When are Debt for Nature Swaps Welfare–Improving?

A Note on Chang and Pillarisetti (1997)

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Chang and Pillarisetti (1997), in a recent article published in this journal, provide sufficient conditions for debt for nature swaps to be welfare–improving for a debtor developing country. They compare a debtor country's utility level with and without a debt for nature swap under two distinct no swap scenarios: (i) the debtor country does not undertake environmental spending in the absence of a swap and (ii) the debtor country undertakes a positive level of environmental spending in the absence of a swap. They compare utility levels under these two scenarios and derive sufficient conditions for debt for nature swaps to be welfare–

improving for the debtor country.

This paper corrects an error made by Chang and Pillarisetti (1997). Specifically, we prove that debt for nature swaps are unambiguously welfare–improving whenever a debtor country undertakes a positive level of environmental spending in the absence of a swap. In this case, debt for nature swaps always result in a first–order welfare gain for the debtor country, namely, the reduction in debt repayment realized from the swap. The proof involves a straightforward application of the Envelope Theorem. In addition, a simpler and more intuitive sufficient condition is derived to show when debt for nature swaps are welfare–improving if a debtor country does not undertake environmental spending in the absence of a swap.

1. The Model

Chang and Pillarisetti (1997) construct a model in which a non–profit organization purchases a portion of a developing country's external debt at a discount from a commercial bank holding the country's debt. The organization and the debtor country, in turn, exchange the purchased discounted debt for an agreed upon amount of financial resources (in local currency) to be allocated by the debtor country to environmental protection activities.

Following Chang and Pillarisetti (1997), the indebted developing country's two period utility can be written as a function of N:

$$U(N) = G - N + \rho\left\{\frac{\delta N(1+r)}{1-t} + B(N)\right\} \tag{1.1}$$

where N is domestic environmental spending, G is intertemporal utility independent of N, ρ is the country's subjective discount factor for future consumption, $0 < \delta < 1$ is the propensity of the organization to purchase the swap, $0 < t < 1$ is the market discount rate on the debt, $0 < r < 1$ is the rate of interest on debt, and $B(N)$ is a strictly concave environmental benefit function.[1]

Define the following indicator function α:

$$\alpha = \begin{cases} 1 & \text{if debt for nature swap} \\ 0 & \text{otherwise} \end{cases} \tag{1.2}$$

Using the indicator function, write the debtor country's utility maximization problem as:

[1] $G = E + D - I + \rho\{\theta F(I) - D(1+r) - H(I)\}$, where E is the country's initial endowment, D is the amount of inherited debt, I is investment, $\theta F(I)$ is total revenue from production (θ is a commodity price and $Y = F(I)$ is a strictly concave production function), $D(1+r)$ is debt repaid in period two, and $H(I)$ is a strictly convex damage function, measuring environmental damages incurred as a result of first period investment.

Since Chang and Pillarisetti (1997) find that the optimal choices of I and N are independent, we can, without loss of generality, restrict analysis to the optimal choice of N. The reader is referred to Chang and Pillarisetti (1997) for analysis pertaining to the optimal choice of I.

$$\max_{N} U(N;\alpha) = G - N + \rho\left\{\alpha\frac{\delta N(1+r)}{1-t} + B(N)\right\} \tag{1.3}$$

The first–order condition for N is given by

$$\frac{\partial U}{\partial N} = -1 + \rho\left\{\alpha\frac{\delta(1+r)}{1-t} + B'(N)\right\} \leq 0 \tag{1.4}$$

The choice of environmental spending, N, clearly depends upon the debt for nature swap parameters. The first term on the right–hand side of (1.4) is the marginal cost of environmental spending, that is, the marginal reduction in resources available for consumption in period one. The second term on the right–hand side of (1.4) is the discounted marginal benefit of environmental spending: the first term in the bracketed expression is the marginal value of the reduction in debt repayment realized in period two if a debt for nature swap occurs while the second term is the direct marginal benefit of environmental spending. Note from (1.4) that a debt for nature swap increases a debtor country's incentive to engage in environmental spending by shifting the marginal benefit curve upward by $\rho\frac{\delta(1+r)}{1-t}$, the marginal value of the debt reduction.

