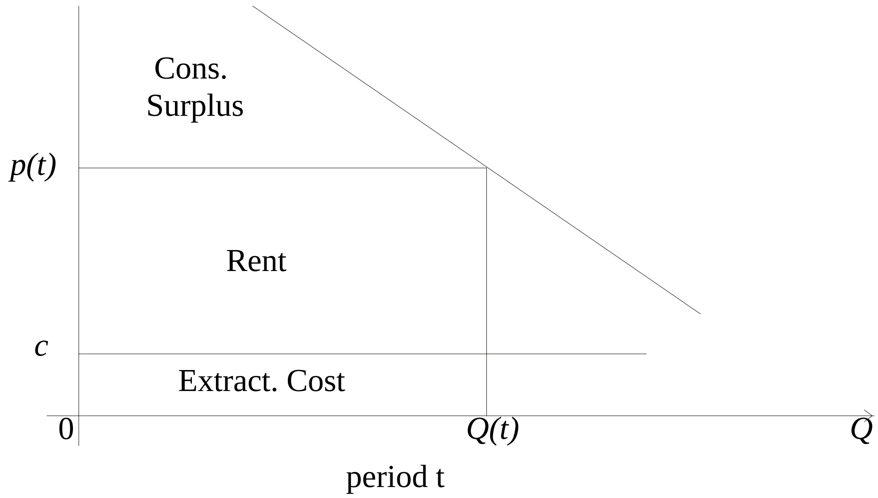
Hotelling [1931]: Industry Extraction Model

Competitive outcome follows from max *PV* of consumer and producer surpluses.

At date *t*, *Q(t)* is extracted from say *Q(t)* tiny firms each with ONE ton of oil.



- Each ton extracted costs $ *c* for extraction.
- *B(Q(t))*= cons. Surplus PLUS rent, at period *t*, PLUS extraction cost.
- Industry extraction path, $Q_0, Q_1, Q_2, ..., Q_T$ emerges from social welfare maximization.

$$W = [B(Q_0) - cQ_0] + \frac{1}{1+r}[B(Q_1) - cQ_1] + \left(\frac{1}{1+r}\right)^2 [B(Q_2) - cQ_2] + ... + \left(\frac{1}{1+r}\right)^T [B(Q_T) - cQ_T]$$

Subject to $Q_0 + Q_1 + Q_2 + ... + Q_T = S_0$

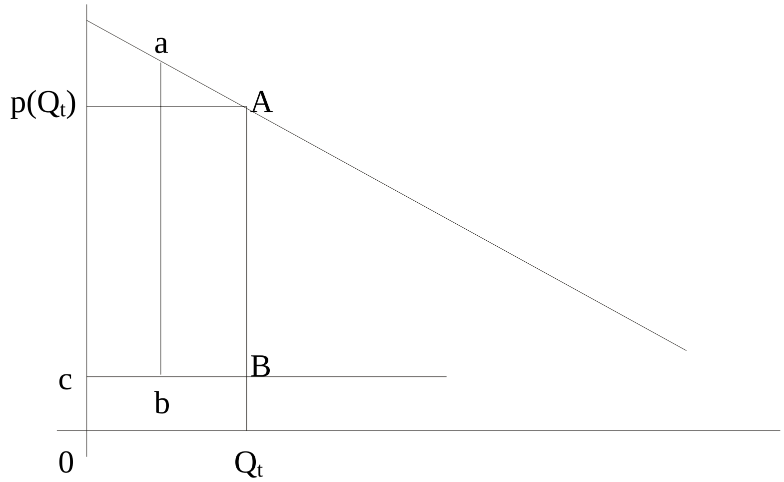
Solution given by,

(a) [marg. "profit", period *t*]= $\frac{1}{1+r}$ [marg. "profit", period *t* +1]

$$\frac{dB(Q_t)}{dQ_t} - c = \frac{1}{1+r}\left[\frac{dB(Q_{t+1})}{dQ_{t+1}} - c\right], \text{ for all } t.$$

