Discounting Climate Change*

by

Partha Dasgupta**

University of Cambridge

April 2007
Revised: October 2007

* The ideas that I apply here were presented in my Plenary Lecture to the World Congress of Environmental and Resource Economists, held in Monterey, California, June 2002, and were explored in Dasgupta (2001: ch. 11). For discussions and correspondence over the years, I am very grateful to Kenneth Arrow, Geir Asheim, John Broome, Richard Cooper, Jeremy Edwards, Peter Hammond, Karl-Göran Mäler, Stephen Marglin, Eric Maskin, James Mirrlees, William Nordhaus, Hashem Pesaran, Amartya Sen, Robert Solow, and Martin Weitzman. I am grateful also to Ken Coutts for his encouragement and for help with the numerical computations. The present version of the paper was much influenced by the comments of Charlie Kolstad and an anonymous referee. It is forthcoming in the *Review of Environmental Economics and Policy*.

** The author is the Frank Ramsey Professor of Economics at the University of Cambridge and Fellow of St John's College.

ABSTRACT

In this paper I offer a fairly complete account of the idea of social discount rates as applied to public policy analysis. I show that those rates are neither ethical primitives nor observables as market rates of return on investment, but that they ought instead to be derived from economic forecasts and society's conception of distributive justice concerning the allocation of goods and services across personal identities, time, and events. The welfare theory is developed in the context of recent empirical work on the economics of global climate change. I argue that prominent books on the subject have been based on too cavaliar an attitude to distributive justice and a Panglossian view of governance in the contemporary world; and that influential articles on social discounting have taken an unusually limited range of economic forecasts into consideration. Such concreteness lends the analysis an air of finality, but I argue that it is a case of misplaced concreteness. I show this by developing the theory of social discounting in the face of future consumption uncertainties. The precautionary motive for saving for climate change is established in the case where future uncertainties are not large. I then show that if the uncertainties associated with climate change and biodiversity losses are large, the formulation of intergenerational well-being we economists have grown used to could lead to ethical paradoxes: an optimum policy may not exist. Various modelling avenues that offer a way out of the dilemma are discussed. It is shown that none of them is entirely satisfactory.

JEL Classification: C61, D53, **D9**, G12

Keywords: utilitarianism, social discount rates, climate change, uncertainty, inequality aversion, risk aversion, rate of time preference, rate of return on investment, precautionary principle, elasticity of marginal well-being, certainty equivalence, risk-free rates.

Prologue

Imagine someone who has been reading articles and watching documentaries on climate change. She is persuaded that rising concentrations of carbon dioxide in the atmosphere is a major contributor to the process. She knows that even though the global warming associated with climate change is slow in comparison to the speed of contemporary economic growth, the carbon concentrations expected to be reached at the end of this century under business as usual haven't been harboured by Earth's atmosphere in the past several million years. This scares her. However, she realises that although the investment required to curb the process - controlling carbon emissions, enlarging sequestration possibilities, and investing in alternative energy technologies - are large, the benefits will be enjoyed only many decades from now. Which is why she is not only anxious about climate change, she is also at a loss to know how to think about the matter.

As our protagonist is a citizen of a functioning democracy, she wants to instruct her political leaders to start discussions with governments of other countries on what, as she sees it, is a global commons problem. That is why she now seeks a grammar that can join her understanding of the way the world works (the ways in which people would choose under various circumstances, the pathways Nature chooses, the consequences of those choices, and so on) to the basis on which alternative global investment policies ought to be evaluated. As carbon emissions involve massive externalities, she realises that in her role as a citizen she shouldn't rely exclusively on her private interests, but should instead adopt something like a *social* point of view, one that would appear reasonable not only to her, but to others as well. This makes her want to take others into account when deliberating over the costs and benefits of alternative investment policies. But she realises that when it comes to climate change, most of those others will be people who are yet to be born. So, she wants to know what contemporary economics has to say about her dilemma.

Our protagonist asks an economist friend to give her a reading list, complaining to him that correspondents even in the most prominent newspapers never write about the questions that are vexing her. The friend assures her that economics does have the conceptual tool she seeks, and that it has already been put to use by contemporary economists for studying the economics of climate change. He gives her three books to read: Cline (1992), Nordhaus (1994), and Stern (2006).

Some days later our protagonist calls her friend to complain. She says she has now read the books, but remains confused. Cline and Stern, she says, urge immediate, strong global action to combat climate change - Stern, she notes, recommends what amounts to an annual expenditure of 2% of the GDP of rich countries. But Nordhaus, she observes, claims that despite the threats climate change poses to the global economy, it would be more equitable and efficient to invest in reproducible and human capital now so as to build up the productive base of economies - including, especially, poor countries - and to put into effect controls on carbon in an increasing, but gradual manner, starting several decades from now. What confounds her, our protagonist remarks, is that Cline and Stern, on the one hand, and Nordhaus, on the other, reach very different conclusions even though they are all agreed that global GDP *per capita* can be expected to continue to grow over the next 100 years and more even under business as usual - at something like 1-2% a year. What, she asks, is going on?

This article offers an account of what she wants to know.

1. Preliminaries: facts and values

When economists study public policy, they take two sets of considerations into account, just as our protagonist did. First, they identify the ways in which the world might work and chart the probable consequences of alternative policies. Secondly, they value those consequences so as to be able to judge the relative merits of the alternative policies. The former set of exercises involves *description*, while the latter involves *evaluation*. Disagreements over the worth of alternative public policies arise when people don't agree on *facts* (e.g., the economic effects of a doubling of carbon concentration in the atmosphere) or when they don't agree on *values* (e.g., the way our well-being ought to be balanced against the well-being of all those future people). Of course, it is common in daily life for *both* facts and values to be subject to dispute.

Reading the many reports on Stern (2006), published in newspapers and magazines at its launch (31 October 2006) - interestingly, reading the book itself - would give one the impression that the case built by the authors for strong, immediate action rests wholly on insights drawn from the new and more refined global circulation models of climate scientists. In fact the conclusions reached by Stern and his co-authors are implications of their choice of a pair of fundamental ethical parameters; they aren't driven so much by the new climatic facts the authors have stressed. It so happens Cline (1992) postulated values for that same pair of parameters that were very close to the ones assumed in Stern's book (see below). In a symposium on his book, Cline (1993:4) summarised his findings in words that reflect a point of view strikingly similar to that in Stern (2006): "My central scenario shows that ... if risk aversion is incorporated by adding high-damage and low-damage cases and attributing greater weight to the former, benefits comfortably cover costs (with a benefit-cost ratio of about 1.3 to 1). Aggressive abatement is worthwhile even though the future is much richer, because the potential massive damages warrant the costs."

In contrast, the figure chosen for one of the ethical parameters in Nordhaus (1994) is so different from the one in Cline (1992) and Stern (2006), that it leads him to advocate the upward-sloping "climate policy ramp" of ever tightening reductions in carbon emissions our protagonist noticed in his work. To explain what those ethical parameters are, it is simplest to begin with basics.

Assume, as all three authors do, that each person's utility depends on his or her consumption level. Assume too, that a generation's well-being is an increasing concave function of the utilities of the members of that generation. By the "fundamental ethical parameters", I mean two things: (i) the tradeoffs that ought to be made between the well-beings of the present and future generations, given that future generations will be here only in the future; and (ii) the tradeoffs that can justifiably be made between the consumptions people enjoy, regardless of the date at which they appear on the scene. Technically, (i) is reflected in the *time discount rate* (we denote it here by δ); and (ii) is reflected in the elasticity of the social weight that ought to be awarded to a small increase in an individual's consumption level (we denote it here by η). We confirm later that δ reflects the way the future is seen through today's telescope, while η is a measure of society's aversion to interpersonal *inequality* and *risk* in consumption. We confirm also that the common ethical framework adopted by the three authors in arriving at a value for η is altogether too narrow, in that they interpret η as the elasticity of marginal utility - which implies in particular that the distribution of utilities within a generation doesn't enter one's conception of social well-being.

