Competitive Firms and the Monopolist Extract the Same: (Stiglitz AER, 1975):

Demand for oil in constant elasticity $p(t) = Q(t)^{-\alpha}; 0 < \alpha < 1$. Zero cost of extraction.

$$TR = pQ = Q^{1-\alpha}$$

$$MR = \frac{dTR}{dQ} = (1-\alpha)Q^{-\alpha} = (1-\alpha)p$$

Hence, $\dfrac{\frac{dMR}{dt}}{MR} = \dfrac{\frac{dp}{dt}}{p}$, and thus extraction paths are the same, given S_0.

Oligopoly (Benchakroun and Long [2006])

Let S_0 be owned by N firms: $S_{01}, S_{02}, S_{03}, \ldots S_{0N} (= S_0)$. Closed loop competition yields current extract $Q_i(t) = \frac{r}{\alpha} S_i(t)$ for $i = 1, \ldots, N$.

$$p(t) = [Q_1(t) + \ldots + Q_N(t)]^{-\alpha}$$

$$\frac{dp}{dt} = -\alpha [Q_1(t) + \ldots + Q_N(t)]^{-\alpha-1}[\dot{Q}_1 + \dot{Q}_2 + \ldots + \dot{Q}_N]$$

$$= -\alpha \frac{p(t)}{Q(t)}[\frac{r}{\alpha}\dot{S}_0 + \frac{r}{\alpha}\dot{S}_1 + \ldots + \frac{r}{\alpha}\dot{S}_N]$$

$$= r \cdot p(t) \frac{[Q_1 + Q_2 + \ldots + Q_N]}{Q}; \quad \text{since } \dot{S}_i = -Q_i$$

$$= r \cdot p(t).$$