QUEEN'S UNIVERSITY

DEPARTMENT OF ECONOMICS

MIDTERM EXAMINATION

Economics 390

February 26, 2009

Instructor: John M. Hartwick

Time Allowed: 75 minutes.

Part I
Do Two (2) (ten marks each).

1) The L.C. Gray problem analyzes the extraction program of say oil for a competitive oil extractor-seller which maximizes profit for the firm. Describe "profit" of the firm and conditions for "profit maximization".

2) An oil extraction firm loses value each period because in each successive period it owns less oil. Explain this "value loss" amount and how this amount shows up in the annual "profit account" of the firms.

3) Kuwait can be viewed as a simple oil extracting competitive, L.C. Gray firm. Explain how Kuwait can maintain a non-declining income stream for itself (the nation), even though it is stuck with less oil in the ground, year after year.

Part II
Do Two (2) (ten marks each).

a) An intuition about monopoly extraction of an oil stock is that there will be a "high" price at each date. Relate this intuition to the pace of extraction under monopoly relative to say a competitive extractive program. Make a careful, precise demonstration of the speed of extraction under monopoly relative to say perfect competition.

b) The great quest in public finance is for government revenue that flows in while leaving the "free market" as UNDISTURBED as possible by taxes. How does Samuelson's taxation scheme for a competitive firm leave the firm's extraction path UNDISTURBED?

c) Nordhaus shifted economists' attention from the issue of running out of oil to viewing oil as a "transition" source of energy. Explain how "cheaper" fusion power will show up in current energy prices (oil price) in the world.

d) Hotelling's r % rule "breaks down" when different sources of oil are not homogeneous. Explain.

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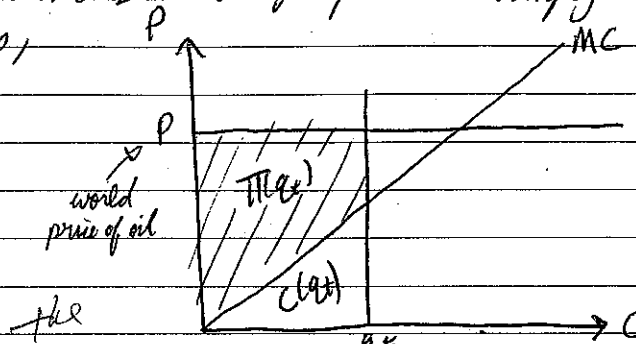
PART I

1) In this scenario, current profit for an L.C. Gray firm is simply total revenues minus total costs. That is,

$\pi(q_t) = P(q_t) - C(q_t)$

- π: profit of firm at extraction level (q_t)
- P: total revenue
- C: total cost at q_t extraction level.



The firm wants to maximize streams of profit given a finite amount of oil supply.

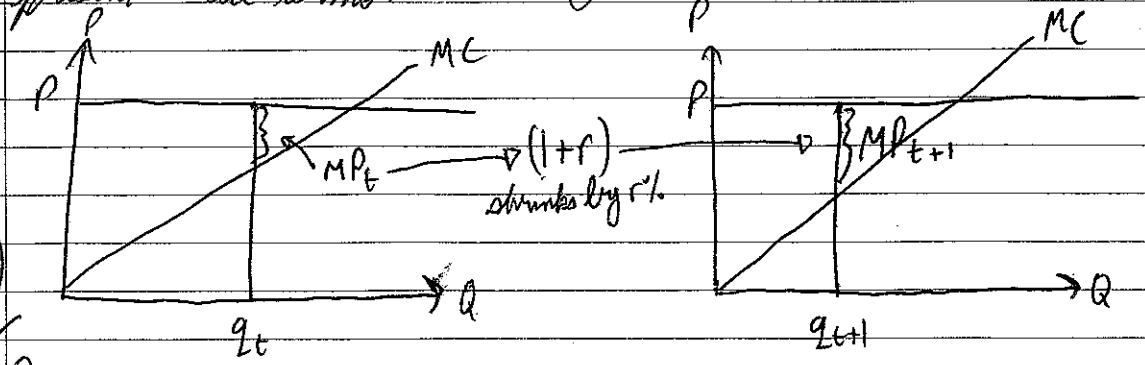
$$\max \quad \pi_0 + \left(\frac{1}{1+r}\right)\pi_1 + \left(\frac{1}{1+r}\right)^2\pi_2 + \cdots + \pi_T\left(\frac{1}{1+r}\right)^T \text{ subject to } q_1 + q_2 + q_3 + \cdots + q_T = S_0$$

