Economics 421 **Winter 2006**
Topics in Macroeconomics **Allen Head**

Notes on the Aghion-Howitt Model of Growth Through Creative Destruction

The exposition here follows that of Aghion's and Howitt's text, *Endogenous Growth Theory*, Chapter 2, sections 2.1–2.5. This part of the book, as well as the original research paper in *Econometrica*, are on reserve at Stauffer Library.

The Aghion and Howitt model illustrates several issues regarding endogenous growth through research and development. First, it introduces the idea of obsolescence. The introduction of new products, while leading to growth of productivity and per capita income, also destroys the value of the old products that are replaced. Second, the model illustrates the importance of agents' beliefs about the future for the growth process. Beliefs alone can be the difference between being on a balanced growth path, oscillating between periods of high and low growth, or even experiencing no growth at all. Finally, the model illustrates that it is possible for the equilibrium growth rate to be higher or lower than that which is socially optimal.

I. The Environment

In many ways the model is similar to the version of the Romer model presented in Jones's text. In particular, we focus on the market equilibrium of an economy with three sectors:

1. a perfectly competitive final output (manufacturing) sector
2. a monopolistic sector producing a single intermediate good
3. a research and development sector comprised of many identical researchers.

Time is continuous and indexed by $\tau \geq 0$. There are two goods, a final consumption good and an intermediate good. There is one basic factor of production, labour. There are a large number (L) of identical consumers who maximize utility over an infinite time horizon.

Preferences:

A representative consumer chooses consumption, c, to maximize:

$$U\left(c(\tau)\right) = \int_0^\infty e^{-r\tau} c(\tau) d\tau, \tag{1}$$

where r is the rate of time preference and interest rate.

Technologies:

The final consumption good is produced using only the intermediate good, according to the production function

$$c(\tau) \leq y(\tau) = A_t x(\tau)^\alpha \qquad 0 < \alpha < 1. \tag{2}$$

Here the productivity parameter, A_t, given by

$$A_t = A_0 \gamma^t \qquad \gamma > 1, \tag{3}$$

is the productivity of the "t^{th}" generation of intermediate good and x is the quantity of the intermediate good employed.

The intermediate good is produced one-for-one using labour. For this reason, we can let x be the quantity of labour employed in production, as well as the quantity of intermediate good produced.

The productivity of the intermediate good is increased randomly at a rate which depends on the amount of effort devoted to research and development. At each point in time, the approximate probability that an innovation occurs right then is given by λn, where $\lambda > 0$ is a parameter, and n is the amount of labour employed in research and development. λn is often referred to as the "Poisson arrival rate" of innovations, and is the expected (or average) number of innovations that occur within a single unit of time.

If the t^{th} innovation has occurred previously, so that productivity is given by A_t, then the $t + 1^{st}$ innovation raises productivity by the factor γ, to

$$A_{t+1} = A_0 \gamma^{t+1} = \gamma A_t. \tag{4}$$

Endowments:

The amount of labour employed in the two sectors (manufacturing and research and development) cannot exceed the total amount available:

$$x + n \leq L. \tag{5}$$

Market Structure:

As noted above, the final goods sector is competitive, and the intermediate good is supplied by a monopolist. This is very similar to the Romer model. The key difference, however, is with regard to the research and development (R&D) sector. This sector is characterized by a "patent race". That is, a measure n of individuals devote their labour to trying to invent the next generation (say the $t + 1^{st}$) of intermediate good. Eventually, one of them succeeds. This lucky researcher can then either become a monopolist, producing and selling his/her innovation, or sell an infinitely lived patent to another firm to produce it monopolistically.

II. Equilibria

"Period t" will refer to the time between the introduction of the t^{th} and $t+1^{st}$ innovations.

As in Romer's model the key issue is the equilibrium division of labour between manufacturing and R&D. Since all individuals are identical, in equilibrium the returns to labour in its two uses must be equalized:

$$w_t = \lambda V_{t+1}. \tag{6}$$

The left-hand side of (6) is the return to working in manufacturing during period t, where w_t is the manufacturing wage. The right-hand side is the expected return to working in the patent race. V_{t+1} is the value of $t+1^{st}$ innovation to the researcher who wins the patent race. The expected length of time until the $t+1^{st}$ innovation appears is λn, and each individual researcher wins the race with probability $1/n$.

