}\right]^{\frac{\alpha}{1-\alpha}}(1-s_R)^{\alpha}(1-s_R)^{1-\alpha} \\ &= \left[\frac{s_K}{d+n+g_A}\right]^{\frac{\alpha}{1-\alpha}}(1-s_R) \qquad (126)\end{aligned}$$

Now, $\tilde{y} = y/A$ and

$$g_A = \frac{\delta s_R L}{A} \quad \Rightarrow \quad A = \frac{\delta s_R L}{g_A} \qquad (127)$$

9. So, income *per capita* along the balanced growth path is:

$$
\begin{aligned}
y(t) &= A(t)\left[\frac{s_K}{d+n+g_A}\right]^{\frac{\alpha}{1-\alpha}}(1-s_R) \\
&= \left[\frac{s_K}{d+n+g_A}\right]^{\frac{\alpha}{1-\alpha}}\frac{\delta s_R}{g_A}(1-s_R)L(t) \qquad (128)
\end{aligned}
$$

Here it is clear that income *per capita* grows with population, $L(t)$.

All parameters other than n; the savings rate, rate of depreciation, technology parameters,and the share of the population employed in R&D affect the level of income *per capita* along the balanced growth path but not its growth rate in the long-run.

Economic behaviour underlying the Romer model

In our study of the neoclassical growth model, we did not devote much attention to individuals' economic behaviour.

This was because we could appeal to two basic results in economic theory:

1. Under certain conditions, the competitive equilibrium is Pareto efficient.
2. Efficient allocations (including the competitive equilibrium) can be computed by solving a "social planning" problem.

Because of these results, we could be confident that the "optimal" balanced growth path was also the "equilibrium" balanced growth path.

In the Romer model, however, we cannot ignore the details of economic behaviour.

Once we depart from perfect competition etc., we can no longer be sure that the equilibrium is efficient. In these cases, the *details* of how economic agents behave makes a difference.

We consider a particular market structure or *decentralization* proposed by Romer. There are three *sectors*:

1. Final goods sector (produces consumption-investment goods)

2. R&D sector (produces ideas)

3. Intermediate goods sector (implements ideas)

Think of the economy as working this way...

i. The R&D sector produces *ideas* which we may think of as plans for new capital goods (*e.g.* machines, equipment, etc.).

ii. Researchers patent these ideas and then sell the exclusive right to produce the new good to a firm in the intermediate goods sector. This firm has a monopoly over the production of this good.

iii. Final goods producers buy the intermediate goods from the firms in this sector and produce goods for consumption and saving.

We will work backwards, starting with the final goods sector.

Imperfect competition in the intermediate goods sector will enable us to deal with increasing returns. The key issue in the economy is the division of labour between R&D and final goods production.

1. Final Goods Sector

The consumption-investment good is produced using labour and intermediate goods:

$$Y = L_Y^{1-\alpha} \sum_{i=1}^{A} x_j^{\alpha} \tag{129}$$

- L_Y: Labour used in producing output
- A: The *variety* of intermediate goods

For fixed A, the technology exhibits constant returns to scale.

As A increases, the economy exhibits a form of increasing returns.

It is useful to interpret A as the *number* of intermediate goods. These goods are indexed by $j \in [0, A]$

$$Y = L_Y^{1-\alpha} \int_0^A x_j^\alpha \, dj \tag{130}$$

Assume that there are a large number of identical final goods producers that behave competitively and solve:

$$\max_{L_Y, x_{j, j\in[0,A]}} L_Y^{1-\alpha} \int_0^A x_j^\alpha \, dj - wL_Y - \int_0^A p_j x_j \, dj. \tag{131}$$

The first order conditions yield:

$$w = (1-\alpha) L_Y^{-\alpha} \int_0^A x_j^\alpha \, dj = (1-\alpha) \frac{Y}{L_Y} \tag{132}$$

$$p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1} \quad \forall j. \tag{133}$$

2. The Intermediate Goods Sector:

Each intermediate good is produced by a single firm which acts as a monopoly and sells to firms in the final goods sector.

