Statistical Appendix for "Canadian City Housing Prices and Urban Market Segmentation"*

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September 5, 2006

ABSTRACT

This statistical appendix complements our paper "Canadian Housing Prices and Urban Market Segmentation." This appendix collects all of our pre-testing results that, because they are now standard, do not warrant to be in the main body of the paper. Also included are robustness checks. Overall, the results in this appendix reflect and support the findings discussed in the related paper.

JEL: C22, C32, R2

Keywords: Housing prices, Cointegration.

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1. Variable Stationarity Tests

This section presents results for stationary tests. Tables 1, 2 and 3 consist of various Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) test statistics with their corresponding p-values.[1] Two versions of each test are performed, one that includes a constant, and the other that includes both constant and trend.

The results in Table 1 suggest that all of the housing price and mortgage rate time series are non-stationary. In the tables we report at a 1% level of significance for all unit root/cointegration tests. Gregory (1994) documents that the Johansen (1988) methodology, which we employ, tends to over-reject the null and hence predict large models.

Table 1
MLS Housing Prices and Mortgage Rates: Stationarity Tests

	Test with Constant		Test with Constant and Trend		Test with Constant		Test with Constant and Trend	
Variable	ADF	P-Value	ADF	P-Value	PP	P-Value	PP	P-Value
CAN	-0.847	0.8050	-1.985	0.6095	-0.433	-3.514	-1.354	0.8740
STJ	0.191	0.9717	-0.988	0.9457	-0.144	0.9449	-1.615	0.7866
HAL	0.215	0.9731	-1.070	0.9339	-0.651	0.8591	-2.789	0.2010
MON	0.544	0.8832	-1.675	0.7615	-0.133	0.9460	-1.110	0.9274
OTT	-1.428	0.5690	-2.657	0.2544	-1.853	0.3586	-2.240	0.4672
TOR	-1.161	0.6900	-1.485	0.8342	-1.667	0.4481	-1.658	0.7687
HAM	-1.840	0.3610	-2.358	0.4020	-1.598	0.4845	-1.425	0.8534
LON	-1.726	0.4180	-1.940	0.6337	-1.309	0.6249	-1.132	0.9236
WPG	0.261	0.9755	-0.795	0.9661	0.243	0.9746	-1.030	0.9399
REG	1.870	0.9985	-0.286	0.9898	0.891	0.9930	-3.096	0.1071
CAL	0.445	0.9831	-3.275	0.0706	1.073	0.9950	-1.968	0.6187
EDM	0.446	0.9831	-3.035	0.1225	1.843	0.9984	-1.251	0.8995
VAN	-1.029	0.7426	-1.976	0.6147	-0.075	0.9519	-2.233	0.4711
ONE	-0.847	0.8050	-1.985	0.6095	-0.433	-3.514	-1.354	0.8740
THREE	0.191	0.9717	-0.988	0.9457	-0.144	0.9449	-1.615	0.7866
FIVE	0.215	0.9731	-1.070	0.9339	-0.651	0.8591	-2.789	0.2010

Note: Augmented Dickey-Fuller and Phillips-Perron test-statistics and the p-values are provided. † is significant at the 1%. The Bayes' Information Criterion (BIC) is used to determine the optimal lag lengths that correspond to these statistics.

[1] Please refer refer to our main paper for variable definitions and descriptions.

Table 2
City-Specific Variables: Stationarity Tests

		Test with Constant and Trend		Test with Constant		Test with Constant and Trend		Test with Constant	
Var.	City/ Prov.	ADF	P-Value	ADF	P-Value	PP	P-Value	PP	P-Value
NPI	STJ	-0.334	0.9205	-1.638	0.7774	0.080	0.9647	-1.044	0.9379
	HAL	-0.112	0.9483	-1.840	0.6851	-1.802	0.3796	-2.976	0.1391
	MON	-0.908	0.7855	-1.877	0.6665	-2.004	0.2850	-1.926	0.6413
	OTT	-0.534	0.8852	-1.597	0.7936	-1.117	0.7083	-1.679	0.7600
	TOR	-1.490	0.5387	-2.282	0.4439	-1.579	0.4941	-1.492	0.8320
	CAL	0.244	0.9746	-4.086†	0.0066	0.781	0.9913	-2.169	0.5072
	EDM	0.145	0.9690	-3.331	0.0613	0.740	0.9906	-1.933	0.6376
	VAN	-1.982	0.2946	-2.435	0.3610	-1.749	0.4062	-2.270	0.4505
UWI	STJ	-2.833	0.0537	-3.055	0.1173	-3.230	0.0183	-3.130	0.0994
	HAL	-4.184†	0.0007	-3.685	0.0234	-7.300†	0.0000	-4.691†	0.0007
	MON	-2.444	0.1296	-3.284	0.0689	-3.981†	0.0015	-3.612	0.0289
	OTT	-3.459†	0.0091	-3.349	0.0586	-4.944†	0.0000	-2.941	0.1492
	TOR	-2.787	0.0601	-2.623	0.2696	-4.066†	0.0011	-2.350	0.4067
	CAL	-1.270	0.6429	-2.860	0.1759	-2.467	0.1237	-3.806	0.0162
	EDM	-1.243	0.6547	-2.657	0.2545	-2.386	0.1457	-3.697	0.0225
	VAN	-1.915	0.3248	-2.466	0.3452	-5.040†	0.0000	-3.147	0.0956
RA	STJ	-4.453†	0.0002	-4.390	0.0023	-7.740†	0.0000	-5.263†	-4.047
	HAL	-2.520	0.1107	-3.254	0.0742	-12.159†	0.0000	-6.450†	0.0000
	MON	-2.703	0.0735	-2.238	0.4687	-10.914†	0.0000	-5.102†	0.0001
	OTT	-3.064	0.0293	-1.586	0.7980	-1.834†	0.0000	-1.938	0.6348
	TOR	-2.976	0.0372	-2.183	0.4992	-10.115†	0.0000	-3.389	0.0529
	CAL	0.640	0.9886	-4.220†	0.0042	-1.493	0.5371	-2.923	0.1548
	EDM	0.176	0.9709	-2.097	0.5478	-1.657	0.4533	-3.184	0.0876
	VAN	-2.105	0.2424	-1.459	0.8429	-7.795†	0.0000	-3.395	0.0520
OA	STJ	-1.630	0.4676	-2.157	0.5143	-3.311†	0.0005	-4.061†	0.0072
	HAL	-1.003	0.7520	-2.019	0.5912	-3.587†	0.0060	-4.714†	0.0007
	MON	-1.550	0.5084	-1.697	0.7521	-4.713†	0.0001	-3.775	0.0179
	OTT	-0.898	0.7886	-1.987	0.6087	-4.315†	0.0004	-4.052†	0.0074
	TOR	-1.450	0.5582	-1.666	0.7656	-3.772†	0.0032	-2.813	0.1921
	CAL	0.420	0.9822	-2.595	0.2823	-1.792	0.3845	-3.828	0.0152
	EDM	-0.760	0.8305	-2.651	0.2570	-2.415	0.1375	-3.464	0.0434
	VAN	-1.707	0.4275	-1.689	0.7558	-3.603†	0.0057	-2.697†	0.2373

Note: Augmented Dickey-Fuller and Phillips-Perron test-statistics and the p-values are provided. † is significant at the 1%. The BIC is used to determine the optimal lag lengths that correspond to these statistics.