A no arbitrage argument can also be used to derive an equation for V_{t+1}. Thinking of V_{t+1} as the cost of acquiring the patent from the researcher who obtains it, rV_{t+1} is the value per unit time of resources required to become the $t+1^{st}$ monopolist. This must satisfy

$$rV_{t+1} = \pi_{t+1} - \lambda n_{t+1} V_{t+1}, \tag{7}$$

where π_{t+1} is the flow of monopoly profits generated by the $t+1^{st}$ innovation and n_{t+1} is *future* effort in R&D (that is the quantity of labour devoted, in period t+1, to producing the $t+2^{nd}$ innovation). The second term on the right-hand side of (7) is the capital loss associated with the obsolescence of the $t+1^{st}$ innovation, which occurs when the $t+2^{nd}$ innovation is invented. Solving (7) for V_{t+1} we have

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda n_{t+1}}. \tag{8}$$

Equation (8) says that the value of the $t+1^{st}$ innovation is equal to the profit stream it generates, π_{t+1}, discounted by the obsolescence-adjusted interest rate, $r + \lambda n_{t+1}$.

As in the Romer model, in order to go further we must solve the profit maximization problem of the successful innovator. Consider the firm that holds the monopoly on the t^{th} innovation:

$$\pi_t = \max_x p_t(x)x - w_t x, \tag{9}$$

where $p_t(x)$ is the inverse demand for the intermediate good. Since the final good sector is perfectly competitive, we know that the intermediate good will be employed until its price equals its marginal value product. Thus, the inverse demand is given by

$$p_t(x) = \alpha A_t x^{\alpha-1}, \tag{10}$$

and (9) can be written

$$\pi_t = \max_x \alpha A_t x^{\alpha} - w_t x. \tag{11}$$

The first-order condition is

$$\alpha^2 A_t x^{\alpha-1} = w_t, \tag{12}$$

which can be solved for x,

$$x = \left[\frac{\alpha^2 A_t}{w_t}\right]^{\frac{1}{1-\alpha}} \equiv \tilde{x}\left(\frac{w_t}{A_t}\right). \tag{13}$$

Here the function $\tilde{x}$ expresses employment in manufacturing as a function of the productivity adjusted wage, w_t/A_t, which will be denoted ω_t. Thus, in general solution of the profit maximization problem gives rise to the function $\tilde{x}(\omega_t)$, which from (13) can be seen to be decreasing in ω_t. As the productivity adjusted wage rises, employment in manufacturing decreases.

Plugging back into (11), profit can be written

$$\pi_t = \left[\frac{1}{\alpha} - 1\right] w_t x \equiv A_t \tilde{\pi}(\omega_t), \tag{14}$$

where the function $\tilde{\pi}$ is also decreasing in ω_t.

Combining (6) and (8), write

$$\omega_t = \frac{\lambda}{A_t}\frac{\pi_{t+1}}{r + \lambda n_{t+1}} \tag{15}$$

$$= \frac{\lambda}{A_t}\frac{A_{t+1}\tilde{\pi}(\omega_{t+1})}{r + \lambda n_{t+1}} \tag{16}$$

$$= \frac{\lambda\gamma\tilde{\pi}(\omega_{t+1})}{r + \lambda n_{t+1}}, \tag{17}$$

where (16) makes use of (14) and (17) uses (4).

Equilibria are characterized by (17) and (5). Inspection of these two equations reveals that they involve more than two unknowns, and so it is natural to expect that there are multiple solutions. These multiple solutions are associated with different equilibria.

1. The Balanced Growth Path

Begin by restricting attention to the steady-state case in which $\omega_t = \omega$ and $n_t = n$ are both constant. Then (17) and (5) may be written

$$\omega = \frac{\lambda\gamma\tilde{\pi}(\omega)}{r + \lambda n} \tag{18}$$

$$n = L - \tilde{x}(\omega) \tag{19}$$

This system can be solved for a unique $\hat{\omega}$ and $\hat{n}$. See Figure 2.1 on p.59 of the Aghion and Howitt text for a diagram illustrating the determination of $\hat{n}$. Steady-state employment in R&D, $\hat{n}$ can be used to characterize the average growth rate along the balanced growth path.

In the steady-state, aggregate output is given by

$$y_t = A_t \hat{x}^{\alpha} = A_t (L - \hat{n})^{\alpha}. \tag{20}$$

Since, $A_{t+1} = \gamma A_t$, $y_{t+1} = \gamma y_t$. This describes the rate at which output changes *as productivity advances* due to innovation. It does *not* describe the rate at which output grows per unit of time. The average growth rate in the steady-state, $\hat{g}$, is given by the expected of change of output per unit time:

$$\hat{g} = \mathrm{E}\left[\ln y(\tau + 1)\right] - \ln y(\tau). \tag{21}$$

Given (20),

$$\ln y(\tau + 1) = \ln y(\tau) + \ln \gamma \epsilon(\tau), \tag{22}$$

where $\epsilon(\tau)$ is the number of innovations that occur between τ and $\tau + 1$. This is a random number, but as noted earlier its expected value is given by the Poisson arrival rate of innovations:

$$\mathrm{E}\epsilon(\tau) = \lambda \hat{n}. \tag{23}$$

Equation (21) can then be written

$$\begin{aligned} \hat{g} &= \ln y(\tau) + \ln \gamma \lambda \hat{n} - \ln y(\tau) \\ \hat{g} &= \lambda \hat{n} \ln \gamma. \end{aligned} \tag{24}$$

Thus the average growth rate along the balanced growth path is determined by the steady-state level of employment in R&D, $\hat{n}$.