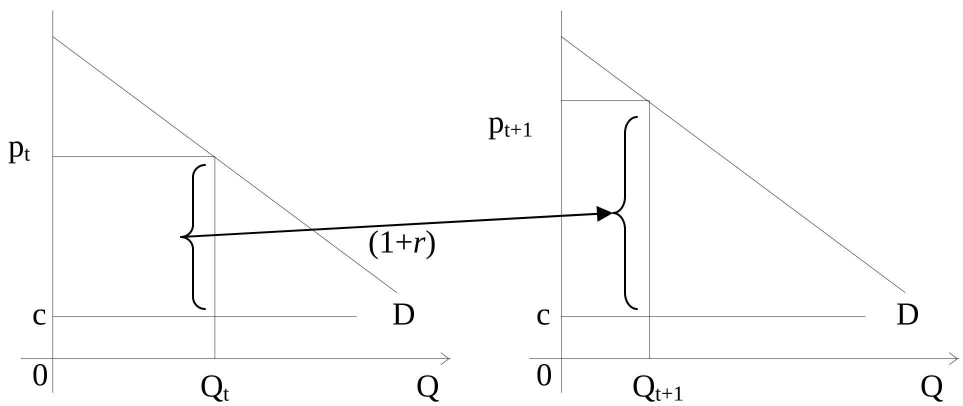
(b) END period condition: marg. "profit" from Q_T equals average "profit".

$$\frac{d[B(Q_T) - cQ_T]}{dQ_T} = \left[\frac{B(Q_T) - cQ_T}{Q_T}\right]$$



Marg. profit is vertical distance AB.
Average profit is vertical distance ab.
Hence, Q_T must be zero for AB=ab.

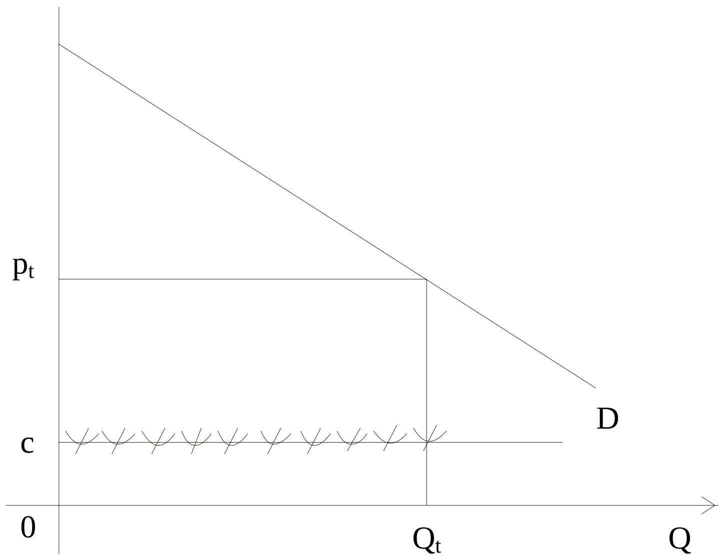
Condition (a) is Hotelling's *r*% Rule.



Period *t*

Period *t+1*

It is useful to keep in mind that this is a competitive outcome with many small price-taking extractors. We illustrate for period *t*.



Hence stock, S_t comprises S_t small, one-ton firms. In each period, Q_t firms "peel off" from S_t leaving $S_{t+1} = S_t - Q_t$. Each firm is indifferent in a sense of present value profit as to which period it extracts in.

Hotelling Monopoly Extraction

In place of max *PV* of social surplus, there is maximize *PV* of genuine profit per period.

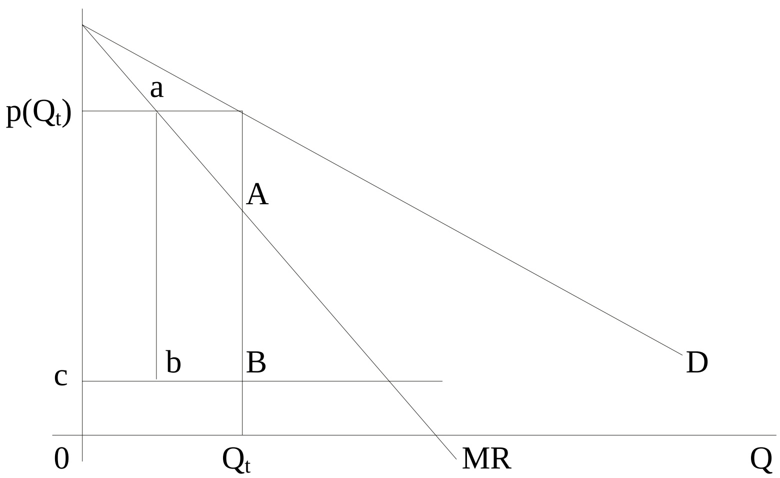
$$\pi(Q_t) = p(Q_t)Q_t - cQ_t$$

In this case, the PV of the profit stream is maximized when;