- $\left(\frac{1}{1+r}\right)$: taking into account the time value of money to maximize profits in present value terms
- S_0 = total amount of oil reserves or extracted by firm.

L.C. Gray creates two conditions for profit maximization for a competitive extractive firm:

I) $MP_1 = \left(\frac{1}{1+r}\right)MP_2$ or $\frac{\partial \pi(q_t)}{\partial q_t} = \left(\frac{1}{1+r}\right)\frac{\partial \pi(q_{t+1})}{\partial q_{t+1}}$, also known as the r% rule.

This means that marginal profits are equal for all extraction periods in present value terms.



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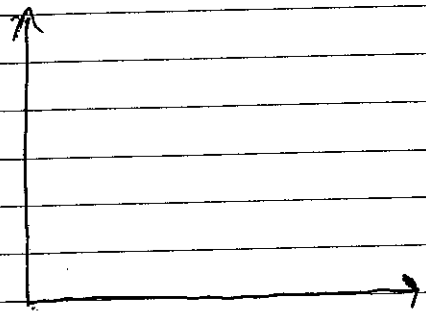
II) The second condition is that $q_T = 0$, that is, the final period of extraction at time T has zero oil left for extraction. L.C. Gray creates this condition by setting marginal profit equal to average profit at time T.

Starting from $q_T = 0$, one can work backwards shrinking MP by r% at each period of extraction to find the profit maximizing quantity for extraction in each period.



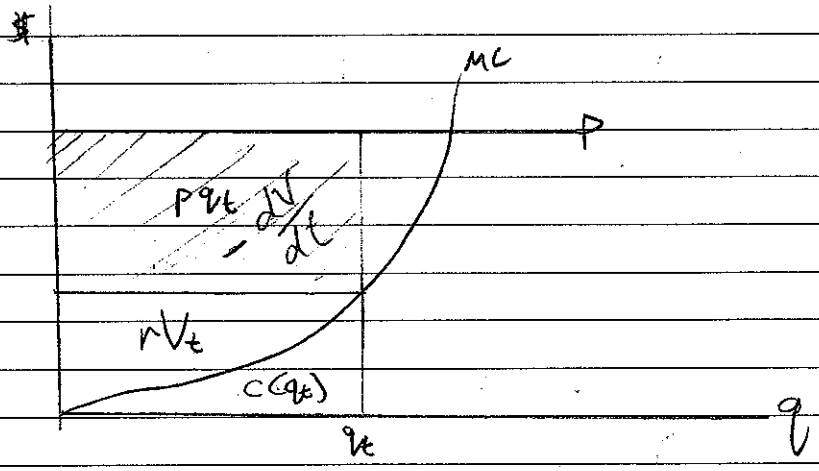
To maximize the shaded area the firm, the firm sets Q_T at the last period T equal to 0 and then uses the r% rule to work backwards to layout an efficient extraction path.

2)

Each period the value of the firm changes because they extract a quantity of oil. However they also gain interest on their stock of oil reserves.

$$V_t - V_{t+1} = [pq_t - C(q_t)] - rV_t \quad \checkmark$$

where $pq_t - C(q_t)$ is the profit from extraction and rV_t is the interest earned on asset



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This also means that the loss in value over time is $\frac{dV_t}{dt} = -[pq_t - C(q_t)] + rV_t$

$-\Pi$

$\Pi = -\frac{dV}{dt} + rV$

but this is a loss so that

$$-\frac{dV_t}{dt} = [pq_t - C(q_t)] - rV_t \quad \checkmark$$

In the profit account the value $pq_t - C(q_t)$ shows up as a profit gained by the firm but it is also a decrease in value of the firm.