Table 3
City-Specific Variables: Stationarity Tests

		Test with Constant and Trend		Test with Constant		Test with Constant and Trend		Test with Constant	
Var.	City/ Prov.	ADF	P-Value	ADF	P-Value	PP	P-Value	PP	P-Value
BP	STJ	-2.771	0.0626	-2.950	0.1465	-6.654†	0.0000	-6.775†	0.0000
	HAL	-2.602	0.0926	-2.604	0.2781	-6.581	0.0000	-6.571	0.0000
	MON	-1.967	0.3014	-2.054	0.5719	-2.534	0.1075	-2.496	0.3298
	OTT	-2.165	0.2190	-2.235	0.4700	-4.156†	0.0008	-4.134†	0.0056
	TOR	-2.309	0.1692	-2.696	0.2376	-2.722	0.0703	-2.985	0.1363
	CAL	-1.689	0.4368	-5.348†	0.0000	-3.027	0.0325	-4.694†	0.0007
	EDM	-1.712	0.4249	-2.816	0.1912	-1.854	0.3542	-2.762	0.2111
	VAN	-3.469†	0.0088	-3.457	0.0442	-4.233†	0.0006	-4.203†	0.0044
LF	STJ	-1.289	0.6343	-2.538	0.3092	-1.304	0.6274	-3.136	0.0978
	HAL	-2.609	0.0912	-2.834	0.1849	-1.237	0.6576	-1.955	0.6255
	MON	0.375	0.9805	-1.745	0.7308	-0.003	0.9582	-1.988	0.6080
	OTT	-0.541	0.8837	-1.587	0.7974	-1.150	0.6947	-2.398	0.3807
	TOR	1.216	0.9961	-0.639	0.9769	1.188	0.9959	-0.836	0.9625
	CAL	0.631	0.9883	-1.549	0.8119	0.666	0.9891	-2.000	0.6013
	EDM	0.503	0.9850	-2.250	0.4617	0.152	0.9695	-3.127	0.0999
	VAN	-0.439	0.9034	-2.266	0.4531	-0.262	0.9307	-2.984	0.1365
GDP	NF	0.877	0.9928	-1.175	0.9155	0.984	0.9941	-1.130	0.9239
	NS	0.563	0.9867	-0.989	0.9456	0.786	0.9914	-1.162	0.9180
	QC	0.111	0.9668	-2.312	0.4272	0.537	0.9859	-1.684	0.7577
	ONT	-0.769	0.8280	-2.107	0.5420	-0.282	0.9280	-1.466	0.8404
	AL	0.384	0.9809	-3.417	0.0492	0.905	0.9932	-3.162	0.0923
	BC	0.325	0.9784	-1.947	0.6297	0.455	0.9834	-4.007	0.0068

Note: Augmented Dickey-Fuller and Phillips-Perron test-statistics and the p-values are provided. † is significant at the 1%. The BIC is used to determine the optimal lag lengths that correspond to these statistics.

2. Systems Cointegration Tests

2.1 Core Cities

Given the results in Section 1 we turn our analysis to various systems of equations consisting of non-stationary variables. Following the approach in the paper, we use the Maximum-Likelihood Trace and Maximum Eigenvalue (λ-max) statistics to determine the cointegrating rank of these systems. We first test the robustness of our results by considering various ways to treat deterministic terms in cointegration relations defined by the system consisting of the existing housing prices for 8 Canadian cities. In particular, we consider three specifications (see Johansen (1994)):

1. Unrestricted Constant: The constant is not restricted to lie in the cointegration space with no deterministic trends in the model.
2. Unrestricted Trend: Both the constant and the trend are not restricted to lie in the cointegration space. This specification allows for quadratic trends in the variables.
3. Restricted Trend: The constant remains unrestricted while the deterministic trend term is restricted to lie in the cointegration space. This framework allows for linear trends in the data, while quadratic trends are ruled out.

It should be noted that all test statistics in this and subsequent sections are corrected for finite sample bias according to Cheung and Lai (1993).

The results for these specifications are listed in Tables 4. We find one significant cointegrating vector at the 1 % level in four of six tests. The order of integration depends somewhat on the specification of the deterministic variables as the model with an unrestricted trend predicts two significant cointegrating relations. However the housing prices do not display a quadratic trend suggesting this is not necessarily the appropriate specification for inference in our paper. In Table 5 statistics related to the normality of the residuals from VAR underlying the system of equations are provided. Since the Trace and λ-max statistics rely heavily on the normality of residuals, it is important to consider them in light of the residuals' distribution relative to the normal distribution. Our results suggest that in some instances the residuals' distribution is significantly different from normality, as a result of relatively high kurtosis. Simulation results by Gonzalo (1994) however show that this is not a critical issue for inference with large samples.

Table 4
Cointegrating Rank Determination for MLS Existing House Prices, Core Cities

Specification	H_0	H_1	Trace Value	Critical	$\lambda_{\max}$ Value	Critical	λ
Unrestricted	$r=0$	$r\geq 1$	224.25†	204.95	60.76	62.80	-
Constant	$r\leq 1$	$r\geq 2$	163.49	168.36	48.27	57.69	0.672
	$r\leq 2$	$r\geq 3$	115.21	133.57	37.28	51.57	0.587
	$r\leq 3$	$r\geq 4$	77.94	103.18	27.03	45.10	0.495
	$r\leq 4$	$r\geq 5$	50.91	76.07	21.14	38.77	0.391
	$r\leq 5$	$r\geq 6$	29.77	54.46	12.70	32.24	0.321
	$r\leq 6$	$r\geq 7$	17.07	35.65	8.78	25.52	0.208
	$r\leq 7$	$r\geq 8$	8.29	20.04	7.75	18.53	0.149
	$r\leq 8$	$r\geq 9$	0.543	6.65	0.543	6.65	0.132
	$r\leq 9$	$r\geq 10$	-	-	-	-	0.010
Unrestricted	$r=0$	$r\geq 1$	262.37†	222.46	77.120†	66.91	-
Trend	$r\leq 1$	$r\geq 2$	185.25†	182.51	50.484	60.81	0.735
	$r\leq 2$	$r\geq 3$	134.77	146.99	38.52	54.48	0.581
	$r\leq 3$	$r\geq 4$	96.25	114.36	30.97	48.17	0.485
	$r\leq 4$	$r\geq 5$	65.28	85.78	23.17	41.58	0.414
	$r\leq 5$	$r\geq 6$	42.11	61.21	21.14	35.68	0.329
	$r\leq 6$	$r\geq 7$	20.97	40.49	10.41	28.83	0.305
	$r\leq 7$	$r\geq 8$	10.56	23.46	8.99	21.47	0.164
	$r\leq 8$	$r\geq 9$	1.57	6.40	1.57	6.40	0.144
	$r\leq 9$	$r\geq 10$		-		-	0.027
Restricted	$r=0$	$r\geq 1$	270.24†	234.41	77.17†	67.88	-
Trend	$r\leq 1$	$r\geq 2$	193.07	196.08	50.86	62.46	0.736
	$r\leq 2$	$r\geq 3$	142.21	158.49	38.55	54.71	0.590
	$r\leq 3$	$r\geq 4$	103.66	124.75	30.99	49.51	0.486
	$r\leq 4$	$r\geq 5$	72.67	96.58	23.64	42.36	0.414
	$r\leq 5$	$r\geq 6$	49.04	70.05	22.98	36.65	0.335
	$r\leq 6$	$r\geq 7$	26.06	48.45	10.43	30.34	0.327
	$r\leq 7$	$r\geq 8$	15.63	30.45	9.10	23.65	0.165
	$r\leq 8$	$r\geq 9$	6.53	16.26	6.53	16.26	0.145
	$r\leq 9$	$r\geq 10$	-	-	-	-	0.106