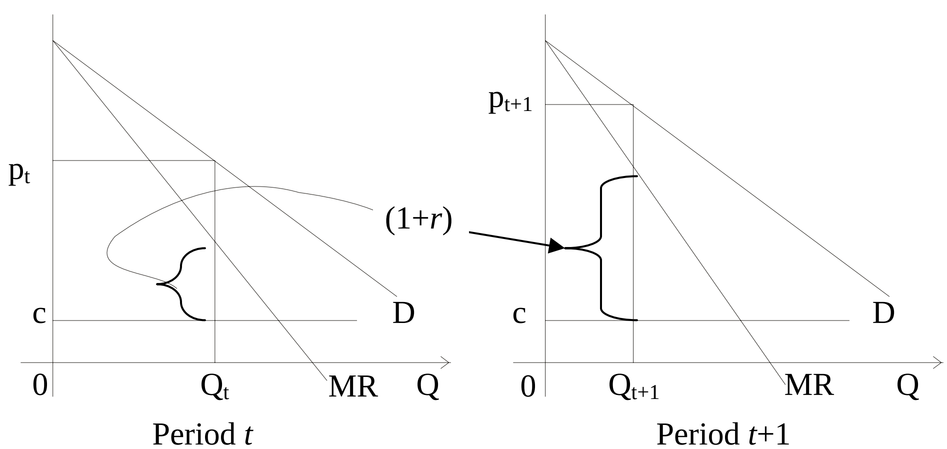
(a) [marginal profit from Q_t] $= \frac{1}{1+r}$[marg. profit from Q_{t+1}]; for all *t*.

(b) end point: marginal profit from Q_T equals average profit from Q_T.

$$\frac{d[p(Q_T)Q_T - cQ_T]}{dQ_T} = \frac{p(Q_T)Q_T - cQ_T}{Q_T}$$



Marginal profit from Q_T is AB and average profit Q_T must be zero for AB=ab. Condition (a) is an *r*% rule for marginal profit, $mr(Q_T) - c$.



$$Q_0 + Q_1 + Q_2 + ... + Q_T = S_0$$

Hotelling with Two Quality Stocks:

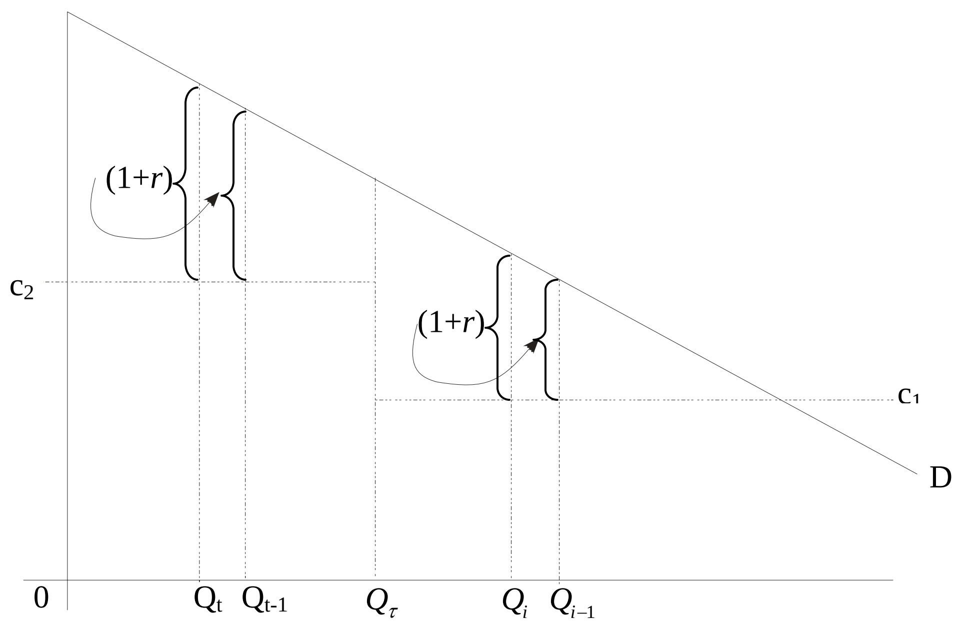
Distinguish quality by "extraction" cost $c_1 < c_2$.