δ and η, as we have defined it above, are fundamental because they help to determine the rates at which society ought to discount changes in future consumption. The other factor that helps to determine those rates is society's forecast of future consumptions. Discount rates on consumption changes combine "values" with "facts".

The ethical viewpoint I explore here is self-consciously anthropocentric. Nature has an intrinsic value, but I ignore it because the three books on the economics of climate change I am responding to ignore it. I don't even accommodate the fact that people care about certain types of natural capital as stocks (e.g., places of scenic beauty or sacred sites), because the books I discuss here don't consider it. Economics can most certainly accept that Nature isn't simply an instrument for human use (Heal, 1998); but we stay close to the minimalist formulations of human motivation in Cline, Nordhaus, and Stern here, because I want to discuss them on their own terms.

2. Consumption Discount Rates: basics

Imagine that society entertains no uncertainty and has made a forecast of future consumption. Imagine also, for simplicity of exposition, that a generation's well-being depends solely on its average consumption level (but see footnote 2). Society now conducts a thought experiment on its forecast by asking how much additional consumption it would demand on behalf of tomorrow's people in payment for a reduction in today's consumption by one unit. We say that the "social rate of discount" between today's and tomorrow's consumptions is that additional consumption demanded, less unity. So, if ρ is that rate, society would demand $(1+\rho)$ units of additional consumption tomorrow as a price for giving up one unit of consumption today; meaning that society regards an additional unit of consumption tomorrow to be worth $1/(1+\rho)$ units of additional consumption today. In order to stress that society is deliberating over a *consumption* swap between today and tomorrow, we say that ρ is the *consumption discount rate*. As would be expected, consumption discount rates play a central role in social cost-benefit analysis (Marglin, 1963; Arrow and Kurz, 1970; Dasgupta *et al*., 1972; Lind, 1982; Arrow *et al*., 1996; Portney and Weyant, 1999).

Any mention of "discount rates", and one thinks immediately of positive numbers. But should society discount future consumption costs and benefits at a positive rate?

There are two reasons why it may think it reasonable to do so. First, an additional unit of consumption tomorrow would be of less value than an additional unit of consumption today if society is impatient to enjoy that additional unit now. Therefore, impatience is a reason for discounting future costs and benefits at a positive rate. Second, considerations of justice and equality demand that consumption should be evenly spread across the generations. So, if future generations are likely to be richer than us, there is a case for valuing an extra unit of their consumption less than an extra unit of our consumption, other things being equal. Rising consumption provides a second justification for discounting future consumption costs and benefits at a positive rate.

A number of questions arise: How should society choose consumption discount rates? How are they related to notions of intergenerational justice and equity? Should they be constant over time or could they depend on date? Do they reflect the "opportunity cost of capital"; if so, how should society determine what that cost is? Can they be inferred from "market observables", such as risk-free interest rates on government bonds? Must consumption discount rates be positive or are there circumstances when they would be negative? And how should we price future consumption when that future is uncertain?

In this paper I discuss tentative answers to those questions. I do this in stages. Section 3 considers a deterministic world. In Sections 4 and 5 I introduce "small" and "large" uncertainties, respectively, in future technology. Unfortunately, even the simplest analytical model of the economics of global climate change (Dasgupta *et al*., 1999) is a lot more complicated than is necessary for our discussion here. So, although climate change motivates this paper - I refer to it repeatedly - the model I use here as my work-horse doesn't contain the phenomenon. Just so that we know how to translate statements in the economic model studied here into corresponding statements in economic models of climate change, we note that, to be concerned about future generations in models of climate change means investing heavily so as to tame that change or to withstand the consumption effects of that change; whereas, to be concerned about future generations in our model translates into high investment rates. Either way, the "present" foregoes consumption in favour of the "future".

3. Intergenerational Well-Being: the deterministic case

As climate change involves the long run, we imagine that population size is constant. And in order to focus on the intergenerational distribution of consumption, we bypass intragenerational issues (but see Section 3.5). Time is assumed to be discrete. When we come to use expression (1), below, in numerical exercises, we will often take the unit of time to be a year. Let t (= 0, 1, 2, ...) denote date and let $U(C_t)$ be the flow of *social well-being* at t if C_t is average consumption at t. The present is $t = 0$. C_t ($t \geq 0$) is to be interpreted as a forecast. We take it that marginal social well-being is positive ($U'(C) > 0$), but declines with increasing consumption ($U''(C) < 0$).[1] We will see below that the extent to which $U'(C)$ declines with increasing C (i.e., the curvature of $U(C)$) plays a crucial role in intergenerational welfare economics.

[1] We write $U'(C) = \mathrm{d}U(C)/\mathrm{d}C$ and $U''(C) = \mathrm{d}U'(C)/\mathrm{d}C$.

If δ ($\geq$ 0) is the *time discount rate*, intergenerational well-being is the present-value of the $U(C_t)$s, namely,

$$U(C_0) + U(C_1)/(1+\delta) + ... + U(C_t)/(1+\delta)^t + ...[2] \quad (1)$$

Although it has become customary in the welfare economics of climate change not only to suppose that a generation's well-being is the sum of personal utilities, but also to infer those utilities from the choices people make in the market place (Cline, 1992; Nordhaus, 1994, 2007; Nordhaus and Boyer, 2000; Stern, 2006; Weitzman, 2007a - see section 3.4), there are several philosophical bases for expression (1) in which U is not necessarily personal utility. For example, Harsanyi (1955) provided an outline of an ethical theory that was independently developed by Rawls (1972) into a far-reaching, contractual theory of justice. Dasgupta and Heal (1979: Ch. 9) applied Harsanyi's theory to the problem of intergenerational justice and showed that it yields expression (1). They also showed that in order to allow for the risk aversion someone might experience when choosing in ignorance of the generation he is to join, U in the Harsanyi-Rawls theory is a concave transformation of personal utility. In contrast, Koopmans (1972) arrived at expression (1) from a set of ethical axioms on rankings over consumption sequences. One of his axioms - "stationarity" - is a near-cousin of the requirement that ethical judgments on consumption sequences be universalizable, which in the present context means that the ranking of a set of alternatives should be the same no matter when the ranking is done. However, in order to allow for considerations of intergenerational equity, the U in expression (1) in Koopmans' theory would be a concave transformation of personal utility. (Rawls, 1972, would call Koopmans' formulation "intuitionist".) In further contrast, Ramsey (1928) interpreted expression (1) - with $\delta = 0$ - in classical utilitarian terms. But he didn't presume that U is to be calibrated from market choices. (Rawls, 1972, would call Ramsey's formulation "teleological".) As I am not

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[2] The time discount rate in expression (1) is constant. Arrow (1999) has appealed to agent-relative ethics to explore the consequences of using a variable time discount rate. The ethics he explored arises from the idea that each generation should award equal weight to the well-beings of all subsequent generations, but should award its own well-being a higher weight relative to that awarded to the subsequent generations. His formulation is a special case of ones involving "hyperbolic" time discounting (see Section 4.1). In this paper I am staying close to Cline (1992), Nordhaus (1994) and Stern (2006), all of whom have used expression (1) as the basis of their studies.