There are three possible equilibrium outcomes for N. First, if $\frac{\partial U}{\partial N}\Big|_{\alpha\in\{0,1\}} < 0$, then $N = 0$, that is, it is never in the interest of the debtor country to undertake

environmental spending and/or to engage in debt for nature swaps. Second, if $\frac{\partial U}{\partial N}|_{\alpha=0} < 0$ and $\frac{\partial U}{\partial N}|_{\alpha=1} = 0$, then a debt for nature swap provides an additional incentive to a debtor country to undertake environmental spending when it otherwise would not. Finally, if $\frac{\partial U}{\partial N}|_{\alpha=0} = 0$, then a debtor country will undertake a positive level of environmental spending whether or not a debt for nature swap is undertaken. Define the possible equilibrium levels of N as follows:

$$N = \begin{cases} N^0 = 0 & \text{if } \frac{\partial U}{\partial N}\Big|_{\alpha\in\{0,1\}} < 0 \\ N^e > 0 & \text{if } \frac{\partial U}{\partial N}|_{\alpha=0} = 0 \\ N^* > 0 & \text{if } \frac{\partial U}{\partial N}|_{\alpha=1} = 0 \end{cases} \tag{1.5}$$

2. Welfare Effects

2.1. No Environmental Funding in Absence of a Debt for Nature Swap

If a debtor country does not undertake environmental spending in the absence of a debt for nature swap, then it is indeed ambiguous whether or not a debt for nature swap is welfare–improving. The additional resource cost of environmental spending may be too high to justify the value of writing down an amount of debt. Nest the debtor country's two period maximal utility as

$$U(\alpha) = G(I^*) + \alpha\left[-N^* + \rho\left\{\frac{\delta(1+r)}{1-t}N^* + B(N^*)\right\}\right] \tag{2.1}$$

where I^* is defined by $\frac{\partial G}{\partial I} = 0$ and N^* is defined by (1.4) when $\frac{\partial U}{\partial N}|_{\alpha=1} = 0$.

Debt for nature swaps are unambiguously welfare–improving if and only if $\frac{dU(\alpha)}{d\alpha} > 0$. By the Envelope Theorem, $\frac{dU(\alpha)}{d\alpha} = \frac{\partial U(\alpha)}{\partial \alpha}$. Taking the partial derivative of (2.1) with respect to α yields:

$$\frac{\partial U(\alpha)}{\partial \alpha} = -N^* + \rho\left\{\frac{\delta(1+r)}{1-t}N^* + B(N^*)\right\} \tag{2.2}$$

A sufficient condition for $\frac{\partial U(\alpha)}{\partial \alpha} > 0$ is

$$\rho\left\{\frac{\delta(1+r)}{1-t} + \frac{B(N^*)}{N^*}\right\} > 1 \tag{2.3}$$

A simpler and more intuitive sufficient condition can be obtained by utilizing the assumption that $B(N)$ is strictly concave. Since $B(N)$ is strictly concave, $B'(0) > \frac{B(N^*)}{N^*}$, and, hence, a sufficient condition for $\frac{\partial U(\alpha)}{\partial \alpha} > 0$ is

$$\rho\left\{\frac{\delta(1+r)}{1-t} + B'(0)\right\} > 1 \tag{2.4}$$

where the left–hand side of (2.4) is the vertical intercept of the marginal benefit

curve and the right–hand side of (2.4) is the vertical intercept of the marginal cost curve. Thus, a simpler sufficient condition for a debt for nature swap to be welfare–improving in this scenario is simply that the marginal benefit curve with a swap cuts the marginal cost curve from above.

The two possible equilibrium outcomes for this case are illustrated in Figure 1. As Figure 1 shows, whether a debt for nature swap is welfare–improving depends upon the value, at the margin, of environmental spending. A swap does not provide a sufficiently strong additional incentive to undertake environmental spending in Figure 1(a), that is, (2.4) fails to hold. In this case, the debtor country will not engage in environmental spending even when debt for nature swaps are possible. In Figure 1(b), the sufficient condition (2.4) holds and hence the country will engage in environmental spending according to the terms of the swap. The welfare gain from a swap is the shaded area depicted in Figure 1(b).

[Insert Figure 1 about here]

Integrating (2.2) yields the utility comparison for this case[2]:

[2]Rearranging (2.5) yields Chang and Pillarisetti's (1997) sufficient condition for a debt for nature swap to be welfare–improving when a debtor country does not undertake environmental spending in the absence of a swap:

$$\rho > \frac{N^*}{\frac{\delta(1+r)}{1-t}N^* + B\left(N^*\right)}$$

$$\begin{aligned}\Delta_{*0} &= U^* - U^0 = \int_0^1 \frac{\partial U(\alpha)}{\partial \alpha} d\alpha \\ &= -N^* + \rho\left\{\frac{\delta(1+r)}{1-t}N^* + B(N^*)\right\} \qquad (2.5)\end{aligned}$$

Note that $\Delta_{*0} > 0$ whenever (2.4) holds.