S_1 tons available at extract cost c_1

S_2 tons available at extract cost c_2

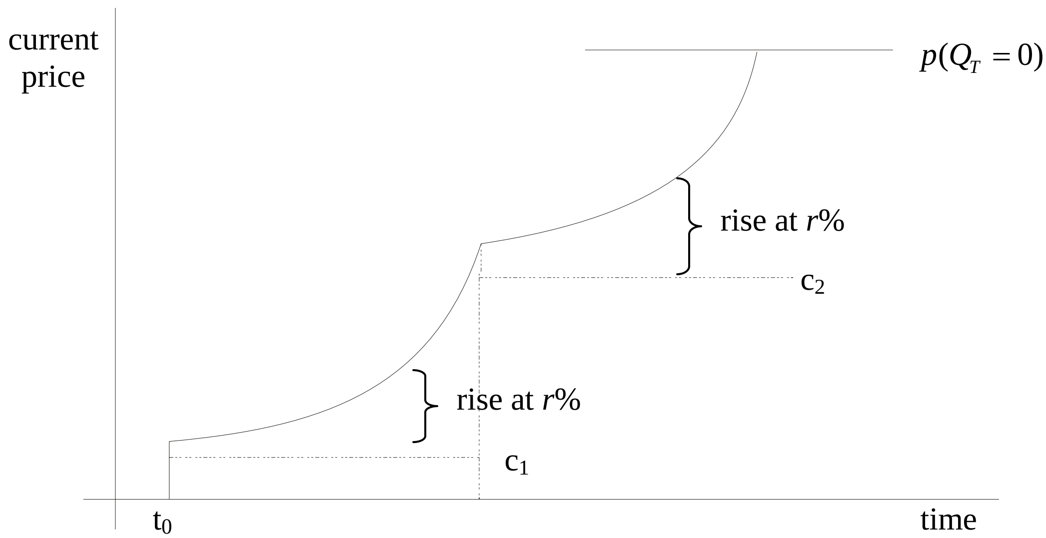
Start at end with $Q_T = 0$ from high cost deposit.