2. Uneven Growth

Return to the system of equations given by (17) and (5). Inspection of these equations reveals that there is a negative relationship between current and future research effort in equilibrium. This makes intuitive sense:

High (Low) "expected" research effort in the future, n_{t+1}, leads current researchers to believe that if they win the patent race, their reign as monopolist will be short (long). Thus the returns to innovating are currently low (high), and n_t is low (high).

Thus beliefs about future research can lead to equilibria in which research effort cycles between low and high levels. When expected future research is high, current research may be low and vice versa. Note that the periods of low research should be longer-lived on average. In the extreme this can lead to cases in which there is no growth. If expected future research is sufficiently high, then no effort will be expended now to invent the next generation of intermediate good. In this case, $n_t = 0$. With $n_t = 0$, no innovation occurs, and the economy remains trapped in period t forever. See Figure 2.3 on p.64 for an illustration of an equilibrium with uneven growth.

The only thing that distinguishes equilibria with uneven growth or no growth at all from the balanced growth path is agents' *beliefs*. These beliefs themselves are an integral part of the equilibrium.

III. Welfare

Now return to consideration of the balanced growth path and consider the relationship between the equilibrium growth rate, $\hat{g} = \lambda\hat{n}\ln\gamma$, and the socially optimal growth rate, denoted g^*. Note that g^* is associated with the socially optimal level of employment in R&D, n^*

First, return to the system of equations that determines the steady-state equilibrium, (18) and (19). Using (14), write

$$\tilde{\pi}(\omega) = \frac{1-\alpha}{\alpha}\omega x. \tag{25}$$

Substitute (25) and (19) into (18) and cancel ω from both sides to obtain

$$1 = \frac{\lambda\gamma\left[\frac{1-\alpha}{\alpha}\right](L-\hat{n})}{r+\lambda\hat{n}} \qquad (26).$$

Note that equation (26) can be solved for $\hat{n}$, the equilibrium level of employment in R&D.

The socially optimal level of employment in R&D is obtained by choosing n^* to solve

$$\max_{n(\tau)} \int_0^\infty e^{-r\tau}y(\tau)d\tau = \int_0^\infty e^{-r\tau}\left[\sum_{t=0}^{\infty} P(t,\tau)A_t(L-n)^\alpha\right]d\tau,$$

where $P(t,\tau)$ is the probability that exactly t innovations have occurred prior to time τ. From the first-order conditions, it can be shown that n^* satisfies

$$1 = \frac{\lambda(\gamma-1)\left[\frac{1}{\alpha}\right](L-n^*)}{r+\lambda n^*} \qquad (27).$$

Now the differences between the equilibrium growth rate, $\hat{g}$, and the socially optimal growth rate, $g^* = \lambda\gamma n^*$, can be seen by noting three differences between (26) and (27):

1. *"Intertemporal Spillover" Effect*

   Comparing the denominators of (26) and (27) note that the private discount rate, $r+\lambda\hat{n}$, exceeds the social discount rate, $r-\lambda(\gamma-1)n^*$. Private innovators see the benefits of innovation as disappearing as soon as their product itself becomes obsolete. From a social point of view, however, an innovation permanently raises productivity, since future innovations will only be improvements. Thus the social discount rate is akin to that derived in the Romer model.

   The implication of this effect is that too little effort is devoted to R&D in the market equilibrium and the equilibrium growth rate is too low.

2. *The "Appropriability" Effect*

   As in the Romer model, the innovator captures only the monopoly profit from his/her innovation, not the entire consumer surplus. This is evident by the term $(1-\alpha)$ in (26). This lowers the incentive to innovate, causing n^* to be greater than $\hat{n}$ and the equilibrium growth rate to be too low.

3. *The "Business Stealing" Effect*

   The innovator views the entire value of his/her innovation as a marginal benefit, ignoring the value of the previous innovation. This is evident from comparison of the γ (gross effect) in the numerator of (26) to the $(\gamma - 1)$ (net effect) in the numerator of (27). From society's point of view, the value of the t^{th} innovation is the *increment* in productivity that it causes. Since the business stealing effect is associated with excessive incentive to innovate, it tends to cause innovation in the market equilibrium to exceed the social optimum, and can cause equilibrium growth to be excessive as well.

Given that the Intertemporal Spillover and Appropriability effects work to make the equilibrium growth rate too low, and the Business Stealing effect works to make it too high, the overall effect is ambiguous. Generally speaking, the smaller the innovation size (γ) and the stronger the monopoly power (the lower α) the stronger is the business stealing effect.