2.2. Environmental Funding in Absence of a Debt for Nature Swap

Contrary to Chang and Pillarisetti (1997), if a debtor country undertakes environmental spending in the absence of a debt for nature swap, then a debt for nature swap is unambiguously welfare–improving.[3] At the margin, the developing country will always benefit from forgiveness of a discrete quantity of debt (a first–order benefit) in return for a marginal addition to environmental spending (a second–order cost). An unambiguous welfare result can be obtained by a straightforward application of the Envelope Theorem.

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[3] Chang and Pillarisetti (1997) state that a debt for nature swap is welfare–improving in this case if and only if the following condition is satisfied:

$$\rho > \frac{N^* - N^e}{\frac{\delta N^*(1+r)}{1-t} + B(N^*) - B(N^e)}$$

The optimal choice of N is defined by

$$-1+\rho\left\{\alpha\frac{\delta\left(1+r\right)}{1-t}+B'\left(N\right)\right\}=0 \tag{2.6}$$

The first–order condition (2.6) implicitly defines $N(\alpha)$. Apply the Implicit Function Theorem to (2.6) to determine how environmental spending changes when a debt for nature swap is undertaken, that is, $\frac{dN}{d\alpha}$:

$$\frac{dN}{d\alpha}=-\frac{\delta\left(1+r\right)}{\left(1-t\right)B''(N)}>0 \tag{2.7}$$

Clearly, $\frac{dN}{d\alpha}>0$ since $B''(N)<0$. Hence, environmental spending will always be higher when a debt for nature swap is undertaken, that is, $N^*>N^e$.

Nest the debtor country's two period maximal utility as

$$U\left(\alpha\right)=G\left(I^*\right)-N\left(\alpha\right)+\rho\left\{\alpha\frac{\delta\left(1+r\right)}{1-t}N(\alpha)+B\left(N\left(\alpha\right)\right)\right\} \tag{2.8}$$

where $N\left(\alpha=0\right)=N^e$, $N\left(\alpha=1\right)=N^*$, and N^e and N^* are defined by (2.6). Debt for nature swaps are unambiguously welfare–improving if and only if $\frac{dU(\alpha)}{d\alpha}>0$. By the Envelope Theorem, $\frac{dU(\alpha)}{d\alpha}=\frac{\partial U(\alpha)}{\partial\alpha}$. Taking the partial derivative of (2.8) with respect to α yields:

$$\frac{\partial U(\alpha)}{\partial \alpha} = \left[-1 + \rho \left\{ \alpha \frac{\delta(1+r)}{1-t} + B'(N(\beta)) \right\} \right] \frac{\partial N}{\partial \alpha} + \rho \frac{\delta(1+r)}{1-t} N(\alpha) \qquad (2.9)$$

The first term on the right–hand side of (2.9) vanishes since (2.6) must hold for all values of α. Hence, (2.9) reduces to

$$\frac{\partial U(\alpha)}{\partial \alpha} = \rho \frac{\delta(1+r)}{1-t} N(\alpha) > 0 \qquad (2.10)$$

which is strictly positive for all parameter values. Thus, a debt for nature swap yields a first–order welfare gain to the debtor developing country. The change in welfare equals the discounted value of the debt reduction obtainable in period two as a result of the swap.

Integrating (2.10) yields the utility change for this case:

$$\begin{aligned} \Delta_{*e} &= U^* - U^e = \int_0^1 \frac{\partial U(\alpha)}{\partial \alpha} d\alpha = \rho \frac{\delta(1+r)}{1-t} \int_0^1 \frac{\partial N}{\partial \alpha} d\alpha \\ &= -\frac{\rho}{B''(N)} \left(\frac{\delta(1+r)}{1-t} \right)^2 > 0 \end{aligned} \qquad (2.11)$$

Note again that $\Delta_{*e} > 0$ for all parameter values. However, the size of the

welfare gain from a debt for nature swap is strictly increasing in ρ, δ, r and t and decreasing in $B''(N)$. The welfare gain from a swap is the shaded area depicted in Figure 2.

[Insert Figure 2 about here]

References

Chang Y. and Pillarisetti J., "Debt for Nature Swaps: A Simple Economic–Ecological Approach", *RISEC: International Review of Economics and Business*, 1997, *44*, pp. 793–806.

Abstract

This paper proves that debt for nature swaps are unambiguously welfare–improving whenever a debtor country undertakes a positive level of environmental spending in the absence of a swap — debt for nature swaps always result in a first–order welfare gain for the debtor country, namely, the reduction in debt repayment realized from the swap. The proof involves a straightforward application of the Envelope Theorem. A simple sufficient condition is also derived to show when debt for nature swaps are welfare–improving if a debtor country does not undertake environmental spending in the absence of a swap. (*Keywords*: debt for nature swaps; *JEL*: F34, O19)