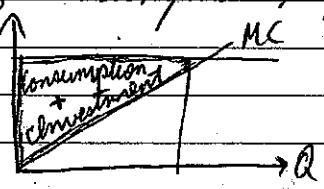
3) Kuwait maintains a non-declining stream of income even given a declining quantity of oil through the use of a Sovereign Wealth Fund.

*Kuwait invests a portion of their income every year into a fund, F, with a rate of return, r, in order to maintain a constant stream of income even once their supply of oil runs out.

The problem is how much ~~of the oil~~ should they invest each year?

This presents a sustainability issue where they must seperate net income into current consumption and current investment.

~~This~~ Their consumption is given by: Consumption = $\underbrace{p(q_t) - C(q_t)}_{\Pi \text{ profit at period } q_t}$ − investment + $\underbrace{\text{investment} \cdot r}_{\text{amount invested into fund}}$ interest earned from fund at rate



They want consumption to be constant through time: $\frac{\partial \text{Consumption}}{\partial \text{time}} = 0$ to be constan

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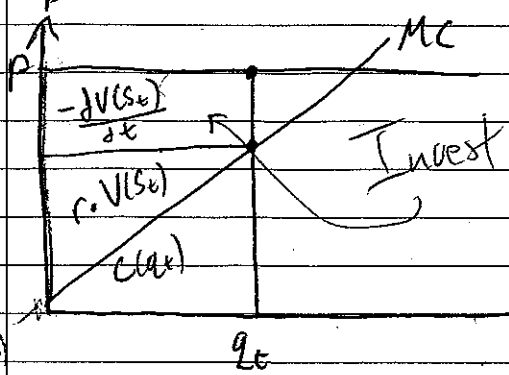
Deriving consumption in this manner yields $\frac{\frac{\partial P_t - C_t}{\partial t}}{P_t - C_t} = r$ (change in marginal profit / marginal profit) = interest rate

Every year, Kuwait loses a value due to extraction equal to $-\frac{\partial V(S_t)}{\partial t}$

also known as economic depreciation. ($V(S_t)$ = present value of firm at time t.)



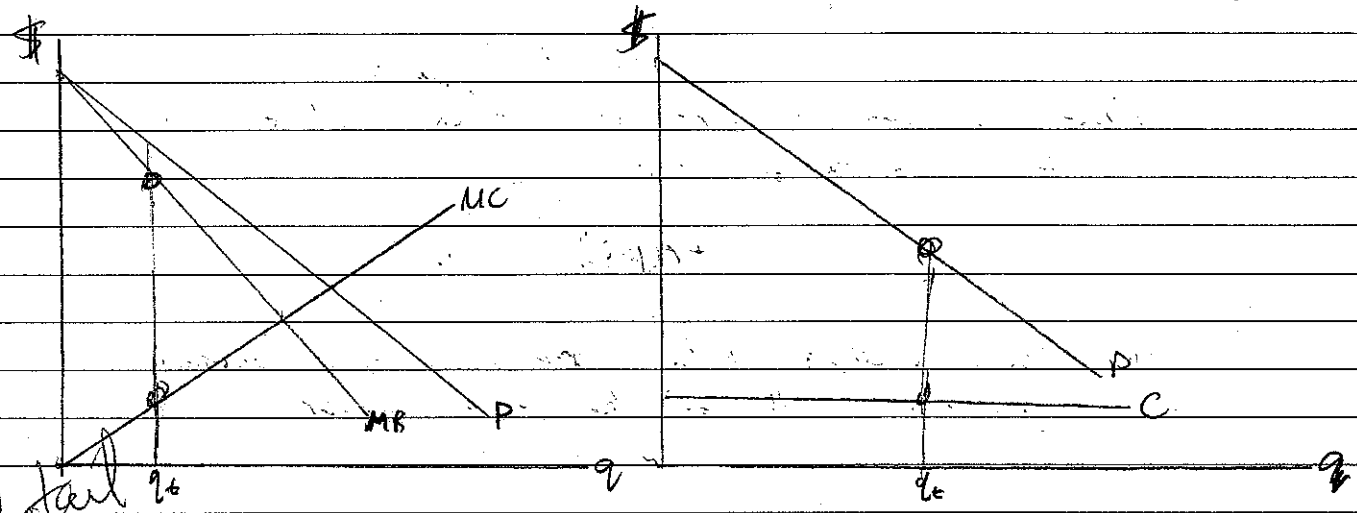
(Yields smooth time expression: $\Pi(q_t) + \frac{dV(S_t)}{dt} = r \cdot$