If we wished to consider intragenerational equity, we would disaggregate each generation and suppose that there are N people at each date (i = 1,2,...,N). Let C_{it} be a forecast of the consumption rate of person i of generation t. Let social well-being at t be $U(C_{1t})+...+U(C_{it})+...+U(C_{Nt})$). In this case intergenerational well-being would be ${}_{t=0}\Sigma^{\infty}\{({}_i\Sigma(U(C_{it}))/(1+\delta)^t)\}$.

restricting myself to classical utilitarianism, let alone utilitarianism founded on revealed preference, we will be able to explore a far wider range of ethical considerations than have been admitted in the recent economics literature.[3]

Admittedly there are huge problems of interpretation if the demographic and normative assumptions underlying expression (1) are taken literally. It can be argued, for example, that to ask, "how much should I save for my children?" involves ethics that are different from those pertinent when we ask, "how should I spread out *my* consumption over time?" Expression (1) encapsulates a framework for addressing the former question and is the one we study in this paper.[4] To have a clean interpretation of expression (1), we imagine that each person lives for many periods, but is regarded as a distinct self in each period.

In expression (1) δ is the *time* discount rate. We now provide a formula for the *consumption* discount rate, ρ_t, defined in Section 2. Let ΔC_t and ΔC_{t+1} denote "small" variations in C_t and C_{t+1}, respectively, and assume that the pair of variations leave the numerical value of expression (1) unchanged. Denote by $g(C_t)$ the percentage rate of change in aggregate consumption between t and t+1.[5] Let η be the absolute value of the percentage change in marginal well-being owing to a percentage change in the level of consumption. η is called the *elasticity of marginal social well-being*. It is a measure of the curvature of $U(C)$.[6] Although there is no obvious reason why η should be independent of C, it simplifies computations enormously to assume that it is. Following Cline, Nordhaus, and Stern, we therefore assume that η is independent of C. The class of Us for which η is constant is given by the form

$$U(C) = C^{(1-\eta)}/(1-\eta), \qquad \text{for } \eta > 0,$$

and $$U(C) = \ln C, \qquad \text{corresponding to } \eta = 1. \tag{2}$$

The larger is η, the greater is the curvature of $U(C)$. Notice that $U(C)$ is bounded above but

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[3] In Dasgupta (2005a) I discuss the philosophical foundations of expression (1) in greater detail.

[4] One way to interpret expression (1) is to imagine that $U(C_t)$ denotes the lifetime utility of someone born at t; that C_t is an aggregate index of his lifetime consumption; and that our ethics do not permit us to peer into his affairs by unravelling that index. If we wished to peer into individuals' lifetime consumptions, an overlapping-generations model would serve the purpose better, toward which the model in Meade (1966) can be put to use.

[5] So, $C_{t+1}/C_t = 1+g(C_t)$. (F1)

[6] Formally, $\eta = -[d(U'(C))/dC]C/U'(C)$; or, in other words, that $\eta = -CU''(C)/U'(C) > 0$. This shows that η is a measure of the curvatiure of $U(C)$.

unbounded below if $\eta > 1$, whereas $U(C)$ is bounded below but unbounded above if $\eta < 1$.[7]

On using expression (1), we obtain

$$1 + \rho_t = (1+\delta)(1+g(C_t))^{\eta}.^{8} \tag{3}$$

Notice the way δ, η, and the forecast, $g(C_t)$, together determine ρ_t. Observe in particular that ρ_t increases with δ, η, and $g(C_t)$, respectively. Equation (3) gives a precise expression to the intuitive reason I offered earlier as to why the present generation would be ethically correct to discount changes in future generations' consumption levels. We will analyse equation (3) in detail presently, but first we derive a useful formula for ρ_t when δ and $g(C_t)$ are both small. So, suppose they are small. Then equation (3) reduces to the approximate form,

$$\rho_t \approx \delta + \eta g(C_t).^{9} \tag{3a}$$

If the interval between dates was to be made smaller and smaller, expression (3a) would be a better and better approximation. Not surprisingly, then, if time were taken to be continuous, expression (3a) would be an exact equality (see, e.g., Arrow and Kurz, 1970).

3.1 The Imperfect Economy

In expression (1), C_t is assumed to be a forecast, nothing more. At this point we do *not* assume that it is an optimum consumption programme for society (but see Section 3.2). The forecast is based on society's reading of technological possibilities, households preferences, current and future government policies, and so forth. To make a forecast requires an understanding of the political economy of society.

Intergenerational welfare economics is frequently conducted in continuous time. Which is why expression (3a) would be more familiar to readers than expression (3). But both are

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[7] Arrow (1965) observed that the simplest U that is bounded at both ends is one for which η is an increasing function of C and is less than 1 at low values of C and greater than 1 at high values of C.

[8] Proof: Because the pair of variations ΔC_{t+1} and ΔC_t leave the numerical value of expression (1) unaltered,

$$U'(C_t)\Delta C_t/(1+\delta)^t + U'(C_{t+1})\Delta C_{t+1}/(1+\delta)^{t+1} = 0. \tag{F2}$$

By definition,

$$\rho_t = -\Delta C_{t+1}/\Delta C_t - 1, \tag{F3}$$

where ΔC_{t+1} and ΔC_t satisfy equation (F2). But from equation (2), we know that $U(C) = C^{(1-\eta)}/(1-\eta)$, where $\eta > 0$. Now use equations (F1) and (F2) in equation (F3) to obtain equation (3) in the text.

[9] Proof: Take the logarithm of both sides of equation (3) and, using the fact that if x is a small number, $\ln(1+x) \approx x$, the approximate equation (3a) follows.

fundamental. They give quantitative expression to the pair of reasons I offered earlier for discounting future consumption gains and losses - namely, "impatience" and "intergenerational equity". The larger is δ, the larger is ρ_t, other things being equal. That much is obvious. So we turn to the influence of η on ρ_t. We have noted that η is an index of the curvature of U. Equations (3) and (3a) say that if $g(C_t) \neq 0$, the larger is η, the larger is the absolute value of ρ_t, other things being equal. This proves

Proposition 1. *η is the index of the aversion society ought to display toward consumption inequality among people*.

It will prove useful to table the most-preferred values of δ and η in Cline (1992), Nordhaus (1994), and Stern (2006).

Cline: $\delta = 0$; $\eta = 1.5$

Nordhaus: $\delta = 3\%$ a year; $\eta = 1$

Stern: $\delta = 0.1\%$ a year; $\eta = 1$

Notice how close the authors are in their choice of η. Notice also how close Cline and Stern are in their specifications of δ. In Section 3.4 we ask why Nordhaus is such an outlier in his choice of δ. Here we note that to say that $\eta = 1$ is to insist that any proportionate increase in someone's consumption level ought to be of equal social worth to that same proportionate increase in the consumption of anyone else who is a contemporary, no matter how rich or poor that contemporary happens to be. It is also to insist that, if in addition $\delta = 0$, any given proportionate increase in consumption today ought to be of equal social worth to that same proportionate increase in consumption at any future date, no matter how rich or poor people will be at that future date. Taken at face value, though, it isn't immediate whether such tradeoffs are ethically reasonable. In Section 3.2 we run more informative tests. They confirm that the pair ($\delta \approx 0$, $\eta = 1$) can recommend bizarre policies in classroom models of consumption and saving.