Note: † Significant at the 1% level.

Table 5
Cointegrating Rank Determination, Residuals Diagnostic Tests, Core Cities

Equation	Jarque-Bera $\chi^2(2)$	P-Value	Skewness $\chi^2(1)$	P-Value	Kurtosis $\chi^2(1)$	P-Value
CAN	$16.856^{\dagger}$	< 0.001	1.827	0.176	$15.030^{\dagger}$	< 0.001
STJ	0.337	0.845	0.011	0.916	0.326	0.568
HAL	0.201	0.904	0.039	0.844	0.163	0.687
MON	$15.147^{\dagger}$	< 0.001	0.419	0.518	$14.728^{\dagger}$	< 0.001
OTT	2.742	0.254	0.265	0.606	2.477	0.116
TOR	1.737	0.420	1.685	0.194	0.053	0.819
CAL	2.054	0.358	0.213	0.644	1.841	0.175
EDM	0.224	0.894	0.111	0.739	0.113	0.737
VAN	$12.984^{\dagger}$	0.001	2.887	0.089	$10.097^{\dagger}$	0.001
ALL	$52.283^{\dagger}$	< 0.001	7.456	0.590	$44.826^{\dagger}$	< 0.001
$LM(1)$	97.816	0.0984				
$LM(4)$	91.158	0.2064				

Note: † Significant at the 1% level. The statistics testing the normality, skewness and Kurtosis of 'ALL' are are distributed $\chi^2(18)$, $\chi^2(9)$, $\chi^2(9)$ respectively. LM statistics and their corresponding p-values testing for the significance of the first and fourth autocorrelation in the residuals of the VAR are also provided.

2.2 Geographic Segments

The next set of tests is concerned with how well defined regional housing markets are in Canada. An important point of note for this section is that if cointegration is found across cities' housing prices *within* a given geographic segment, then we only need to include one of the cities in models that focus on relationships among existing housing prices *across* cities in different regions of Canada. We present results for three parts of Canada: Southern Ontario (Toronto, Hamilton, London), the Prairies (Regina, Calgary, Edmonton) and Western Canada (Calgary, Edmonton, Vancouver).

The results in Tables 6, 7 and 8 show that only Southern Ontario has evidence of a cointegrated region. However, the results for Southern Ontario must be considered with caution, as the residuals for the Toronto equation are highly skewed (see Table 9). Once again kurtosis is significantly high in Southern Ontario and the Prairies, but the residuals for each region are well-behaved otherwise. Finally, in Table 10 we assess cointegration in each region using the Engle-Granger (1987) two-step approach. These results conform with our Maximum Likelihood findings as Southern Ontario yields I(0) residuals, implying cointegration across the housing prices of the cities in this region. The residuals for the Prairies and Western Canada models on the other hand are I(1) implying no cointegration among the cities' housing prices in these regions.

Table 6
Cointegrating Rank Determination for MLS Existing House Prices, Geographic Segments: Southern Ontario

Specification	H_0	H_1	Trace Value	Critical	$\lambda_{\max}$ Value	Critical	λ
Unrestricted Constant	$r=0$	$r\geq 1$	34.582	35.65	22.763	25.52	-
	$r\leq 1$	$r\geq 2$	11.819	20.04	8.284	18.63	0.253
	$r\leq 2$	$r\geq 3$	3.535	6.65	3.535	6.65	0.101
	$r\leq 3$	$r\geq 4$	-	-	-	-	0.044
Unrestricted Trend	$r=0$	$r\geq 1$	50.463†	40.49	25.190	28.83	-
	$r\leq 1$	$r\geq 2$	25.273†	23.46	19.554	21.47	0.276
	$r\leq 2$	$r\geq 3$	5.719	6.40	5.719	6.40	0.222
	$r\leq 3$	$r\geq 4$	-	-	-	-	0.071
Restricted Trend	$r=0$	$r\geq 1$	52.729†	48.45	25.329	30.34	-
	$r\leq 1$	$r\geq 2$	27.400	30.45	20.534	23.65	0.277
	$r\leq 2$	$r\geq 3$	6.866	16.26	6.866	16.26	0.231
	$r\leq 3$	$r\geq 4$	-	-	-	-	0.084

Note: † Significant at the 1% level.

Table 7
Cointegrating Rank Determination for MLS Existing House Prices, Geographic Segments: Praries

Specification	H_0	H_1	Trace Value	Critical	$\lambda_{\max}$ Value	Critical	λ
Unrestricted Constant	$r = 0$	$r \geq 1$	30.303	35.65	18.979	25.52	-
	$r \leq 1$	$r \geq 2$	11.323	20.04	8.462	18.63	0.216
	$r \leq 2$	$r \geq 3$	2.861	6.65	2.861	6.65	0.103
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.036
Unrestricted Trend	$r = 0$	$r \geq 1$	27.519	40.49	17.997	28.83	-
	$r \leq 1$	$r \geq 2$	9.521	23.46	7.645	21.47	0.206
	$r \leq 2$	$r \geq 3$	1.877	6.40	1.877	6.40	0.093
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.024
Restricted Trend	$r = 0$	$r \geq 1$	35.675	48.45	19.177	30.34	-
	$r \leq 1$	$r \geq 2$	16.498	30.45	11.841	23.65	0.218
	$r \leq 2$	$r \geq 3$	4.656	16.26	4.656	16.26	0.141
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.058

Note: † Significant at the 1% level.

