Preliminary

PhD Labor Economics
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G31.1701.001 FALL 2001

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II. Assignments

A1. Estimate Mincer's HCEF using CPS data. Graph 'return to education' over time
A2. Estimate a selection-correction model of earnings for married women.
A3. write a program to iterate on $w^\star$ in the basic search model. Confirm $dw^\star/d\beta$ numerically.
A4. Pick a published 'natural experiment' result. Define it as a dynamic experiment.
A5. ??

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IV. Human Capital Interpretation of Wages and Earnings

A man educated at the expense of much labour and time to any of those employments which require extraordinary dexterity and skill, may be compared to one of those expensive machines. The work which he learns to perform, it must be expected, over and above the usual wages of common labour, will replace to him the whole expense of his education, with at least the ordinary profits of an equally valuable capital (Smith, Wealth of Nations, page 101.)

IV.A Becker: The Investment Value of Education

Time and energy is put into creating *human capital*, skills embodied in the person through ability and knowledge. Opportunity costs, time, and patience are important factors in making the decision.

Framework

Benefits of Further Education

- Additional earnings over a career
- Utility from education and higher satisfaction from jobs

Costs of Further Education

- tuition, books, extra expenses not included in tuition
- disutility from schooling
- foregone (delayed) earnings while in school

Simplifying Assumptions

- Utility and disutility from education cancel out
- Hours of work (including work in acquiring education) are fixed (but can differ with educational attainment)
- The income streams associated with different amounts of education are known with certainty
- Individuals can borrow and lend money at the real interest rate r

IV.A.1 Flat earnings; no discounting

A one-time decision is made at age 18 ($t = 1$). The decision is to choose either H (stop at high school) or U. The career lasts until age 65 ($T = 48$). U takes 4 (or E) years and costs \$$c$ each year. Let $W_H(t)$ denote the realized earnings at age t if education stops at high school, and $W_U(t)$ the corresponding earnings for university.

$$\begin{aligned} W_H(t) &\equiv w_H \quad \text{for all} t \\ W_U(t) &\equiv \begin{cases} -c & \text{if } t < 4 \\ w_U & \text{if } t \geq 4 \end{cases} \end{aligned} \tag{1}$$

The value of the choices (H, U)

$$PV_H = \sum_{t=1}^{T} W_H(t) = Tw_H$$
$$PV_U = \sum_{t=1}^{T} W_U(t) = Ec + (T-E)w_U. \tag{2}$$

The decision $d \in \{H, U\}$ is simple:

$$d = \begin{cases} U & \text{if } PV_U > PV_H \\ H & \text{if } PV_U \leq PV_H. \end{cases} \tag{3}$$

We can also write this rule using the value of the difference between the two streams on a year-to-year basis:

$$D(t) \equiv PV_U(t) - PV_H(t)$$
$$PV_D = \sum_{t=1}^{T} D(t) \tag{4}$$

So we can also say

$$d = \begin{cases} U & \text{if } PV_D > 0 \\ H & \text{if } PV_D \leq 0. \end{cases} \tag{5}$$

IV.A.2 Rising earnings

Suppose that $W_H(t)$ and $W_U(t)$ vary with age. Without discounting the value of the two choices.

$$PV_H = \sum_{t=}^{T} W_H(t)$$
$$PV_U = c() + \sum_{t=}^{T} W_U(t) \tag{6}$$
$$D(t) = PV_U(t) - PV_H(t)$$

Decision rule still applies.

IV.A.3 Rising earnings and discounting

$\beta = 1/(1+r)$ equals the yearly discount factor given the interest rate r. For example, one dollar received next year is worth b dollars today. One dollar received in two years is worth β^2 dollars today. Why? One dollar today is worth $(1+r)$ tomorrow. So $1/(1+r)$ dollars today are worth $(1+r)/(1+r) = 1$ dollars tomorrow.

$$PV_H = \sum_{t=1}^{T} \beta^{???} W_H(t)$$
$$PV_U = c() + \sum_{t=1}^{T} \beta W_U(t) \tag{7}$$

Decisions rule (???) still applies

IV.A.4 When to stop school

Framework

- $W(t,e)$: yearly earnings at age t with e years of education.
- $D(a,e) = W(a,e) - W(a,e-1)$: Difference in earnings at age a between e and $e-1$ years of education
- $PV(e,b)$: Present discounted value of net earnings *after leaving* school with e years instead of $e-1$ years (marginal benefit of education).

$$PV(e,\beta) = \beta D(e+6,e) + \beta^2 D(e+7,e) + ... + \beta^{65-e-5} D(65,e). \tag{8}$$