For computational purposes, it helps to assume that expression (3a) is a good approximation. I summarise the points it makes:

(a) ρ_t is not a primary ethical object, it has to be derived from an overall conception of intergenerational well-being and the consumption forecast: consumption discount rates cannot be plucked from air. (b) Just as growing consumption provides a reason why discount rates in use in social cost-benefit analysis should be positive, declining consumption would be a reason why they could be negative. Example: Suppose $\delta = 0$, $\eta = 2$, and $g(C_t) = -1\%$ per year. Then $\rho_t = -2\%$

per year. Such reasoning assumes importance when we come to discuss that people in the tropics, who are in any case very poor, will very likely suffer greatly from climate change under business as usual (Section 3.5). The reasoning takes on an interesting application when we come to consider uncertainty in future consumption (Sections 4 and 5).[10] (c) If intertemporal external diseconomies are substantial, as is the case with climate change under business as usual, both ρ_t and private rates of return on investment could be positive for a period of time, even while the social rate of return on investment is negative.[11] (d) Only in a fully optimizing economy (Section 3.2) is it appropriate to discount future consumption costs and benefits at the rate that reflects the direct opportunity cost of capital. In imperfect economies ρ_t should be used to discount consumption costs and benefits, but the capital deployed in projects ought to be revalued so as to take account of the differences between ρ_t and the various rates of return on investment (Section 3.3). Note though that the revalued cost of capital would be less than the price of consumption if the social rate of return on investment in that form of capital is less than ρ_t. (e) Unless consumption is forecast to remain constant, social discount rates depend on the numeraire: $\rho_t = \delta$ if and only if $g(C_t) = 0$. (f) If $g(C_t)$ varies with time, so does ρ_t. For example, suppose it is forecast that long-run consumption growth is not sustainable but will decline at a constant rate of 1% a year - from the current figure of 2% a year to zero. Suppose $\delta = 0$ and $\eta = 2$. In that case ρ_t will decline over time at 1% a year, from a current-high 4% a year, to zero. Note though that the "hyperbolic" discounting that comes with a declining value of $g(C_t)$ doesn't lead to time inconsistency over project evaluation.

The point estimate of consumption growth under business as usual in Stern (2006) is $g(C_t) = 1.3\%$ a year. Using this in equation (3a), we find that:

$\rho_t = 2.05\%$ a year for Cline

$\rho_t = 4.30\%$ a year for Nordhaus

$\rho_t = 1.40\%$ a year for Stern

4.3% a year may not seem very different from 1.4% a year, but is in fact a lot higher when it is

[10] I have friends in the US who find illustrations involving negative economic growth to be unrealistic. In fact a number of countries in sub-Saharan Africa suffered from negative growth during the period 1970-2000. What discount rates should government project evaluators there have chosen in 1970 if they had an approximately correct forecast of the shape of things to come?

[11] See Dasgupta *et al*. (1999). This parallels the well-known fact that if the external disbenefits arising from someone's use of a commodity are large enough, the commodity's shadow price will be negative even when its market price is positive.

put to work on the economics of the long run. Just how much higher can be seen from the fact that the present-value of a given loss in consumption, owing, say, to climate change 100 years from now, if discounted at 4.3% a year is *seventeen* times smaller than the present-value of that same consumption loss if the discount rate used is 1.4% a year. The moral is banal: If the time horizon is long, even small differences in consumption discount rates can mean large differences in the message cost-benefit analysis gives us. The reason Cline (1992) and Stern (2006) have recommended that the world spends substantial sums today to tame climate change, while Nordhaus (1994) has recommended a far more gradualist investment policy can be traced to the difference in their choice of δ. Nordhaus (2007) confirms this by using Stern's specifications for δ and η in the climate-change model he has developed over the past two decades.

In contrast to these authors, I suggest below that, while it is reasonable to set $\delta \approx 0$, values for η larger than unity should be considered, perhaps in the range [2,4].

3.2 The Fully Optimum Economy

To see why, let us put ethics to work on a production model of the economy. In a full optimum, the C_t that is chosen from the set of all technologically feasible C_ts maximizes expression (1). We want to uncover how the optimum C_t varies with η. We would then gain a feel for what are ethically reasonable values of η. For example, if a particular choice of η requires great sacrifices from earlier generations - in the form of very low consumption - in order that later generations will be able enjoy very high consumption, the η in question would not capture the idea of intergenerational equity. It has proved unfruitful to test ethical intuitions in the "integrated assessment models" of climate change in Nordhaus' and Stern's books, because it isn't possible to track what is influencing what in huge computer runs. Simple classroom production models are far better at informing us how η affects the relative ethical merits of alternative consumption paths. And the simplest production structure by far is the pure capital model, in which output is a fixed proportion of wealth. By wealth I mean not only reproducible capital, but also human capital (skills, knowledge, and health) and those types of natural capital whose stocks generate a flow of production services (e.g., ecosystem services). The rate of return on investment is taken to be a positive constant, r.

If our model economy were to enjoy exogenous productivity growth, consumption could be made to increase faster than any constant exponential rate. There is no evidence such patterns of growth has ever been experienced over any extended period of time. So I assume there is no exogenous technological change. In any case, we shouldn't expect exogenous productivity growth in our model: as no capital asset is left out from the production function, accounting for economic growth doesn't leave behind a "residual". If labour productivity rises in our model economy, it is because of investment in various forms of capital.

A more common way to model production is to assume that reproducible capital and labour are imperfect substitutes; and that labour is a fixed factor, enjoying exogenous productivity growth. The problem is that it isn't possible to solve analytically for optimum consumption when the latter isn't very close to its long-run steady state. Mirrlees (1967) studied the sensitivity of optimum consumption to δ and η outside steady state, but he had to take recourse to numerical methods. Moreover, Mirrlees' general findings are not at variance with those I report below. That r is constant in the model I pursue allows me to offer a complete account of optimum consumption. In Sections 4 and 5 we will find that the model offers me an easy route for studying the effect of future uncertainty on today's investment decision.

Following Cline, Nordhaus, and Stern, I suppose that $\eta \geq 1$. Consumption is assumed to take place at the beginning of each period. Writing K_t for wealth at t, the economy's accumulation process can therefore be expressed as

$$K_{t+1} = (K_t - C_t)(1+r), \qquad K_0 \ (> 0) \text{ is given.} \tag{4}$$

In a fully optimum economy, the C_t that society chooses maximizes expression (1), subject to the accumulation equation (4). But infinite sums, as is the case with expression (1), needn't converge. So, we need to identify those specifications of our model economy under which an optimum C_t exists. Now, the parameters that specify the economy we are studying here completely are r, δ, and η. Let us begin by pretending that an optimum C_t exists and determine the condition it must satisfy. A simple argument then shows that the optimum C_t must satisfy,

$$\rho_t = r, \qquad \text{for all } t \geq 0.^{12} \tag{5}$$

In other words, r is the consumption discount rate in a fully optimum economy. I conclude that it is only in a fully optimum economy that the direct opportunity cost of capital ought to be used for discounting future benefits and costs.

[12] Proof: If ρ_t is less than r, society would be advised to save a bit more at t. But to save a bit more at t is to consume a bit less at t, and this tilts consumption more toward the remaining future, which in turn raises ρ_t. Alternatively, if ρ_t exceeds r, society would be well advised to save a bit less at t. But to save a bit less at t is to consume a bit more at t, and that tilts consumption more toward t, which in turn lowers ρ_t. It follows that along the optimum C_t, $\rho_t = r$.

What does an optimum C_t look like? Using equations (3) and (5), we note that C_t grows at the compound rate, g, where

$$C_{t+1}/C_t - 1 = g = [(1+r)/(1+\delta)]^{1/\eta} - 1. \qquad (6)$$

It is possible to show from equation (4) that along the optimum path wealth, K_t, grows at that same rate.[13]

Equation (6) tells us that, along the optimum, consumption growth is positive if r exceeds δ, but is negative if r is less than δ. In other words, optimum consumption grows if Nature's productivity exceeds societal impatience, but declines if r is less than δ. The interesting case is where r exceeds δ.[14] In that case the fully optimum economy grows at a positive rate. Note too that if r and δ are both small, then equation (6) reduces to the approximate equation (3a). In what follows, we assume that $r > \delta$.