Thus, in order to maintain a constant stream of income Kuwait offsets this loss in value due to economic depreciation by investing an equal amount each year

Part II

a)

The monopolist extraction differs from competitive extraction such that the monopolist maximizes the marginal revenue minus the marginal cost of the firm, which is the Monopoly profits



a) Monopoly b) Competition

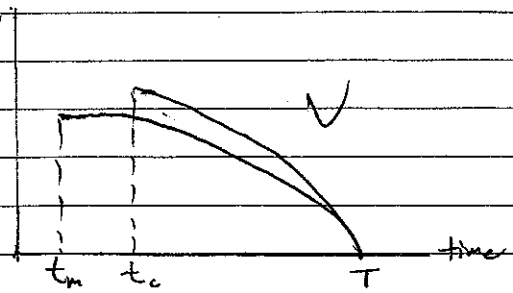
$$V_{monopoly} = [MR_t - MC_t] + \frac{1}{1+r}[MR_{t+1} - MC_{t+1}] + \dots \left(\frac{1}{1+r}\right)^{T-t}[MR_T - MC_T]$$

$q_t + q_{t+1} + \dots q_T = S_0$

Because the monopolist faces rising marginal costs, it can get the most profits by extracting at low levels of quantity. Also because the monopoly firm is a price setter, it can produce at low levels of q and charge a high price, unlike the competitive firms who face a strict price schedule.

Thus the Monopoly firm has an incentive to extract a low quantity at every period and this is what it will do.

Given that the monopoly firm and a competitive firm both have the same initial stock of oil, the monopoly firm will take a larger number of periods to extract all the oil.



The monopoly firm also follows the r% rule in mapping out the efficient extraction path

$$MR_t - MC_t = \frac{1}{1+r}(MR_{t+1} - MC_{t+1})$$

Part II

b) Samuelson's taxation scheme for a competitive firm leaves the firm's extraction path undisturbed (known as tax neutrality) by imposing two conditions:

1) Break for economic depreciation: a deductible amount from profit equal to the value lost from economic depreciation

$$D_t = \frac{-dV(S_t)}{dt}$$

2) Debt payment deductibility: interest payments to debt are deductible.

↳ this is equivalent to imposing a smaller interest rate, $r(1-\tau) < r$.

His results show that the value of the firms before tax equals the value of the firm after tax (tax neutrality). However, since the r is smaller under the tax, firms will take longer to exhaust their resources, but, discounted the present values in the Samuelson tax and no tax case are equal. 10/10

The values of the firms are defined as:

No tax

$$V(S_t) = \pi_0 + \left(\frac{1}{1+r}\right)\pi_1 + \left(\frac{1}{1+r}\right)^2\pi_2 + \cdots + \left(\frac{1}{1+r}\right)^T\pi_T$$

Regular tax

$$V^t(S_t) = (1-\tau)\pi_0 + \left(\frac{1}{1+r}\right)(1-\tau)\pi_1 + \cdots$$, this is not equal to the no tax case. $V(S_t) > V^t(S_t)$

