Efficiency, Optimality, and Competitive Market Allocations

Pareto Efficient Allocations

- An allocation of resources is *Pareto* efficient if it is not possible to reallocate resources in economy to make one or more persons better off without making at least one other person worse off.

- Three conditions must be met for an allocation to be *Pareto* efficient:
  - Consumption efficiency
  - Production efficiency
  - Product-mix efficiency
Efficiency in Consumption

- Efficiency in consumption requires that the allocation of available goods be distributed so that individuals marginal rates of utility substitution (MRUS\(i\)) are equal:

\[
\text{MRUS}^A = \text{MRUS}^B
\]

where MRUS\(i\) at any point is the slope of an individual’s indifference curve at that point:

\[
\text{MRUS}^i = \frac{\partial U}{\partial X^i} \frac{\partial U}{\partial Y^i}
\]

Efficiency in Consumption

- Move from a to b is a Pareto improvement since individual A is strictly better off and individual B is no worse off.

- Allocation b is also Pareto efficient since no reallocations away from b will improve one individual’s well-being without making other individual worse-off.
Many efficient allocations

- Many different allocations of X and Y consumption levels are allocatively or Pareto efficient.
- Contract curve is locus of Pareto efficient allocations that satisfy the condition for consumption efficiency.

Efficiency in Production

- Three conditions required for efficiency in production:
  1. Efficient input choice for a single firm
  2. Efficient input choice across firms
  3. Efficient output choice across firms
Production Efficiency
1. Efficient input choice for a single firm

- Efficiency in production requires that the available inputs be allocated between the production of two goods X and Y in a firm such that the marginal rates of technical substitution (MRTS) are equalized between goods:

\[ \text{MRTS}_X = \text{MRTS}_Y \]

where MRTS at any point is |slope| of firm’s isoquant map at that point:

\[ \text{MRTS}_X = \frac{\partial \varepsilon / \partial L}{\partial \varepsilon / \partial K} = \frac{MP_L}{MP_K} \]

- Move from a to be is a Pareto improvement because substituting K for L in production of X, holding X constant, leads to a higher level of production of Y.

- Allocation b is Pareto efficient because any reallocation of inputs at this point will increase the production of one good but lead to a reduction in the production of the other good.
Many efficient allocations

- There are many different allocations of K and L that are allocatively efficient. The locus of points shows the family of allocations that satisfy the condition for efficient input choice within a firm.

Production Possibilities Frontier

- PPF is locus of all possible combinations of X and Y that correspond to efficient allocations of inputs.
- \(|\text{slope}|\) of PPF is marginal rate of transformation (MRT) of X for Y.

\[
\text{MRT} = \frac{\text{MC}_X}{\text{MC}_Y}
\]
Production Efficiency

2. Efficient input choice across firms

Efficiency in production requires that input resources be allocated across firms so that marginal physical product (MP) of any input in the production of a particular good is the same no matter which firm produces that good:

\[ MP_L^1 = MP_L^2 \text{ and } MP_K^1 = MP_K^2 \]

Production Efficiency

3. Efficient output choice across firms

Efficiency in production requires that firms producing the same outputs must operate at those points on their respective PPFs at which their marginal rates of product transformation (MRT) are equalized:

\[ MRT^1 = MRT^2 \]
Product-Mix Efficiency

- Product-mix efficiency requires that an allocation of inputs and goods among firms and individuals be such that producers marginal rate of transformation of X for Y is equal to consumers marginal rate of utility substitution of X for Y:

\[ \text{MRUS} = \text{MRT} \]

- Condition ties together preferences and productive capabilities: end goal of production must be satisfaction of individual preferences.

Product-Mix Efficiency

- Simple case of one consumer and one producer

- Allocation b is only allocation which satisfies condition for product-mix efficiency such that the utility of consumer is maximized and resources are fully utilized.
Summary

- Notice that there is an essential similarity in all three conditions for *Pareto* efficiency.

- The study of *Pareto* efficient allocations is a study of marginal trade-offs.

- Only when the trade-off rates are the same for all economic agents will resources be allocated efficiently.

- The “benefits” and “costs” of any reallocation must always be examined.

An efficient allocation is not unique

- There are an indefinitely large number of allocations of X and Y between individuals A and B that satisfy $MRUS^A = MRUS^B$ and there are an indefinitely large number of X and Y output combinations that satisfy $MRTS_X = MRTS_Y$.

- UPF is locus of all possible combinations of $U^A$ and $U^B$ that correspond to efficient allocations (i.e., allocations that satisfy all three conditions for *Pareto* efficiency).
Social Choice

- How should society choose between Pareto efficient allocations?

- Pareto criterion and compensation criterion can only provide partial rankings of efficient allocations and hence cannot provide a basis for choosing which allocation is best from a social point of view.

- Pareto efficient allocations can be ranked using a SWF.

- Pareto optimal allocation is the (unique) Pareto efficient allocation that maximizes social welfare.

Efficiency and Optimality

- Pareto optimum lies on UPF hence all of the necessary conditions for efficiency hold.