Table 8
Cointegrating Rank Determination for MLS Existing House Prices, Geographic Segments: Western Canada

Specification	H_0	H_1	Trace Value	Critical	$\lambda_{\max}$ Value	Critical	λ
Unrestricted Constant	$r = 0$	$r \geq 1$	17.021	35.65	9.496	25.52	-
	$r \leq 1$	$r \geq 2$	7.525	20.04	6.436	18.63	0.115
	$r \leq 2$	$r \geq 3$	1.089	6.65	1.089	6.65	0.079
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.014
Unrestricted Trend	$r = 0$	$r \geq 1$	24.491	40.49	14.164	28.83	-
	$r \leq 1$	$r \geq 2$	10.327	23.46	8.375	21.47	0.166
	$r \leq 2$	$r \geq 3$	1.952	6.40	1.952	6.40	0.102
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.025
Restricted Trend	$r = 0$	$r \geq 1$	30.471	48.45	15.281	30.34	-
	$r \leq 1$	$r \geq 2$	15.191	30.45	9.483	23.65	0.178
	$r \leq 2$	$r \geq 3$	5.708	16.26	5.708	16.26	0.114
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.071

Note: † Significant at the 1% level.

Table 9
Cointegrating Rank Determination, Residuals Diagnostic Tests, Geographic Segments

Region	Equation	Jarque-Bera $\chi^2(2)$	P-Value	Skewness $\chi^2(1)$	P-Value	Kurtosis $\chi^2(1)$	P-Value
S. Ontario	TOR	$39.842^{\dagger}$	< 0.001	$12.553^{\dagger}$	< 0.001	$27.289^{\dagger}$	< 0.001
	HAM	1.462	0.481	1.343	0.247	0.119	0.72979
	LON	3.144	0.208	1.951	0.162	1.193	0.27479
	ALL	$44.448^{\dagger}$	< 0.001	$15.847^{\dagger}$	0.001	$28.601^{\dagger}$	0.00000
	$LM(1)$	12.6476	0.17921				
	$LM(4)$	7.7118	0.56344				
Prairies	REG	1.352	0.50853	1.011	0.31457	0.341	0.55920
	CAL	$34.013^{\dagger}$	< 0.001	1.597	0.20631	$32.416^{\dagger}$	< 0.001
	EDM	3.817	0.14832	0.767	0.38101	3.049	0.08077
	ALL	$39.182^{\dagger}$	< 0.001	3.376	0.33721	$35.806^{\dagger}$	< 0.001
	$LM(1)$	4.6832	0.861				
	$LM(4)$	13.119	0.157				
West	CAL	7.960	0.01869	4.914	0.02664	3.046	0.08095
	EDM	0.000	0.99997	0.000	0.99423	0.000	0.99770
	VAN	0.169	0.91902	0.000	0.99678	0.169	0.68112
	ALL	8.129	0.22882	4.914	0.17819	3.215	0.35971
	$LM(1)$	14.0356	0.12106				
	$LM(4)$	$23.6579^{\dagger}$	0.00488				

Note: † Significant at the 1% level. The statistics testing the normality, skewness and Kurtosis of 'ALL' are are distributed $\chi^2(6)$, $\chi^2(3)$, $\chi^2(3)$ respectively. LM statistics and their corresponding p-values testing for the significance of the first and fourth autocorrelation in the residuals of the VAR are also provided.

Table 10
Cointegration Test (by OLS) for MLS Existing House Prices, Geographic Segments

	Test with Constant		Test with Constant and Trend		Test with Constant		Test with Constant and Trend	
Variable	ADF	P-Value	ADF	P-Value	PP	P-Value	PP	P-Value
S.Ontario	$-3.802^{\dagger}$	0.003	-3.785	0.017	$-4.819^{\dagger}$	0.000	$-4.814^{\dagger}$	0.000
Prairies	-3.327	0.014	-3.241	0.0765	-3.066	0.0292	-3.087	0.1093
West	-2.069	0.2572	-2.021	0.5898	-2.281	0.1783	-2.193	0.4940

Note: Augmented Dickey-Fuller and Phillips-Perron test-statistics and the p-values are provided. † is significant at the 1%.

2.3 All Cities and Mortgage Rates

To conclude our multivariate analysis of Canadian housing prices we present three additional models. These include (1) all Canadian cities with the Canadian aggregate index, (2) the one, three and five year mortgage rate and (3) all Canadian cities with the five year mortgage rate. It should be noted that for Southern Ontario, only Toronto is included since the housing prices in this region are cointegrated. We provide 3 sets of results for each model based on specifications mentioned above: unrestricted constant, unrestricted trend and restricted trend.

Including the additional cities, Winnipeg and Regina, with the 'core cities' from above, makes little difference in our results as illustrated in Table 11. There are no significant cointegrating vectors according to the Trace and λ-max statistics at the 1% level amongst all the average existing housing prices of the Canadian cities and the aggregate housing price index. Further, we find one cointegrating relation between the one, three and five year mortgage rate in Table 13. Table 15 shows that the introduction of the five year mortgage rate into the system with the housing prices yields one additional cointegrating vector.[2] This is to be expected if there is a stable, long run relationship relating housing prices and mortgage rates. The following section looks at this issue more closely with both ML and OLS-based methods for cointegration. Residual normality statistics can be found in Tables 12, 16 and 16. These conform with normal distribution on the whole, with the exception of high kurtosis in the Canada equation in the "all cities" model and with the residuals from the mortgage rates model.

[2]Note for this model we replace the Canadian aggregate index with the five year mortgage rate.

Table 11
Cointegrating Rank Determination for MLS Existing House Prices, All Cities

Specification	H_0	H_1	Trace Value	Critical	$\lambda_{\max}$ Value	Critical	λ
Unrestricted Constant	$r = 0$	$r \geq 1$	273.15	293.44	63.64	75.95	-
	$r \leq 1$	$r \geq 2$	209.51	247.18	42.14	69.09	0.720
	$r \leq 2$	$r \geq 3$	167.37	204.95	37.60	62.80	0.569
	$r \leq 3$	$r \geq 4$	129.78	168.36	31.88	57.69	0.529
	$r \leq 4$	$r \geq 5$	97.90	133.57	26.19	51.57	0.471
	$r \leq 5$	$r \geq 6$	71.71	103.18	19.41	45.10	0.408
	$r \leq 6$	$r \geq 7$	52.30	76.07	19.06	38.77	0.322
	$r \leq 7$	$r \geq 8$	33.26	54.46	15.51	32.24	0.317
	$r \leq 8$	$r \geq 9$	17.75	35.65	8.69	25.52	0.267
	$r \leq 9$	$r \geq 10$	9.06	20.04	7.34	18.63	0.160
	$r \leq 10$	$r \geq 11$	1.72	6.65	1.72	6.65	0.137
	$r \leq 11$	$r \geq 12$	-	-	-	-	0.034
Unrestricted Trend	$r = 0$	$r \geq 1$	295.53	312.58	70.00	78.51	-
	$r \leq 1$	$r \geq 2$	225.52	263.94	48.73	72.96	0.753
	$r \leq 2$	$r \geq 3$	176.79	222.46	38.389	66.91	0.623
	$r \leq 3$	$r \geq 4$	138.40	182.51	30.83	60.81	0.536
	$r \leq 4$	$r \geq 5$	107.58	146.99	25.09	54.48	0.460
	$r \leq 5$	$r \geq 6$	82.50	114.36	24.13	48.17	0.395
	$r \leq 6$	$r \geq 7$	58.36	85.78	18.60	41.58	0.383
	$r \leq 7$	$r \geq 8$	39.76	61.21	17.56	35.68	0.311
	$r \leq 8$	$r \geq 9$	22.20	40.49	12.13	28.83	0.296
	$r \leq 9$	$r \geq 10$	10.09	23.46	7.66	21.47	0.215
	$r \leq 10$	$r \geq 11$	2.43	6.40	2.43	6.40	0.142
	$r \leq 11$	$r \geq 12$	-	-	-	-	0.047
Restricted Trend	$r = 0$	$r \geq 1$	314.08	327.45	71.16	79.23	-
	$r \leq 1$	$r \geq 2$	242.92	279.07	48.82	73.73	0.759
	$r \leq 2$	$r \geq 3$	194.10	234.41	39.40	67.88	0.623
	$r \leq 3$	$r \geq 4$	154.70	196.08	32.09	62.46	0.545
	$r \leq 4$	$r \geq 5$	122.61	158.49	30.82	54.71	0.474
	$r \leq 5$	$r \geq 6$	91.79	124.75	24.69	49.51	0.460
	$r \leq 6$	$r \geq 7$	67.10	96.58	19.24	42.36	0.390
	$r \leq 7$	$r \geq 8$	47.86	70.05	17.57	36.65	0.319
	$r \leq 8$	$r \geq 9$	30.29	48.45	14.51	30.34	0.296
	$r \leq 9$	$r \geq 10$	15.78	30.45	8.49	23.65	0.252
	$r \leq 10$	$r \geq 11$	7.29	16.26	7.29	16.26	0.156
	$r \leq 11$	$r \geq 12$	-	-	-	-	0.136