Samuelson tax

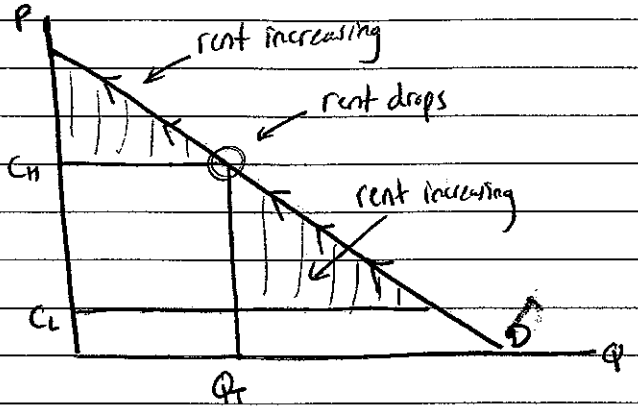
$$V^{ST}(S_t) = \pi_0 + \tau D_0 + \left(\frac{1}{1+r(1-\tau)}\right)\left[(1-\tau)\pi_1 + \tau D_1\right] + \cdots$$ where $V(S_t) = V^{ST}(S_t)$

↳ the smaller interest rate means the future value of cash flows are worth more.

(c)

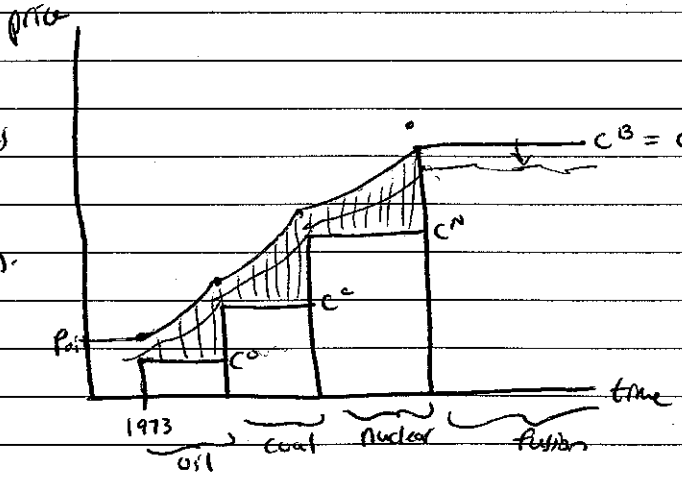
~~The Hotelling model~~

Nordhaus modified the Hotelling model of competitive industry to include a backstop technology:



Nordhaus proposed that $Q_T = 0$ is no longer a doomsday prediction of when the world will run out of energy and we will all freeze to death etc. He says that Q_T is now a future time when we switch from oil (low cost) to some expensive substitute (high cost). Even though the substitute is expensive, the lack of oil has pushed the price up so high that it is economical to switch over to the more costly substitute.

NOTE: The price of energy remains smooth; it is the rent that changes



Blue lines are rent paths.

Nordhaus constructed a price path for oil using real data. He tried to predict the 1973 price of oil based on future expected energy prices. His value was actually lower than the true 1973 price. He concluded this was possible for two reasons:

(1) He was using a Hotelling competitive industry model when in reality the oil industry is monopolistic so prices are higher.

(2) There were few speculators in the 1970s oil mkt b/c it was very thin so the actions of an individual firm had distortionary effects on the whole industry.

→ PTO

Note also on the previous diagram that the price path remains smooth. As time passes and we move to more costlier energy sources, it is the rent that shrinks with each transition not the price that jumps. This is because expectations of future energy costs are reflected in current prices.