- The intermediate goods firms acquire the sole right to produce their goods by buying a patent. The price of this patent represents a *fixed cost* of production.

- Once a firm has acquired a patent for an intermediate good it can convert one unit of capital into one unit of that good.

- Think of these intermediate goods as

  i. specific machines or types of equipment

  ii. specialized services (management, technical consulting etc.)

The maximization problem of the producer of intermediate good j:

$$\max_{x_j} p_j(x_j)x_j - rx_j - F \tag{134}$$

i. $p_j(x_j)$ is the demand *function* for intermediate good j

ii. F is the fixed cost of acquiring the patent on good j

iii. r is the price of capital, which the firm takes as given.

iv. The production function, $x_j = k_j$ has been substituted in.

Note:

1. The capital market is competitive.
2. The patent lasts forever.
3. This problem is the same for all j (so we can drop it)

The first order condition for this problem is:

$$p'(x)x + p(x) - r = 0 \tag{135}$$

Note that F does not appear because it is *sunk* when production begins.

Rearranging we have:

$$p'(x)\frac{x}{p(x)} + 1 = \frac{r}{p(x)} \tag{136}$$

$$p = \frac{1}{1 + p'(x)\frac{x}{p}} r = \frac{1}{1 - \epsilon} r. \tag{137}$$

where $\epsilon = -p'(x)\frac{x}{p}$ is the elasticity of demand for x.

From the first-order conditions for the final goods producers' problems we have the demand for good x_j (for any j):

$$p(x) = \alpha L_Y^{1-\alpha} x^{\alpha-1} \tag{138}$$

$$\begin{aligned} \epsilon &= -p'(x)\frac{x}{p(x)} && (139)\\ &= -(\alpha-1)\alpha L_Y^{1-\alpha} x^{\alpha-2}\frac{x}{\alpha L_Y^{1-\alpha} x^{\alpha-1}} && (140)\\ &= 1-\alpha && (141) \end{aligned}$$

so the price of an (*i.e.* any) intermediate good is given by:

$$p = \frac{1}{\alpha} r. \tag{142}$$

In this case we say that the price is a constant mark-up over marginal cost:

$$p = \underbrace{\frac{1}{\alpha}}_{\text{mark}-\text{up}} \times \underbrace{r}_{\text{marginal cost}} \tag{143}$$

1. Since $\alpha < 1$, price is greater than marginal cost. *Monopolistic competition* of this form avoids the problems associated with increasing returns stemming from fixed costs. In equilibrium, firms can charge a price high enough to compensate them for the cost of the patent.

2. The prices of all intermediate goods are the same. So, all final goods producers will demand the same quantity of each intermediate good and all will be produced in the same quantity:

$$x_j = x \qquad \text{for all } j. \tag{144}$$

Since each intermediate goods firm produces the same quantity, x, and sells it at the same price, p, they will all have the same profit, π:

$$\begin{aligned} \pi &= px - rx \\ &= \frac{1}{\alpha}\alpha px - \alpha px \qquad \text{because} \quad p = \frac{1}{\alpha}r \Leftrightarrow r = \alpha p \\ &= \left(\frac{1}{\alpha} - 1\right)\alpha px \\ &= (1-\alpha)px. \end{aligned} \tag{145}$$

Again using the demand function, $p(x)$, we have:

$$\begin{aligned} \pi &= 1-\alpha)\alpha L_Y^{1-\alpha} x^{\alpha-1} x \\ &= \alpha(1-\alpha)L_Y^{1-\alpha} x^{\alpha}. \end{aligned} \tag{146}$$

Now, if all intermediate goods are produced in the same quantity we have:

$$Y = L_Y^{1-\alpha} \int_0^A x^\alpha \, dj = L_Y^{1-\alpha} A x^\alpha \tag{147}$$

so that profit may be written:

$$\pi = \alpha(1-\alpha)\frac{Y}{A} \tag{148}$$

This equation will be of use when we consider the research sector.