Note: † Significant at the 1% level.

Table 12
Cointegrating Rank Determination, Residuals Diagnostic Tests, All Cities

Equation	Jarque-Bera $\chi^2(2)$	P-Value	Skewness $\chi^2(1)$	P-Value	Kurtosis $\chi^2(1)$	P-Value
CAN	18.779†	< 0.001	1.174	0.279	17.605†	< 0.001
STJ	1.250	0.535	1.015	0.314	0.235	0.628
HAL	1.872	0.392	0.407	0.523	1.464	0.226
MON	8.260	0.016	0.202	0.653	8.058	0.004
OTT	1.956	0.376	0.000	0.988	1.955	0.162
TOR	9.519†	0.008	9.436†	0.002	0.083	0.772
WPG	1.290	0.524	0.058	0.810	1.232	0.267
REG	0.052	0.974	0.032	0.859	0.020	0.888
CAL	0.960	0.619	0.116	0.734	0.844	0.358
EDM	0.588	0.745	0.291	0.590	0.298	0.585
VAN	5.220	0.076	0.090	0.764	5.131	0.024
ALL	49.745†	< 0.001	12.819	0.305	36.925†	< 0.001
$LM(1)$	161.770	0.001				
$LM(4)$	133.991	0.198				

Note: † Significant at the 1% level. The statistics testing the normality, skewness and Kurtosis of 'ALL' are are distributed $\chi^2(22)$, $\chi^2(11)$, $\chi^2(11)$ respectively. LM statistics and their corresponding p-values testing for the significance of the first and fourth autocorrelation in the residuals of the VAR are also provided.

Table 13
Cointegrating Rank Determination for the 1, 3 and 5 Year Mortgage Rate

Specification	H_0	H_1	Trace Value	Critical	$\lambda_{\max}$ Value	Critical	λ
Unrestricted Constant	$r = 0$	$r \geq 1$	65.20†	35.65	46.40†	25.52	-
	$r \leq 1$	$r \geq 2$	18.81	20.04	13.62	18.63	0.403
	$r \leq 2$	$r \geq 3$	5.19	6.65	5.19	6.65	0.141
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.056
Unrestricted Trend	$r = 0$	$r \geq 1$	74.74†	40.49	47.19†	28.83	-
	$r \leq 1$	$r \geq 2$	27.56	23.46	14.02	21.47	0.403
	$r \leq 2$	$r \geq 3$	13.54	6.40	13.54	6.40	0.141
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.056
Restricted Trend	$r = 0$	$r \geq 1$	75.50†	48.45	47.25†	30.34	-
	$r \leq 1$	$r \geq 2$	28.25	30.45	14.68	23.65	0.408
	$r \leq 2$	$r \geq 3$	13.57	16.26	13.57	16.26	0.144
	$r \leq 3$	$r \geq 4$	-	-	-	-	0.140

Note: † Significant at the 1% level.

Table 14
Cointegrating Rank Determination, Residuals Diagnostic Tests,
1, 3 and 5 Year Mortgage Rate

Equation	Jarque-Bera $\chi^2(2)$	P-Value	Skewness $\chi^2(1)$	P-Value	Kurtosis $\chi^2(1)$	P-Value
ONE	20.668†	< 0.001	0.238	0.625	20.429†	< 0.001
THREE	0.530	0.767	0.002	0.966	0.528	0.467
FIVE	4.789	0.091	2.608	0.106	2.182	0.140
ALL	25.987†	< 0.001	2.848	0.416	23.139†	< 0.001
$LM(1)$	17.899	0.0364				
$LM(4)$	15.462	0.0790				

Note: † Significant at the 1% level. The statistics testing the normality, skewness and Kurtosis of 'ALL' are are distributed $\chi^2(6)$, $\chi^2(3)$, $\chi^2(3)$ respectively. LM statistics and their corresponding p-values testing for the significance of the first and fourth autocorrelation in the residuals of the VAR are also provided.

Table 15
Cointegrating Rank Determination for MLS Existing House Prices, All Cities, w/ 5 Year Mortgage Rate