A macroeconomic variable for which we all have an intuitive feel is the saving rate. So, let us use equations (4) and (5) to determine the optimum saving rate. Because consumption takes place at the beginning of each period in our model economy, the saving rate at t is aggregate saving at t as a proportion of wealth at t, namely, $(K_t-C_t)/K_t$. And because both C_t and K_t grow at the same rate, the optimum saving rate must be a constant. So, our search for the optimum saving rate boils down to a search for that constant rate of saving that maximizes expression (1). Writing the optimum saving rate as s^*, routine calculations show that,

$$s^* = (1+r)^{-(\eta-1)/\eta}/(1+\delta)^{1/\eta}. \qquad (7)$$

Recall from equation (4) that *net* saving is zero if $s = (1+r)^{-1}$, which implies that C_t is constant if the saving rate equals $(1+r)^{-1}$. Note furthermore that a saving rate of unity is the worst possible consumption programme, because it implies that C_t is zero at all time. We therefore seek to identify conditions under which s^* in expression (7) is not nonsensical (i.e., $s^* < 1$). We have already assumed that $r > \delta$. This means $s^* > (1+r)^{-1}$. We now assume that either (i) $\eta = 1$ and $\delta > 0$, or (ii) $\eta > 1$ and $\delta \geq 0$. In either case, $s^* < 1$, implying that an optimum consumption programme exists. So we have,

Proposition 2. *The optimum saving rate is a decreasing function of η and δ. If, holding δ and r constant, larger and larger values of η are admitted, s* declines to $(1+r)^{-1}$.*

The first part of Proposition 2 explains the sense in which η and δ, are fundamental

[13] For proof, see Dasgupta (2001 [2004]: Appendix).

[14] In Section 3.4 I argue that in a deterministic world δ should be set equal to zero.

ethical parameters. The second part describes a limiting case. Solow (1974) observed that in one interpretation of Rawls (1972), $\eta = \infty$. What Proposition 2 says is that, to assume $\eta = \infty$ is to display infinite inequality aversion.

Citing consumer behaviour (Section 3.4), Nordhaus (2004) and Stern (2006) are in agreement that $\eta = 1$, which, on using equation (7) implies that $s^* = 1/(1+\delta)$. But in that case s^* is independent of r, a fact that should alone set off alarm bells that $\eta = 1$ reflects bad ethics. To see how bad the ethics is, let us follow Stern by setting $\delta = 0.1\%$ a year. Then $s^* = 1/1.001$. Is this large or small? To investigate, notice that because net saving is zero if $s = 1/(1+r)$, we must normalise round that figure. Moreover, the maximum possible rate of saving is 1, which implies that the range of non-negative saving rates is $[(1+r)^{-1}, 1]$. Since the saving-wealth ratio is $(K_t-C_t)/K_t$, its normalised value is $[(K_t-C_t)/K_t-(1+r)^{-1}]/[1-(1+r)^{-1}]$. It is then easy to confirm that the normalised saving-wealth ratio is none other than the more familiar ratio of aggregate saving to aggregate output (rK_t).[15]

Suppose $r = 4\%$ a year. At $\delta = 0.1\%$ a year, the optimum saving-output ratio is approximately 97%. This is an absurdly high saving rate. Never mind that future generations will be vastly richer: the present generation should not object! $\eta = 1$ doesn't reflect much inequality aversion.

If we are to smooth intergenerational consumption, larger values of η have to be admitted. Figures in the range [2,4] suggest themselves. And if we are forced to go empirical on the matter, I can cite Hall (1988), who estimated η to be broadly in the range [2,4] from consumer behaviour in the US. For simplicity of computation, imagine that the unit interval of time is sufficiently small, so that we can assume time to be continuous. It is then easy to confirm that if $\eta = 2$, the optimum saving rate is approximately 37%, and that if $\eta = 3$, it is approximately 25%. These are far more palatable figures.

3.3 Capital Revaluation in the Imperfect Economy

Imagine that because of imperfections in the capital market, the saving rate doesn't equal s^*, but is a constant, s, such that $(1+r)^{-1} < s < s^*$. (The latter inequality implies that the economy

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[15] Proof: Re-write equation (4) as

$$K_{t+1} - K_t = rK_t - (1+r)C_t,$$

which says that a consumption level of C_t at the beginning of the period t is equivalent to the consumption level $(1+r)C_t$ at the end of that period. As aggregate saving from aggregate output at the end of the period is $(rK_t-(1+r)C_t)$, the ratio of saving to output is $(rK_t-(1+r)C_t)/rK_t$, which, as is easily confirmed equals the normalised saving-wealth ratio.

is underinvesting for the future, while the former inequality implies that the economy enjoys growth.) Consumption grows at the rate (s(1+r)-1), as do wealth and output. Let ρ be the consumption rate of discount along that growth path. Using equation (3) leads us to conclude that,

$$1 + \rho = (1+\delta)s^{\eta}(1+r)^{\eta}. \tag{8}$$

Because $s < s^*$, we know from equations (7) and (8) that $\rho < r$.

The question arises as to what rate one ought to use to discount future consumption costs and benefits in investment projects. It is frequently argued that, as r is the productivity of capital, the correct discount rate to use in social cost-benefit analysis is r. To use ρ as the discount rate runs the risk that relatively low-yielding projects will crowd out high-yielding ones, or so the argument continues. The argument is wrong. In the imperfect economy we are studying, r is not the social rate of return on investment. So, investment needs to be revalued in social cost-benefit analysis.[16] Let P_k be the shadow price of capital relative to consumption *numeraire*. P_k is the social opportunity cost of capital: when a unit of capital is invested in a project, P_k is the present discounted value of the flow of displaced consumption. Routine calculations yield,

$$P_k = (1-s)(1+\delta)/[(1+\delta)-(s(1+r))^{1-\eta}].^{17} \tag{9}$$

Note first that $P_k = 1$ if $s = s^*$, which confirms that at a full optimum, consumption and investment are equally valuable at the margin. However, in our imperfect economy, as $s < s^*$, $P_k > 1$. Moreover, from equation (9) we conclude that the smaller is s, the bigger is P_k, other things being equal. So, even though we would use ρ to discount future costs and benefits, a project would have to be high yielding to pass the cost-benefit test. Of course, it may be that the project evaluator chooses investment as *numeraire* (as did Little and Mirrlees, 1969). In that case consumption would have to be revalued at $1/P_k$. Choice of *numeraire* has no bearing on project selection.

3.4 Revealed Preference and Calibration, or, How Should Society Select δ and η?

Because capital is productive, later generations enjoy a natural advantage over earlier

[16] See Marglin (1963) and Dasgupta *et al*. (1972). Among economists writing on climate change, only Cline (1992) has mentioned the need to revalue capital in imperfect economies.

[17] Proof: a marginal additional unit of capital at $t = 0$ yields a small change in consumption, ΔC_t, equal to $(1-s)(s(1+r))^t$. Using equation (3) to identify the consumption discount rate ρ, the present value of that small change, from 0 to ∞, is the expression for P_k. (Note that, because $s > (1+r)^{-1}$, the present value exists.) Equation (9) is due to Marglin (1963).

generations. The expression for s^* (equation (7)) says that if $\delta = 0$ and $\eta < \infty$, the optimum policy for each generation is to save so that future generations can be wealthier. That way, or so the ethical reasoning goes, advantage can be taken of the productivity of capital. The lower is η, the larger is the optimum saving rate. Net positive saving ensures that consumption increases indefinitely, implying that generations in the distant future will be far better off than the those alive now. If this is in conflict with our immediate intuition regarding distributive justice, we have the choice of considering larger values of either δ, or η, or both.

Philosophers have argued that societal impatience is ethically indefensible. They say that to set $\delta > 0$ is to favour policies that discriminate against the utilities of future generations merely on the grounds that they are not present today.[18] They also say that values frequently in use among economists, ranging as they do between 2-3% a year, are way too high.