Note also that if the variety or number of intermediate goods increases *and* they continue to be produced in the same quantity, x, the technology exhibits *increasing returns to variety*.

In equilibrium, the total demand for capital by intermediate goods producers must equal the supply of capital, K:

$$\begin{aligned}
\int_0^A x_j \, dj &= K && (149) \\
Ax &= K \qquad \text{since} \quad x_j = x && (150) \\
x &= \frac{K}{A} && (151)
\end{aligned}$$

$$\begin{aligned}
Y &= L_Y^{1-\alpha} A x^{\alpha} \\
&= L_Y^{1-\alpha} A \left(\frac{K}{A}\right)^{\alpha} = K^{\alpha} \left(A L_Y\right)^{1-\alpha} && (152)
\end{aligned}$$

This is exactly the aggregate technology that we started with.

3. The Research Sector

Any person can decide whether to work in the research sector or in the final goods industry.

Working in research is like being an independent inventor. You come up with an idea, and then you patent it and sell the patent to an intermediate goods producer.

Imagine an auction taking place with potential producers bidding for the patent. What will he price of the patent be?

P_A: The present discounted value of the profits that the patent will generate.

We want to know what happens to P_A over time. Does it grow? etc.

Why is this important?

- P_A determines the return to working in the research sector. We need to compare this to the wage (return to working in production) to understand the allocation of labour across the two sectors in which it is employed.

Why is this difficult?

- We know profits at a point in time (derived earlier), but we don't know how to discount.

For this, we make use of a "no-arbitrage" argument:

That is, we relate the price of a patent to the return on another asset, capital:

An agent in this economy that wants to make an investment has two options:

1. save (create physical capital)

2. buy a patent and become and intermediate goods producer

In equilibrium, the return to these two activities must be equal.

Let $\$P_A$ be the amount of money to be invested. Then:

- The return to purchasing capital is given by: rP_A
- The return on the patent is: $\pi + \dot{P}_A$

  i. Flow profit, π

  ii. Capital gain, $\dot{P}_A$

So, our no-arbitrage argument requires that in equilibrium:

$$\begin{aligned} rP_A &= \pi + \dot{P}_A \\ r &= \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} \end{aligned} \tag{153}$$

Now, r is constant (and proportional to Y/K). So, if $\frac{\dot{P}_A}{P_A}$ is constant, it must be that $\frac{\pi}{P_A}$ is constant as well.

$$\pi = \alpha(1-\alpha)\frac{Y}{A}$$

We know that

$$\frac{\dot{A}}{A} = \frac{\dot{y}}{y} \quad \Rightarrow \quad \frac{y}{A} \quad \text{constant}$$

Now,

$$\ln\left(\frac{Y}{A}\right) = \ln y + \ln L - \ln A$$

$$\frac{\dot{\overline{Y/A}}}{Y/A} = \underbrace{\frac{\dot{y}}{y} - \frac{\dot{A}}{A}}_{0} + \frac{\dot{L}}{L} = n \tag{154}$$

So, $\frac{Y}{A}$, π, and P_A must grow at the rate of population growth, n:

$$r = \frac{\pi}{P_A} + n \tag{155}$$

$$P_A = \frac{\pi}{r-n} \tag{156}$$

This is the price of a patent along the balanced growth path.

We have now described a *decentralized* environment for the Romer model:

1. The aggregate production function exhibits:

i. constant returns to K and L_Y

ii. increasing returns once A is taken into account as an input.

2. Increasing returns require imperfect competition (profits):

After paying for labour and capital, intermediate goods producers must have something left over to compensate the inventor of their good.

Note that these firms' "profits" are all used to buy the patent. The value of the monopoly is therefore "capitalized".