Specification	H_0	H_1	Trace Value	Critical	$\lambda_{\max}$ Value	Critical	λ
Unrestricted	$r = 0$	$r \geq 1$	313.11†	293.44	88.20†	75.95	-
Constant	$r \leq 1$	$r \geq 2$	224.91	247.18	50.58	69.09	0.829
	$r \leq 2$	$r \geq 3$	174.33	204.95	41.21	62.80	0.636
	$r \leq 3$	$r \geq 4$	133.12	168.36	26.89	57.69	0.561
	$r \leq 4$	$r \geq 5$	106.22	133.57	26.42	51.57	0.416
	$r \leq 5$	$r \geq 6$	79.99	103.18	22.81	45.10	0.408
	$r \leq 6$	$r \geq 7$	57.17	76.07	17.76	38.77	0.366
	$r \leq 7$	$r \geq 8$	39.42	54.46	16.16	32.24	0.299
	$r \leq 8$	$r \geq 9$	23.26	35.65	12.23	25.52	0.276
	$r \leq 9$	$r \geq 10$	11.04	20.04	8.66	18.63	0.217
	$r \leq 10$	$r \geq 11$	2.37	6.65	2.37	6.65	0.159
	$r \leq 11$	$r \geq 12$	-	-		-	0.046
Unrestricted	$r = 0$	$r \geq 1$	328.81†	312.58	87.41†	78.51	-
Trend	$r \leq 1$	$r \geq 2$	241.40	263.94	49.91	72.96	0.826
	$r \leq 2$	$r \geq 3$	191.49	222.46	43.21	66.91	0.631
	$r \leq 3$	$r \geq 4$	148.28	182.51	37.04	60.81	0.579
	$r \leq 4$	$r \geq 5$	111.24	146.99	26.27	54.48	0.523
	$r \leq 5$	$r \geq 6$	84.97	114.36	22.58	48.17	0.409
	$r \leq 6$	$r \geq 7$	62.38	85.78	19.09	41.58	0.363
	$r \leq 7$	$r \geq 8$	43.29	61.21	17.68	35.68	0.317
	$r \leq 8$	$r \geq 9$	25.61	40.49	12.78	28.83	0.298
	$r \leq 9$	$r \geq 10$	12.84	23.46	10.38	21.47	0.225
	$r \leq 10$	$r \geq 11$	2.46	6.40	2.46	6.40	0.187
	$r \leq 11$	$r \geq 12$	-	-	-	-	0.048
Restricted	$r = 0$	$r \geq 1$	348.08†	327.45	89.00†	79.23	-
Trend	$r \leq 1$	$r \geq 2$	259.08	279.07	52.04	73.73	0.831
	$r \leq 2$	$r \geq 3$	207.04	234.41	43.28	67.88	0.647
	$r \leq 3$	$r \geq 4$	163.76	196.08	38.74	62.46	0.579
	$r \leq 4$	$r \geq 5$	125.02	158.49	26.37	54.71	0.539
	$r \leq 5$	$r \geq 6$	98.66	124.75	23.99	49.51	0.410
	$r \leq 6$	$r \geq 7$	74.67	96.58	21.73	42.36	0.381
	$r \leq 7$	$r \geq 8$	52.94	70.05	17.70	36.65	0.352
	$r \leq 8$	$r \geq 9$	35.24	48.45	15.77	30.34	0.298
	$r \leq 9$	$r \geq 10$	19.47	30.45	11.20	23.65	0.271
	$r \leq 10$	$r \geq 11$	8.26	16.26	8.26	16.26	0.201
	$r \leq 11$	$r \geq 12$	-	-		-	0.152

Note: † Significant at the 1% level.

Table 16
Cointegrating Rank Determination, Residuals Diagnostic Tests, All Cities w/ 5 Year Mortgage Rate

Equation	Jarque-Bera $\chi^2(2)$	P-Value	Skewness $\chi^2(1)$	P-Value	Kurtosis $\chi^2(1)$	P-Value
STJ	0.010	0.995	0.006	0.936	0.004	0.949
HAL	0.680	0.712	0.173	0.677	0.506	0.477
MON	1.876	0.391	1.000	0.317	0.876	0.349
OTT	2.956	0.228	0.000	0.944	2.956	0.086
TOR	0.216	0.897	0.102	0.750	0.115	0.735
WPG	0.191	0.909	0.077	0.781	0.114	0.736
REG	0.288	0.866	0.191	0.662	0.097	0.755
CAL	4.91	0.101	1.790	0.181	2.801	0.094
EDM	1.935	0.380	0.961	0.327	0.974	0.324
VAN	0.637	0.727	0.637	0.425	0.000	0.989
FIVE	0.098	0.952	0.095	0.758	0.003	0.954
ALL	13.478	0.919	5.032	0.930	8.446	0.673
$LM(1)$	166.430	0.004				
$LM(4)$	135.512†	0.174				

Note: † Significant at the 1% level. The statistics testing the normality, skewness and Kurtosis of 'ALL' are are distributed $\chi^2(22)$, $\chi^2(11)$, $\chi^2(11)$ respectively. LM statistics and their corresponding p-values testing for the significance of the first and fourth autocorrelation in the residuals of the VAR are also provided.

3. Pairwise Cointegration Tests

We now provide additional evidence for cointegration (or lack thereof) through simpler bi-variate models based on housing prices and mortgage rates. Specifically, in this section we make use of the ML and OLS based approaches employed above to assess long-run relationships between *pairs* of time series. The ML results are finite-sample corrected and are based on the model with an unrestricted constant. We extend the pairwise results for housing prices in our complementary paper to include all of the cities available. Moreover, we look at the pairwise housing price - mortgage rate relationships as well. *A prior*, our intuition suggests that neighboring cities' housing prices are more likely to be cointegrated if local cities are 'competitors' in their respective housing markets. Furthermore, if the negative long run relationship between average existing housing prices and mortgage rates is clear, then pairwise analysis should unveil this statistically.

The results reflect the findings from the system-based analysis above and those reported in our paper. It follows from Tables 17, 18 and 19 that there is very little cointegration amongst all possible pairwise relationships in the cities' existing housing prices, for all ML and OLS-based tests. Furthermore, most of the significant cointegrating relations involve Edmonton, whose housing prices are borderline stationary, thus making this result unsurprising. Turning to the various ML and OLS based pairwise tests in Table 20, we see again a lack of cointegration between the individual cities' prices and each of the one, three and five year mortgage rates. In fact each of the five bi-variate tests for cointegration in this table suggest that *none* of the time series for existing housing prices and mortgage rates linked in the long run. This contrasts our finding of one additional cointegrating vector in our systems-based analysis above, leaving the statistical relationship between housing prices and mortgage rates unclear. It also relates to our mixed/unclear FM-OLS results that relate mortgage rates and housing prices reported in the paper and with the DOLS results in the following section. At this point we can conclude that the time series properties the data do not lend themselves to a simply-defined long-run relationship between housing prices and mortgage rates in the (fragmented) Canadian housing market.

Table 17
Pairwise Cointegration Tests for MLS Existing House Prices
Trace and Maximium Eigenvalue Tests

Test		HAL	MON	OTT	TOR	WPG	REG	CAL	EDM	VAN	CAN
ML: Trace	STJ	19.100	9.642	22.032†	3.403	5.952	5.871	11.228	6.531	18.821	9.495
	HAL	-	15.555	15.013	7.571	20.161†	11.713	13.570	17.880	17.454	23.607
	MON	-	-	17.438	4.896	5.863	9.547	12.571	18.849	32.964†	21.368†
	OTT	-	-	-	10.764	13.822	17.549	17.967	18.938	25.838†	22.898†
	TOR	-	-	-	-	12.650	5.597	23.797†	38.598†	19.839	11.563
	WPG	-	-	-	-	-	3.436	11.971	19.848	18.858	23.257†
	REG	-	-	-	-	-	-	20.571†	27.192†	9.153	10.071
	CAL	-	-	-	-	-	-	-	10.950	4.928	11.196
	EDM	-	-	-	-	-	-	-	-	5.548	14.656
	VAN	-	-	-	-	-	-	-	-	-	28.411†
ML: Max	STJ	18.606	9.643	19.323†	2.964	5.950	4.741	11.125	6.072	17.975	9.460
	HAL	-	15.445	13.155	7.571	19.045†	10.057	13.225	15.437	17.454	23.381†
	MON	-	-	13.794	4.804	5.610	5.384	12.519	18.791†	31.750†	21.059†
	OTT	-	-	-	8.510	10.645	12.773	17.770	18.156	20.078†	20.952†
	TOR	-	-	-	-	11.938	3.950	23.133	37.377†	17.227	10.633
	WPG	-	-	-	-	-	3.289	11.922	17.920	17.768	23.257†
	REG	-	-	-	-	-	-	19.483†	25.103†	7.429	7.742
	CAL	-	-	-	-	-	-	-	10.903	4.618	10.743
	EDM	-	-	-	-	-	-	-	-	4.868	14.385
	VAN	-	-	-	-	-	-	-	-	-	25.622†

Note: † Significant at the 1% level. The top and bottom panels are Trace and λ-max statistics for cointegration between city-pairs. The corresponding 1 % critical value for the Trace tests is 20.04 and 18.63 for the λ-max tests.