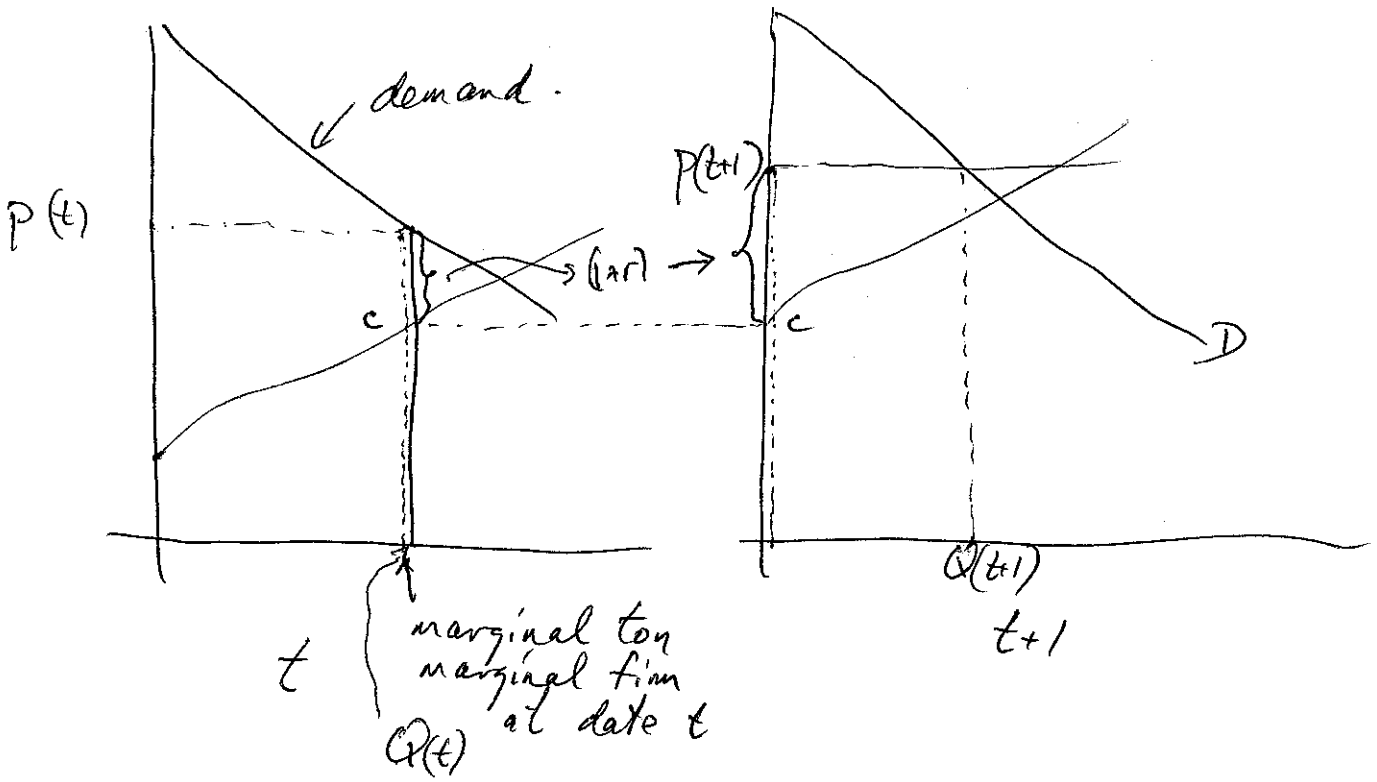
Scientists have forever been trying to create fusion power. When two scientists claimed they had done so in a "cheap" manner, the current price of energy dropped because expectations about future energy costs dropped. Of course the two scientists were proven wrong, and the energy prices jumped back up again. This example reflects how future expectations of energy costs effect current energy prices.

(d) heterogeneous stock?

Competitive - 1 ton per firm.

Assume all stock is a column, closer to surface, the ton is cheaper to extract.

Each firm must be in zero profit arbitrage across dates.



Marginal firm at t says, maybe I will wait one period. Then profit must be

$$p(t) - c = \frac{1}{1+r}\left[p(t+1) - c\right]$$

for c her cost of extraction for her ton.

At Q(t+1), rent per ton is LESS than (1+r) rent at Q(t).