I find their argument hard to rebut. Admittedly, the ethical axioms Koopmans (1972) imposed on infinite consumption streams implies time discounting, but the axioms don't say how large the discount rate ought to be. Koopmans' axioms are consistent with very, very low values of δ.[19] In contrast, to assume $\delta = 2\%$ a year, as is routinely done in the economics literature, is to say that the well-being of the next generation (35 years down the road) ought to be awarded half the weight we award our own well-being. Justifying that is difficult. But once we accept the philosophers' argument, we are obliged to turn to the second part of Proposition 2, which tells us that η is an index of aversion to consumption inequality. The problem is that we have very little prior understanding of what η implies as regards intergenerational saving. That's why it is legitimate to conduct sensitivity analyses on equation (3) by varying η. Such exercises are thought experiments, and thereby resemble laboratory tests. They give us a sense of how the interplay of facts and values in complicated worlds tells us what we should do. Rawls (1972) called the termination of iterative processes involving such thought experiments, "reflective equilibria".

To illustrate, consider an optimizing society. We know that that the rate of growth of

[18] Ramsey (1928: 261) famously wrote that to discount future well-beings is "ethically indefensible and arises merely from the weakness of the imagination." That is, of course, not an argument; merely an expression of one's beliefs. Broome (1992) contains a summary of the arguments that support Ramsey's position.

[19] Possible extinction of the human race offers a reason for $\delta > 0$, but that is a different reason for positive time discounting. We discuss that in Section 4. We should also bear in mind that infinite-horizon deterministic models are mathematical artifacts: we know Humanity will not survive forever.

optimum consumption, $g(C_t)$, satisfies equation (6), which revealed that δ and η play similar roles in determining the character of the *optimum* C_t: the larger is η or δ (subject to $r > \delta$), the more even is the intergenerational distribution of optimum consumption - which is another way of stating Proposition 2. But the *reasons* δ and η play similar roles should matter; and the reasons differ. As moral philosophers have observed, if we try to achieve greater equality in consumption by increasing δ, we run into a problem of intergenerational inequity. It seems to me we should experiment instead with η.

Even as I compose this paper, I realise that doing welfare economics is a delicate matter. There is a fine dividing line between ethical thinking and authoritarian impulses. It is all well and good for the ethicist to assume the high moral ground and issue instructions like a philosopher-king or a Whitehall Mandarin, but social ethics contains an irremediably democratic element. If others aren't persuaded by the conclusions ethicists have reached, the policies they recommend ought to take those others' ethical viewpoints into account. I personally don't know how to justify $\delta > 0$ in a determimistic world; but if the protagonist for whom I am writing this paper is not persuaded by me, her view should count equally and we should conduct sensitivity tests on δ as well.[20]

Nordhaus (1994, 2007) takes the position that δ and η ought to be calibrated to be consistent with market interest rates, observed values of $g(C_t)$, and rates of private and public saving and investment. This is an interesting, democratic move; but it seems to me there is a problem with the stance when the object of study is climate change. There are two unknowns that Nordhaus must determine (δ and η), but only one equation, $\rho_t = \mathrm{r}$, that relates them. So he is forced to estimate one of the unknowns from other types of data. There is then a problem of consistency in the ways the parameters have been estimated in the different studies. Moreover, while an accumulation process described by our pure capital model may do for exercises in the classroom, climate change under "business as usual" involves a massive global commons problem. For all we know, social rates of return on investment in energy intensive activities are negative today. But the market wouldn't tell us that it is, because private rates would perforce be positive (why else would anyone invest?). There is every reason to believe that observed values of $g(C_t)$ are not the ones society would have chosen had they been able to decide collectively on

[20] In this context, Arrow (1963) can be interpreted as an attempt to discover an aggregator function of individual *ethical* preferences. It isn't an accident that the title of his classic is "Social Choice and Individual *Values*". I have explored that interpretation in Dasgupta (2005b).

the basis of the sort of consideration we have been exploring here. So there is a serious possibility that observed behaviour offers a wrong basis for calibrating δ and η. (I have developed this line of argument further in Dasgupta, 2001, 2007a.)

But in relying exclusively on revealed preference, Nordhaus has been consistent. Cline and Stern would appear not to have bothered at all about consistency. They chose η on the basis of estimates obtained from consumer behaviour, but ignored consumer behaviour entirely when it came to the choice of δ and sought the advice of moral philosophers instead. This is neither good economics nor good philosophy.

3.5 Consumption Smoothing Among Whom?

It can be argued that $\eta = 3$ flies against the face of revealed preference on foreign aid in the contemporary world. Schelling (1999) has very reasonably noted that the rich world's moral posturing over the problem of global climate change doesn't square with its reluctance to increase foreign aid to poor countries beyond the very small proportion of income allocated to it today. If average consumption in the contemporary poor and rich worlds are C_p and C_r, respectively, and N_p and N_r are the sizes of their populations, world well-being today would be $(N_pU(C_p) + N_rU(C_r))$. Now, N_p exceeds N_r (remember, $N_p \approx 3N_r$) and C_r far exceeds C_p (remember, $C_r \approx 20C_p$). Schelling didn't argue that climate change shouldn't be taken seriously, but rather that it would be more equitable and efficient to invest in reproducible and human capital now, so as to build up the productive base of economies - including, especially, poor countries - and divert funds to meet the problems of climate change at a later date, when people are a lot richer.

It seems to me though that there is a reason why people in the rich world could justifiably translate their concerns about equity into doing a lot more for "tomorrow's them" than "today's them". And it has to do with incentives, governance, and responsibility. We contemporaries should be anxious over the plight of future generations caused by climate change because we are collectively responsible for amplifying that change; the rich world especially so. If future generations inherit a hugely damaged Earth, it is we who would be in part responsible. In contrast, it isn't possible to trace the source of absolute poverty in today's poor countries solely to inequities in the global trading system. There are many other reasons why the world's poorest countries continue not to progress. Governance is one of those reasons; and our protagonist, whom I introduced at the beginning of this paper, could be forgiven for maintaining that, while she does join public demonstrations against the inequities of the global trading system, there isn't much she can do about bad governance in other places. Interfering in foreign countries' affairs violates other principles of international justice, such as respecting the autonomy of nations.

Matters are different *within* countries. The rich in Western democracies have been paying a lot more than a mere 2% of their incomes for redistributive purposes. Our protagonist contributes significantly to protect and promote her fellow citizens' well-being. Stern (1976) calibrated η on the basis of income tax rates in the UK when applied to the timeless model of optimum income taxation due to Mirrlees (1971), and arrived at a specification of $\eta = 2$. That said, climate change is predicted to inflict far more damage to the people in the tropics (the poor world) than to the temperate zone (the rich world). Today's rich world, which has been and continues to be the site of the largest emissions of carbon per person, has a particular obligation toward tomorrow's people in today's poor world. Increasing η from 1 to, say, 3 would accentuate that obligation.[21]

I don't believe what I have offered is anything like an air-tight argument. All I have done is to draw attention to ethical principles that create an asymmetry between tomorrow's and today's "them". There is little evidence that a concern for future generations is a case of misplaced ethics.

4. Intergenerational Well-Being: future uncertainty

Yaari (1965) showed that if Humanity is subject to a constant exogenous risk of extinction - say at the hazard rate δ per year - each generation could reasonably pretend that there is no chance of extinction, but discount future utilities at the hazard rate. Stern (2006) has justified the choice of $\delta = 0.1\%$ a year on that very basis.

4.1 Uncertain Constant Growth Rates

Humanity faces many other risks and uncertainties. One particular risk is over future consumption, conditional on Humanity being around. Suppose that intergenerational well-being under uncertainty is the *expected value* of expression (1). Weitzman (2001, 2007a) has modelled that uncertainty by imagining that $g(C_t)$ is an uncertain constant. Let j denote a sample path and g_j the constant growth rate in consumption along that path. Equation (3a) then tells us that, if δ and the g_js are all small, the consumption discount rate along j is $\rho_j = \delta + \eta g_j$.