Table 18
Pairwise Cointegration Tests for MLS Existing House Prices
Augmented Dickey-Fuller Tests

Test		HAL	MON	OTT	TOR	WPG	REG	CAL	EDM	VAN	CAN
OLS: ADF (nt)	STJ	-1.566	-2.188	-2.550	-1.711	-1.957	-1.777	-2.601	-2.688	-1.212	-1.885
	HAL	-	-1.302	-1.773	-0.973	-1.606	-2.727	-3.055	-2.932	-1.365	-1.124
	MON	-	-	-3.076	-0.943	-2.214	-1.540	-2.615	-3.220	-1.570	-1.561
	OTT	-	-	-	-3.658	-2.801	-2.499	$-3.459^{\dagger}$	$-4.572^{\dagger}$	-2.172	-3.153
	TOR	-	-	-	-	-0.730	-1.140	-2.067	-2.801	-1.819	-1.425
	WPG	-	-	-	-	-	-1.699	-2.425	-2.164	-0.802	-0.806
	REG	-	-	-	-	-	-	-2.660	-2.324	-0.667	-0.272
	CAL	-	-	-	-	-	-	-	-1.785	-1.231	-1.532
	EDM	-	-	-	-	-	-	-	-	-1.411	-2.161
	VAN	-	-	-	-	-	-	-	-	-	-1.893
OLS: ADF (t)	STJ	-1.609	-2.366	-3.307	-2.242	-2.018	-1.645	-2.400	-2.293	-1.277	-1.968
	HAL	-	-1.378	-3.119	-1.328	-1.623	-2.667	-2.818	-2.499	-1.443	-1.208
	MON	-	-	-3.328	-1.391	-2.078	-1.483	-2.409	-3.030	-1.574	-1.551
	OTT	-	-	-	-3.735	-2.563	-2.483	-3.267	$-4.609^{\dagger}$	-2.194	-3.147
	TOR	-	-	-	-	-0.644	-1.046	-1.880	-2.576	-1.798	-1.690
	WPG	-	-	-	-	-	-1.688	-2.287	-2.038	-0.849	-0.655
	REG	-	-	-	-	-	-	-2.588	-2.093	-0.890	-0.819
	CAL	-	-	-	-	-	-	-	-1.504	-1.636	-2.356
	EDM	-	-	-	-	-	-	-	-	-1.822	-2.983
	VAN	-	-	-	-	-	-	-	-	-	-1.914

Note: † Significant at the 1% level. (nt) and (t) stand for tests without trend and with a trend respectively. Optimal lag length for each test is determined by the BIC.

Table 19
Pairwise Cointegration Tests for MLS Existing House Prices
Phillips-Perron Tests

Test		HAL	MON	OTT	TOR	WPG	REG	CAL	EDM	VAN	CAN
OLS: PP (nt)	STJ	-3.574†	-4.026†	-3.164	-1.560	-3.429†	-2.156	-2.156	-2.787	-2.689	-3.408
	HAL	-	-2.873	-3.191	-0.966	-2.814	-4.372†	-3.345	-3.633†	-2.572	-1.918
	MON	-	-	-3.202	-1.236	-5.108†	-1.850	-2.219	-2.454	-2.112	-1.986
	OTT	-	-	-	-1.775	-3.241	-2.429	-3.453†	-3.790†	-3.285	-2.600
	TOR	-	-	-	-	-1.293	-1.443	-2.649	-2.542	-7.950†	-2.086
	WPG	-	-	-	-	-	-1.993	-1.996	-2.271	-1.731	-1.702
	REG	-	-	-	-	-	-	-2.940	-2.782	-1.355	-0.876
	CAL	-	-	-	-	-	-	-	-2.159	-0.945	-1.462
	EDM	-	-	-	-	-	-	-	-	-0.680	-1.205
	VAN	-	-	-	-	-	-	-	-	-	-2.801
OLS: PP (t)	STJ	-3.592	-4.220†	-3.588	-2.152	-3.524	-2.066	-2.300	-2.251	-2.592	-3.522
	HAL	-	-2.930	-3.563	-1.413	-2.763	-4.275†	-3.017	-3.062	-2.440	-1.949
	MON	-	-	-3.284	-1.781	-5.104†	-1.788	-1.961	-1.995	-1.996	-1.979
	OTT	-	-	-	-1.885	-2.967	-2.277	-2.973	-3.149	-2.998	-2.489
	TOR	-	-	-	-	-1.200	-1.293	-2.220	-1.943	-7.233†	-2.331
	WPG	-	-	-	-	-	-1.970	-1.889	-2.023	-1.730	-1.686
	REG	-	-	-	-	-	-	-2.924	-2.565	-1.492	-1.227
	CAL	-	-	-	-	-	-	-	-1.987	-1.167	-1.986
	EDM	-	-	-	-	-	-	-	-	-0.871	-1.654
	VAN	-	-	-	-	-	-	-	-	-	-2.950

Note: † Significant at the 1% level. (nt) and (t) stand for tests without trend and with a trend respectively. Optimal lag length for each test is determined by the BIC.