Select $Q_T, Q_{T-1}, Q_{T-2}, ...$ for r% rule backwards

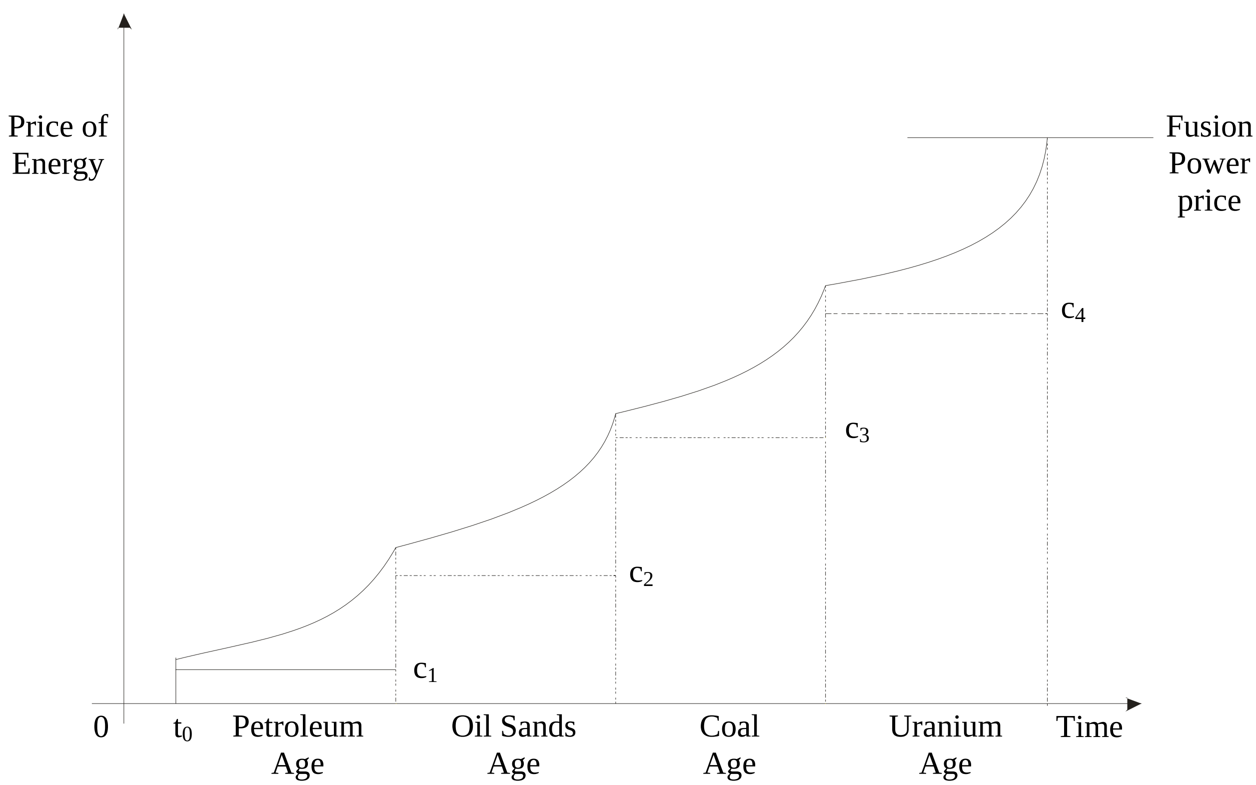


Working backwards until $Q_T, Q_{T-1}, Q_{T-2}, ..., Q_0 (= S_2)$, then a JUMP down in rent, and backwards in low cost deposit, $Q_\tau, Q_{\tau-1}, Q_{\tau-2}, ...Q_0 (= S_0)$. Q_0 defines initial quantity from low cost deposit, and initial price.

Herfindahl Diagram:



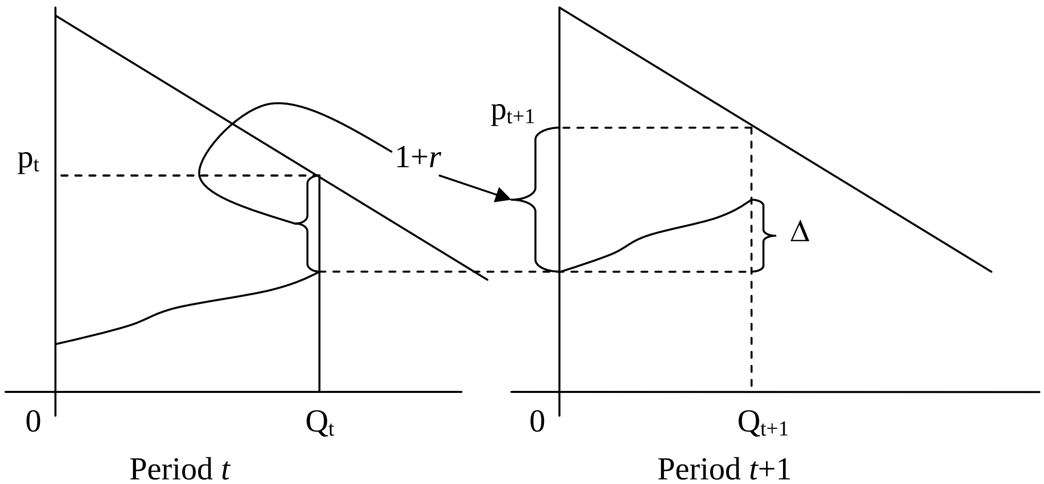
Nordhans [1974] employed multi-quality Hotelling Model to analyze a world energy future.



Smoothly Declining Quality

Lower quality takes the form of higher unit extract cost. We index each ton by its address S_t and quality $C(Q_t, S_t)$. It costs $ C to extract Q_t tons at "address" S_t in the deposit. S_t means S_t tons remaining.

Zero profit arbitrage on marginal ton extracted at period t,



Arbitrage Condition: $\left[p_t - C_Q(Q_t, S_t)\right] = \left(\frac{1}{1+r}\right)\left[p_t - C_Q(Q_{t+1}, S_{t+1}) - \Delta\right]$

Δ is the STOCK SIZE (stock address) EFFECT.

Δ is $-\frac{\partial C(Q_{t+1}, S_{t+1})}{\partial S_{t+1}}$.

$\frac{\partial C}{\partial S_{t+1}}$ is negative, since the less remaining in the ground, the higher cost it is to extract Q_{t+1} tons.

$Q_0 + Q_1 + Q_2 + \ldots + Q_T = S_0$