Assume that there are a finite number of possible g_js. Let π_j be the subjective probability that g_j will prevail. Then π_j is also the subjective probability that ρ_j is the appropriate consumption discount rate. Weitzman (2007a) has shown that society can equivalently pretend

[21] Barrett (2003) contains an interesting discussion of those obligations.

that there is no risk, but use a time varying consumption discount rate α_t, where

$$\alpha_t = - \ln({}_j\Sigma(\pi_j \exp(-\rho_j t)))/t. \tag{10}$$

Equation (11) provides a justification for hyperbolic discounting in a societal context. It implies that the certainty-equivalent consumption discount rate should decline over time from $\alpha_0 = {}_j\Sigma(\pi_j\rho_j)$ to the limit, $\alpha_\infty = \min\{\rho_j\}$.

But there is a problem with this line of reasoning. I know of no reason why we should be required to restrict the state space to constant growth paths. Presumably, future consumption is uncertain because the production process is stochastic. So we should model the stochastic process explicitly. In what follows we study optimum consumption plans when future output is uncertain. The analysis will yield both stochastic and risk-free consumption discount rates along the optimum. As in Section 3.3, our analysis can be extended to imperfect economies.

4.2 Consumption Discount Rates in an Uncertain Production Economy

Levhari and Srinivasan (1969) studied optimum policies in a world where, at each date, r in the pure capital model of Section 3 (equation (4)) is drawn independently from the same probability distribution.[22] Imagine specifically that $(1+r)$ is a random draw from an urn in which, in each period, $\ln(1+r)$ is distributed independently, identically, and normally, with mean μ and variance σ^2. We take it that μ and σ^2 are known.

Let $\bar{r}$ be the expected value of $\tilde{r}$. Assume $\bar{r} > \delta$. Obviously, $\bar{r}$ is a function of μ and σ; as is the variance of $\tilde{r}$.[23] Assume that $\eta \geq 1$. Levhari and Srinivasan showed that the optimum saving rate is

$$s^{**} = (1+r)^{-(\eta-1)/\eta}\exp[(\eta-1)\sigma^2/2]/(1+\delta)^{1/\eta}, \tag{11}$$

provided the parameters μ, σ, η, and δ assume values for which $s^{**} < 1$. For the moment, let us suppose they do.[24] Notice that if $\eta > 1$, uncertainty in future productivity is a reason for saving

[22] The subsequent, asset-pricing literature (e.g., Brock, 1982) has explored models that are more general than the one studied by Levhari and Srinivasan (1969). I use the Levhari-Srinivasan formulation to illustrate my points because of its simplicity and because its findings are directly comparable to those discussed in textbooks on asset pricing (e.g., Cochrane, 2005), where asset prices are taken to be exogenous stochastic variables.

[23] It is easy to show that,

$$1+\bar{r} = \exp(\mu+\sigma^2/2), \tag{F4}$$

and

$$\mathrm{var}(1+\tilde{r}) = \mathrm{var}(\tilde{r}) = (\exp(\sigma^2)-1)\exp(2\mu+\sigma^2). \tag{F5}$$

[24] Using equation (F4) in equation (11), we confirm that $s^{**} = s^*$ (equation (7)) if $\sigma = 0$.

more; but that if $\eta = 1$, uncertainty has no effect on s^{**}. $\eta = 1$ is a bad assumption, no matter how one looks at it.

In the finance literature, the certainty-equivalent consumption discount rate is called the risk-free rate. Let that rate be ρ_t, and let C_t^{**} be the uncertain consumption at t along the optimum path. It can be shown that, provided δ, $\bar{r}$, and σ are small,

$$\rho_t \approx \rho = \delta + \eta E(g(C_t^{**})) - \eta^2 \mathrm{var}(g(C_t^{**}))/2,^{25} \tag{12}$$

where $E(g(C_t^{**}))$ is the expected value of $g(C_t^{**})$ and $\mathrm{var}(g(C_t^{**}))$ is the variance of $g(C_t^{**})$. From equation (11) it is clear that both $E(g(C_t^{**}))$ and $\mathrm{var}(g(C_t^{**}))$ are constants.[26] It follows that the risk-free rate is a constant, not hyperbolic. The third term on the right hand side of equation (12) shows that an increase in uncertainty reduces the consumption discount rate, other things being equal. This feature of ρ_t is related to summary point (b) of Section 3.1: an increase uncertainty raises the downside risk that the economy will hit very low consumption levels in the future.

In Dasgupta (2007b) I argued that Stern and his co-authors ought to have tested the sensitivity of their recommendations to the choice of η on grounds that without running tests, it isn't possible to tell whether η in the range 2-4 would make their integrated assessment model recommend a greater immediate concern for global climate change (i.e., do more now to ease the problem than would be recommended by $\eta = 1$) or a less immediate concern (i.e., do less now to ease the problem than would be recommended by $\eta = 1$). As they did not conduct such a test, it will be instructive to summarise what s^{**} (equation (11)) tells us.

Proposition 3. *η is not only an index of inequality aversion, it is also an index of risk aversion. At the saving rate s^{**}, future generations can be expected to be richer than the present generation. Because of the growth effect, larger values of η recommend earlier generations to save less for the future (the equity motive). However, as future productivity is uncertain, larger values of η recommend earlier generations to save more (the precautionary motive). The combined effect depends on the parameters η, δ, μ, and σ.*

We noted earlier that economists working on climate change have tended to set $\eta = 1$. We

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[25] The proof, which makes use of the assumption that $(1+r)$ is drawn from a lognormal distribution, is similar to the one that was used to arrive at equation (3a). The equation is familiar in the theory of finance (Cochrane, 2005: p. 10). Notice that if $\sigma = 0$, equation (12) reduces to equation (3a).

[26] It is simple to confirm that $E(g(C_t^{**})) = (1+\bar{r})s^{**}$.

found that η = 3 yields more reasonable recommendations about saving rates in the deterministic version of the pure capital model we studied in Section 3.2. Equation (11) says that whether society ought to save more for the future or less if η = 3 than it ought to if η = 1 depends on whether σ^2 is greater or less than $2\ln((1+\bar{r})/(1+\delta))/3$. That μ and σ contribute to the answer in opposition to one another is exactly what intuition should have told us.

It will prove instructive to experiment with values of η higher than 1. As before, assume $\bar{r}$ = 4% a year. Suppose the uncertainty in question is not large. Specifically, let us assume that σ/μ = 1. It follows that the (normalised) saving ratio is 39% if η = 2, and is 29% if η = 3. On comparing these figures with the corresponding numbers we arrived at for σ = 0 (Section 3), we observe that if η = 2, the uncertainty in future productivity of capital is a reason for raising the saving ratio from 37% to 39%; whereas, if η = 3, that same uncertainty is a reason for raising the saving ratio from 25% to 29%. The precautionary motive for saving is non-negligible even if the uncertainty is relatively small.

Interestingly, the precautionary motive increases rapidly with uncertainty. To see this, let us persist with $\bar{r}$ = 4% a year, but suppose now that σ/μ = 2. It is then simple to confirm that if η = 3, the (normalised) saving ratio is 51%, which is a considerable step up from 29%.

5. Large Uncertainties

All that said, we shouldn't believe in any model that explicitly models risk when the horizon extends 100-200 years into the future. We simply don't know what the probabilities are, meaning that "uncertainty" is a lot hazier than "risk".

Applying this belief to the Levhari-Srinivasan model, we should acknowledge that there would be uncertainty over the values of μ and σ. Estimating those parameters poses the problem that we are required to make a forecast of future realizations of $\tilde{r}$ over the indefinite future, but have data on only a finite number of its past realizations. Worse, we will continue to observe only a finite number of realizations. Pesaran *et al*. (2007) and Weitzman (2007a,b) have argued that the probability distributions over the uncertain μ and σ can plausibly have a *thick* lower tail, implying that a long, long run of low realizations of $\tilde{r}$ would not be improbable. In the context of global climate change, this reasoning becomes especially relevant, because we have little-to-no *usable* record from a world where the mean global temperature was, say, 3 degrees Celsius above the current level.