Table 20
Pairwise Cointegration Tests for MLS Existing House Prices and Mortgage Rates

Test		STJ	HAL	MON	OTT	TOR	WPG	REG	CAL	EDM	VAN	CAN
ML: Trace	ONE	13.347	12.147	8.960	11.186	13.367	10.863	19.734	10.766	17.375	10.350	11.256
	THREE	13.592	13.568	8.860	11.617	10.168	10.168	18.845	18.240	15.692	9.446	10.487
	FIVE	13.813	14.338	9.341	12.201	10.582	10.582	18.719	17.607	15.017	9.544	10.562
ML: Max	ONE	13.261	11.962	8.810	10.239	10.495	10.851	17.501	10.185	15.501	10.158	10.908
	THREE	13.502	13.243	8.760	10.912	10.019	10.019	16.170	16.746	13.601	9.163	10.170
	FIVE	13.697	13.980	9.212	11.475	10.323	10.323	15.795	16.700	12.790	9.248	10.224
OLS: ADF (nt)	ONE	-3.008	-2.348	-2.100	-2.344	-2.220	-1.847	-3.099	-3.901	-3.631	-3.709	-2.741
	THREE	-2.814	-2.884	-1.892	-2.098	-2.148	-1.641	-2.870	-4.212†	-3.259	-3.203	-3.203
	FIVE	-2.803	-2.771	-1.798	-2.071	-2.154	-1.666	-2.839	-4.278	-3.350	-3.152	-3.047
OLS: ADF (t)	ONE	-1.903	-1.499	-1.523	-1.523	-1.901	-1.172	-1.612	-1.896	-1.764	-2.431	-1.729
	THREE	-1.938	-1.975	-1.497	-1.471	-1.727	-1.128	-1.631	-2.256	-1.618	-2.340	-2.340
	FIVE	-1.963	-1.906	-1.437	-1.477	-1.768	-1.207	-1.639	-2.271	-1.690	-2.299	-2.404
OLS: PP (nt)	ONE	-1.901	-2.526	-2.384	-1.624	-2.004	-1.518	-1.685	-1.298	-0.912	-1.838	-1.711
	THREE	-2.200	-2.948	-2.770	-1.873	-2.188	-1.774	-1.924	-1.502	-0.936	-2.075	-2.075
	FIVE	-2.324	-3.098	-2.895	-1.952	-2.297	-1.912	-2.029	-1.506	-0.917	-2.095	-2.197
OLS: ADF (t)	ONE	-2.321	-2.997	-2.542	-1.959	-2.087	-1.882	-2.335	-2.069	-1.711	-2.384	-2.043
	THREE	-2.498	-3.288	-2.844	-2.096	-2.204	-2.013	-2.430	-2.063	-1.603	-2.444	-2.444
	FIVE	-2.577	-3.391	-2.936	-2.134	-2.289	-2.106	-2.484	-2.017	-1.543	-2.413	-2.369

Note: † Significant at the 1% level. (nt) and (t) stand for tests without trend and with a trend respectively. The top two panels are trace and λ-max statistics that test for cointegration between cities' existing housing prices and mortgage rates via Maximum Likelihood. The bottom four panels are OLS-based tests: Augmented Dickey-Fuller and Phillips Perron tests.

4. City-Level Housing Price Determinants: DOLS

This section provides complementary estimates for city-specific housing price determinants for the 8 core cities in our sample. These are based on Stock and Watson (1993) Dynamic Ordinary Least Squares (DOLS), a least-squares approach for obtaining consistent long-run parameter estimates relating existing housing prices for a city to its local determinants. In the paper, we have ensured via OLS-based methods that for each city, the MLS housing prices are cointegrated with the explanatory variables, NPI, UWI, BP, LF, GDP and R. This allows us to adopt DOLS as an alternative method of estimation. Further, from Tables 2 and 3 above we see that all of the independent variables are non-stationarity for all 8 cities. Hence the DOLS approach requires us to include leading and lagged differences to control for the unit root in each of these independent variables. It follows that the model estimated for city j is of the following form:

$$\begin{aligned} MLS_t^j = X_t^j\Theta' + \lambda^i t \quad &+ \sum_{q=-p}^{q=p} \delta_q^j \Delta NPI_{t-q}^j + \sum_{q=-p}^{q=p} \delta_q^j \Delta UWI_{t-q}^j + \sum_{q=-p}^{q=p} \delta_q^j \Delta BP_{t-q}^j \\ &+ \sum_{q=-p}^{q=p} \delta_q^j \Delta LF_{t-q}^j + \sum_{q=-p}^{q=p} \delta_q^j \Delta GDP_{t-q}^j + \sum_{q=-p}^{q=p} \delta_q^j \Delta R_{t-q} + \eta_t^j \end{aligned} \tag{1}$$

where $X_t^j = [1, NPI^j, UWI^j, BP^j, LF^j, GDP^j, R]$ and $\Theta = [\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7]$. The parameter p is chosen so as to ensure the residuals from Equation 1 are not autocorrelated.

From our results in Table 21, we are able to contrast the Fully Modified Ordinary Least Squares (FM-OLS) estimates from our paper. Qualitatively similar conclusions can be drawn. The strong relationship between existing housing prices and the price of new homes prevails in the DOLS estimates, like those from FM-OLS. With the exception of Calgary, the results suggest a unitary price elasticity of substitution between new and existing prices. Similarly, the positive relationship between union wage levels and existing housing prices is predicted by both models. Both DOLS and FM-OLS generally give a positive relationship between the level of existing housing prices and the number of building permits and the size of the municipal labour force. However, FM-OLS gives significant estimates for the BP coefficients while DOLS does not. The opposite is true for the LF coefficients. Finally, DOLS, like FM-OLS, yields mixed results for the GDP and R coefficients,

Table 21
City-Specific Estimates via DOLS

City	Regressors						
	NPI	UWI	BP	LF	GDP	R	$LM(1)$
STJ	0.9489*	0.1995	0.0723*	-0.2394	0.0423	-0.0029	0.793
(p=2)	(0.2139)	(0.2087)	(0.0317)	(0.2580)	(0.0895)	(0.0057)	
HAL	0.9469*	0.7561*	0.0276	0.9250*	1.0168*	-0.0299*	1.082
(p=1)	(0.2396)	(0.2580)	(0.0277)	(0.4321)	(0.2754)	(0.0059)	
MON	1.0394*	-0.2045	0.0680*	2.0288*	-0.2303	-0.0094	0.566
(p=2)	(0.1095)	(0.1643)	(0.0247)	(0.4595)	(0.1961)	(0.0053)	
OTT	1.3502*	0.07272	-0.0238	0.4201*	0.3198*	-0.0166*	2.013
(p=1)	(0.0804)	(0.0646)	(0.0173)	(0.1863)	(0.0936)	(0.0031)	
TOR	0.5731*	3.8829*	0.4927*	2.0966	-0.6336	0.0302	0.897
(p=5)	(0.1919)	(1.5799)	(0.2202)	(1.1709)	(1.2742)	(0.0529)	
CAL	2.0843*	0.2349	0.0154	0.3678	0.1440	0.0039	1.895
(p=4)	(0.5411)	(0.2969)	(0.0388)	(0.2997)	(0.5111)	(0.01180)	
EDM	1.0461*	0.2297	0.0412	2.3667*	0.6244	0.0130*	0.434
(p=4)	(0.2328)	(0.2138)	(0.0302)	(0.3709)	(0.4563)	(0.0062)	
VAN	0.7242*	1.6062*	0.0542	0.8426	0.9069	0.0294*	0.465
(p=3)	(0.1658)	(0.2622)	(0.0560)	(0.6498)	(0.6899)	(0.0141)	

Note: Standard errors are in parentheses. * is significant at the 5% level. p corresponds to the number of leading and lagged differences included the city DOLS equation for the non-stationary variables. LM(1) corresponds to the Breusch-Godfrey LM test for first-order autocorrelation in the residuals.

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