- A fourth condition must also hold at Pareto optimum:

\[
\frac{W_A}{W_B} = \frac{U_A^B}{U_B^A} = \frac{U_Y^B}{U_Y^A}
\]

- Slopes of UPF and social indifference curve \(W_1\) must be equal.
Efficiency and Optimality

While allocative efficiency is a necessary condition for optimality, it is not generally true that moving from an inefficient allocation to an efficient allocation improves social welfare. Such a move could result in a lower level of social welfare.

- Allocation b is inefficient while allocation c is efficient.
- But allocation b gives a higher level of social welfare than allocation c.
- The move from b to c is not a Pareto improvement, even though it is a move from an inefficient allocation to an efficient allocation.

Allocation in a Market Economy

- The conditions for Pareto efficiency require that the rate of trade-offs between any 2 goods, X and Y, should be the same for all economic agents.
- In a perfectly competitive economy, the relative price ratio \( P_x/P_y \) between 2 goods provides this common rate of trade-off to which all agents will adjust.
- Since prices are treated as fixed parameters in both individuals’ utility-maximizing decisions and firms’ profit-maximizing decisions, all trade-off rates between X and Y will be equalized to the rate at which X and Y can be traded in the market, that is, \( P_x/P_y \).
- Thus, an efficient allocation will be achieved in a competitive market.
Consumers

- Utility maximization results in choice of consumption bundle \( b = (X^*, Y^*) \).
- At \( b \), budget line is tangent to \( U \), the maximum attainable indifference curve, hence \( P_x/P_y = \text{MRUS}(X^*, Y^*) \).
- Since consumers face the same prices, this condition will hold for individuals A and B:
  \[
  \frac{P_x}{P_y} = \text{MRUS}^A = \text{MRUS}^B
  \]
- Thus, consumption efficiency condition is satisfied in a market economy.

Producers

- In the production of a particular good, say \( X \), a profit-maximizing firm will use inputs, \( L \) and \( K \), up until point at which the marginal value product of each input (MVP) is equal to the competitive input price.
- (3) will hold for any good produced by firm, hence, allocative efficiency in input choice within a firm is attained at a competitive equilibrium.

\[
\begin{align*}
\frac{P_x}{P_L} &= \frac{\partial X}{\partial L} = P_L \quad (1) \\
\frac{P_x}{P_K} &= \frac{\partial X}{\partial K} = P_K \quad (2) \\
\frac{\partial X}{\partial L} &\div \frac{\partial X}{\partial K} = \text{MRTS}_X \quad (3)
\end{align*}
\]
Producers

Since $P_X$ and $P_L$ are given by the market, (4) will hold for each firm producing $X$, hence each firm has same $MP_L$ in the production of $X$.

Condition will hold for all input choices $j$ in production of any good $k$ by all firms at a competitive equilibrium:

$$\frac{P_j}{P_k} = \frac{MP_{j1k}}{MP_{j2k}}$$

Hence, allocative efficiency in input choice across firms is attained at a competitive equilibrium.

Rearranging (1)

$$\frac{P_L}{P_X} = \frac{\partial X}{\partial L} = MP_L$$

Producers

Profit-maximizing firm chooses output levels to equate marginal revenue (output price) to marginal cost of production.

(7) will hold for any firm producing $X$ and $Y$ in a competitive economy.

Since output prices are given, firms MRT are equalized, hence allocative efficiency in output choice across firms is attained at a competitive equilibrium.
Product-mix

- An efficient product-mix will be attained in a competitive economy with utility-maximizing consumers and profit-maximizing firms.

- Since relative price ratio quoted to consumers is the same ratio that market presents to firms, MRUS shared by all individuals will be identical to MRT shared by all firms:

\[
\frac{P_X}{P_Y} = MRUS = MRT
\]

- Conditions holds for any pair of goods, hence an efficient mix of goods will be consumed and produced.

- Thus, a competitive equilibrium is Pareto efficient.

Efficiency, Optimality, and Markets

- The efficient allocation that is achieved by a competitive market may or may not be optimal from a social welfare point of view.

- While the market equilibrium will be on the contract curve, the point on the contract curve that is “chosen” by the price system is necessarily related to the initial endowments with which the individuals start.

- Different initial endowments will result in different points on the contract curve being chosen.
First Welfare Theorem

- The **First Welfare Theorem** states that, under certain conditions, a competitive market economy will attain an efficient allocation of resources.

- In other words, a competitive equilibrium is *Pareto* efficient.

- What conditions must hold to obtain this extraordinary decentralization result?

**Conditions for FWT to hold**

1. Complete set of markets for all goods and services produced and consumed.
2. Large number of participants in all markets.
3. All utility and production functions are well-behaved.
4. All economic agents have perfect information.
5. Private property rights are fully assigned in all resources and commodities.
6. No externalities exist.
7. All goods and services are private goods, that is, there are no public goods.
Welfare Economics and Environmental Economics

- Welfare Economics provides the theoretical starting point (foundation) of Environmental Economics.

- Environmental economists apply the tools and concepts of Welfare Economics to:
  - identify how and why competitive markets fail to allocate environmental resources and services efficiently in a market system
  - design public policies that will correct the source(s) of market failure and improve economic efficiency