In the Levhari-Srinivasan model, however, μ and σ are assumed to be known. So, with one hand tied behind our back, let us interpret the econometrician's message as being that σ is "large". By assumption, $\ln(1+\tilde{r})$ is normally distributed, but that implies that it is *thin*-tailed. So we are studying a thin-tailed distribution with "large" variance. In Section 4 we supposed that the values of the parameters, δ, η, μ, and σ fall within a range for which s^{**} is less than 1. However, equation (11) says that $s^{**} \geq 1$ if

$$\sigma^2/2 \geq \ln(1+\delta)/\eta(\eta-1) + \ln(1+\bar{r})/\eta, \tag{13}$$

then $s^{**} \geq 1$. As $s^{**} \geq 1$ is nonsensical, we can summarise the finding as

Proposition 4. *If σ satisfies inequality (13), no optimum policy exists*.

How large must the uncertainty be for inequality (13) to be satisfied? Let σ^* be the value of σ at which (13) is an equality; implying that, for values of σ in excess of σ^*, inequality (13) holds strictly. Suppose, as earlier, that δ = 1% a year, η = 3, and $\bar{r}$ = 4% a year. Routine computations show that $\sigma^* \approx 0.17$. Now, when $\bar{r}$ = 4%, the value of μ that corresponds to $\sigma^* \approx 0.17$ is approximately 0.024; which implies a coefficient of variation, σ^*/μ, of approximately 7. This is large, but perhaps not remarkably so. And yet, no optimum policy exists. Suppose $\eta = 2$ instead. We should expect σ^*/μ to be larger than 7. To confirm this, note first $\sigma^* \approx 0.22$. Now, when $\bar{r}$ = 4%, the value of μ that corresponds to $\sigma^* \approx 0.22$ is approximately 0.15; which implies a coefficient of variation, σ^*/μ, of approximately 15.

Proposition 4 holds that if σ satisfies (13), then, for any saving rate, there is a higher saving rate for which the expected value of intergenerational well-being is higher. But at 100% saving rate no one ever consumes anything. We therefore have a contradiction.

Another way to interpret Proposition 4 is to say that if σ satisfies inequality (13), the problem of optimum saving, when formulated in terms of expected well-being over an infinite horizon, is so inadequately posed as to defy an answer. Consumption discount rates cannot be defined and social cost-benefit analysis of projects becomes meaningless. To be sure, for any value of σ, no matter how large, one can always choose η to be sufficiently close to 1 to ensure that inequality (14) does *not* hold. It may explain why Stern (2006) didn't notice that there could be a problem with the existence of an optimum climate policy in the integrated assessment model he and his co-authors worked with. But as values of η close to 1 carry with them serious ethical deficiencies, choosing a figure for η close to 1 would not be a legitimate way out of the dilemma. To do so would be a technical fix, nothing more. So we search for more defendable escape routes from the ethical dilemma.

The integrated assesment models that Nordhaus (1994) and Stern (2006) have worked with consider only a finite number of scenarios, implying that the downside risks associated with climate change are bounded. In the context of our model here, we could ensure the existence of an optimum programme by truncating the normal distribution of $\ln(1+r)$ on the left. But there is no ready recipe for determining where we should perform the truncation.

Another escape route would be to abandon the assumption that $U(C)$ is unbounded below (i.e., $\eta \geq 1$, for very low consumption levels) and assume instead that no matter how greatly the economy were to be hit by bad luck, the loss in well-being people would suffer from is bounded. It isn't clear, however, that such a position is defendable.

So we turn to two assumptions underlying expression (1) that are surely artifacts: the constant hazard rate (δ) for Humanity's extinction and an infinite horizon. One way to ensure that the ethical framework we invoke doesn't have contradictions no matter how high σ is would be to to abandon the infinite time horizon. But the choice of a terminal date would at best be arbitrary. That is why economists have avoided working with finite time horizon models.

Another possible way out would be to continue to postulate an infinite time horizon, but formalise Humanity's extinction process in terms of a hazard rate that increases in an unbounded fashion over time at a sufficiently high rate. The problem is that we have little intuition on how to formulate that in a way that is scientifically reasonable.

6. Avoiding Misplaced Concreteness

The (linear) model economy we have worked with in this paper is of the utmost simplicity. And yet it has yielded several insights that would be of relevance if we were to study the economics of climate change. The concentration of carbon dioxide in the atmosphere is currently 380 p.p.m. (parts per million), a figure which ice cores in Antarctica have revealed to be in excess of the maximum that had been reached during the past 650,000 years. And if there is one truth about Earth we all should be made to memorize, it's that the system is driven by interlocking non-linear processes running at differing speeds. Doing little about climate change would involve Earth crossing an unknown number of tipping points (formally, separatrices) in the global climate system.[27] We have no data on the consequences if Earth were to cross those tipping points. They could be good, on the other hand they could be disastrous. And even if we did have data, they would probably do us little good, because Nature's processes are irreversible.

[27] See Lenton *et al*. (2007).

One implication of Earth system's deep non-linearities is that estimates of climatic parameters based on observations from the recent past are unreliable for making forecasts about the state of the world at concentration levels of 560 p.p.m. or more. The uncertainties are therefore enormous.

Climate change has been taken seriously by all economists who have studied the science since the late 1970s. Even the now-famous "hockey-stick", displayed by time series of carbon concentration in the atmosphere, appeared some time ago (Bolin, 1989: fig. 5). Moreover, the Second Assessment Report (1996) of the Intergovernmental Panel on Climate Change should have made us acknowledge climate change to be one of the most significant environmental issues facing Humanity. To be critical of the economics of climate change that has been on offer over the years is not to understate the harm Humanity is inflicting on itself by degrading the natural environment - not only in regard to the stock of carbon in the atmosphere, but also in regard to so many other environmental matters besides, such as ecosystem losses. But the cause is not served by choosing parameter values that they yield currently desired answers.

In any event, Proposition 4 reveals the limitation of overly formal analyses of the economics of climate change. (We should add to that the economics of biodiversity loss.) Nevertheless, in view of the possibility that advancements in global sequestration technologies and technologies using alternative sources of energy may be prove to be harder to realise than is currently hoped, it is possible to believe that Humanity should invest a lot more in reducing climate change than the mere 2% of the GDP of rich countries proposed by Stern (2006). One can hold such a belief even while being unable to justify it from formal modelling.

Economics helps us to realise what we are able to say about matters that will reveal themselves only in the distant future. Simultaneously, it helps us to realise the limits of what we are able to say. And that too is worth knowing, for limits on what we are able to say are not a reason for inaction. Climate change and biodiversity losses are two phenomena that are probably not amenable to formal, quantitative economic analysis. We economists should have not pressed for what I believe is misplaced concreteness. Certainly, we should not do so now.

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Glossary of Symbols

C_t: aggregate consumption rate at t.

$U(C_t)$: flow of social well-being at t if aggregate consumption is C_t.

δ: Time discount rate. It is often referred to as the "pure rate of time preference", although, as I argue in Section 3.4, this is misleading, because in the case of social choice, δ is a purely ethical parameter.

η ($=-CU''(C)/U'(C)$): elasticity of the flow of marginal social well-being. In the framework being adopted here, it is also the elasticity of the social weight that ought to be awarded to a small increase in aggregate consumption.

ρ: consumption discount rate. It is often called the "social rate of discount".

r: social rate of return on investment in the deterministic model.

P_k: shadow price of capital relative to consumption *numeraire*$_k$ P = 1 in a fully optimum economy.

$\bar{r}$: expected social rate of return on investment in the model with uncertainty, where $\ln(1+\tilde{r})$ is distributed normally with mean μ and variance σ^2.

σ_r^2: variance of the social rate of return on investment, $\tilde{r}$.