Note: after e years of schooling started at age 6, the person must be 6+e years old; after e-1 years, the person must be e+4 years old.

$$MC(e) = c + W(e+4,e-1).$$

- The internal rate of return for e years of education is the value $I(e)$, such that $\beta^\star = 1/(1+I(e))$ and

$$PV(e, 1/(1+I(e))) = MC(e).$$

Notes

- If $PV(e,b) > MC(e)$ then lifetime net earnings are increased by staying in school another year.
- If we can measure $D(a,e)$ and $MC(e)$, then the decision depends upon the interest rate r that the person uses in the calculation. The greater the value of r the more *impatient* the person is.
- If the person uses $I(e)$ to calculate $PV(e,b)$ they will be indifferent to going to another year of school.
- Given an interest rate r, the value $e^\star$ such that $I(e^\star) = r$ is (under some important conditions) the optimal years of education to earn.
- The IRR for an investment may not exist, and there may be multiple solutions.
- As long as costs are paid up front and benefits remain positive during the life of the investment the IRR is a well-defined concept.

Figure 9.2 Internal rate of return equals market interest rate

IV.B Mincer's Human Capital Earnings Function

Framework

Key Assumptions

- Competitive Wages: wage equal to their value marginal product (VMP)
- Schooling and on-the-job training: A person's VMP depends on their level of human capital, which is acquired through education and OTJ training.

Simplifying Assumptions

- Acquired skills last forever (no depreciation)
- There are constant returns to skill acquisition
- A person has a fixed career length ($t = 0, 1, 2, ..., T$)
- Total time working and engaging in OTJ is constant.

The First Period Working

- p_s The increase in ln(VMP) for an additional year spent in school.
- $W_s = W_0 exp(p_s S)$ potential wage rate just after finishing s years of school (called time 0) **if no OTJ training occurs in time 0**
- k_0 fraction of time spent on the job acquiring skills but not contributing to the firm's product. With compeition firms don't pay workers for the time spent acquiring skills. They only pay for $(1 - k_0)$.
- $w_0 = (1 - k_0)W_s$ observed wage at time 0
- p_j The increase in human capital per unit of time spent doing OTJ

The Second Period Working

- $W_1 = W_s(1 + pk_0)$ potential wage at time 1 **if no OTJ training occurs in time 1**
- k_1 fraction of time spent doing OTJ training in period 1. (We are assuming constant hours on-the-job, but allowing the amount of time working and learning OTJ to vary.)
- $w_1 = (1 - k_1)W_1$: Actual wages at time 1
- It is generally optimal to set $k_1 < k_0$. Why? This implies $w_1 > w_0$

The t^{th} period working

- k_t: *fraction of time spent doing OTJ in period t.*
- $W_t = W_{t-1}(1 + p_j k_t)$: *potential wages at any time t* **if no OTJ occurs in time t**
- $w_t = (1 - k_t)W_t$: *Actual wages at time t*
- *It is optimal to set $k_t < k_{t-1}$. This implies $w_t > w_{t-1}$.*
- *Wages grow with time spent working. For two reasons: (1) an increase in the stock of human capital, which raises wages; (2) a decrease in the amount of investment in further human capital, which raises the amount of productive time spent working.*
- *Note that if x is small number (close to 0) then $ln(1 + x)$ is approximately equal to x*

$$\ln(w_t) = \ln(W_t) + \ln(1 - k_t) = \ln(W_0) + p_s S + p_j(k_0 + k_1 + ... + k_{t-1}) + \ln(1 - k_t) \tag{9}$$

IV.B.1 Mincer's Equation

Supose that training decreases linearly over the labour market career (t=0,1,2,...,T)

$$k_t = k_0(1 - t/T) \tag{10}$$

After some manipulation this leads to

$$ln(w_t) = ln(W_0) + p_s S + (p_j k_0)t - (p_j k_0/2T)t^2 + ln(1 - k_t) \tag{11}$$

The natural logarithm of wages with t years of experience depends on years of schooling, t and t^2 The coefficients on these variables are not items we can directly measure. But this equation suggest that one can use **multiple regression** to estimate these coefficients from data on wages, schooling, and OTJ experience.

For the purposes of regression analysis, this equation is usually re-written in a form called **Mincer's Equation** or the **Human Capital Earnings Function**:

$$ln(w) = \beta_0 + \beta_1 S + \beta_2 X + \beta_3 X^2 + u \tag{12}$$

where

1. $\beta_0 = ln(W_0)$: Earnings of the person if they have acquired no human capital
2. $\beta_1 = p_s > 0$: Return to Education as a human capital investment.
3. $X = t$: Years of labour market eXperience. In many sources of data actual experience (# of years worked in the past) is not directly measured. It is typical to approximate this with ...
4. $X = Age - S - 6$: **potential labour market experience**.
5. $\beta_2 > 0$ and $\beta_3 < 0$: Coefficients on eXperience and eXperience squared. These depend on the return to OTJ training.
6. u: This term captures all other aspects of a persons wage-earning capacity (the residual in a linear regression).

Qualitative Predictions of Mincer's HCEF

- ln($wages$) are linearly related to years of schooling.
- ln($wages$) are concave in experience.

Stated another way: does applying the Mincer Equation to labour market data leads to a valid estimate (from econometrics, an unbiased estimate) of the returns to schooling?

The short but naive answer is *yes*.

Table 9.2 Estimated Returns to Schooling and Experience

Notes

- Why might the age-profile for women be flatter than for men?
- At what point in people's career wages peak?
- Suppose we estimate this equation for another year (1980, say). Where does inflation come into play? (Hint: think about the ln(w) specification.)
- The value of R^2 can be interpreted as saying that about 13 percent of the variation in ln(wages) is explained by education and potential labour market experience using the Mincer equation. The rest of the variation across people is captured by the error term u.. What other factors do you think would help explain differencs in wages across individuals?

These are fairly typical results. In the following sense: simple estimates on the returns to education suggest a coefficient of around 0.07, although in some countries and in other time period the returns are much different

Table 9.1 Estimates of the Private Returns to Schooling in Canada, 1985

How did Vallaincourt calculate these values?

- Estimated several linear regression equations using Cdn data
- Calculated predicted earnings for people by age and education level (and B.A. major)
- Collected information on tuition costs, costs of education, etc.
- Used a spreadsheet program (?) to calculate the internal rate of return

V. Sequential Choice: Basic Job Search

Basic Assumptions

- ▹ a simple *work* or *stay unemployed* decision
- ▹ a decision to work is permanent
- ▹ a known number of periods T to search
- ▹ a constant value of continuing to search
- ▹ a known and constant distribution of wage offers

V.A One Period (T) Decision

A person is unemployed coming into period T. The person values staying at home at some value α. The person has a wage offer in hand, w_T. The choice: stay unemployed (U) or work (W). The values of the two choices are:

$$\begin{aligned} V_T^U &= \alpha \\ V_T^W(w_T) &= w_T. \end{aligned} \tag{13}$$

The optimal choice (or *optimal decision rule*) is: work if $V_T^W(w_T) \geq V_T^U$, or if $w_T \geq \alpha$. This defines the lowest wage offer, the *reservation wage*, that will be accepted by the person in period T:

$$w_T^\star = \alpha. \tag{14}$$

Given the optimal decision rule, we can determine the *indirect value* of of entering period T unemployed with the offer w_T in hand:

$$\begin{aligned} V_T(w_T) &= \max\{V_T^U, V_T^W(w_T)\} \\ &= \max\{\alpha, w_T\} \\ &= \max\{w_T^\star, w_T\} \end{aligned} \tag{15}$$

V.B Two Period (T and T-1) Decision

Enter T-1 unemployed, with wage offer w_{T-1} in hand. If the job offer is accepted, the job lasts today and tomorrow. Can either take the job or stay at home today and search again tomorrow. Tomorrow's wage offers are drawn from the distribution $F(w)$:

$$\begin{aligned} F(w) &= Prob(\text{wage offer} < w) \\ f(w) &= F'(w) = \text{density of wage offers} \end{aligned} \tag{16}$$

Income tomorrow is discounted by the rate β, $0 \le \beta < 1$. An unemployed person makes a decision in period T-1 to accept the job offer or not to maximize discounted expected income.

The values of the parameters $(T, \alpha, \beta, F())$ define the *basic search model*.

Given the distribution of wage offers expected in time T, we can write down the expected income of arriving in period T unemployed:

$$\begin{aligned} EV_T &= \int_0^\infty V_T(w_T) f(w_T) dw_T \\ &= \alpha F(\alpha) + \int_\alpha^\infty w_T f(w_T) dw_T \\ &= w_T^\star F(w_T^\star) + \int_{w_T^\star}^\infty w_T f(w_T) dw_T \end{aligned} \tag{17}$$

The value taking of the two choices at T-1 are then

$$\begin{aligned} V_{T-1}^W(w_{T-1}) &= w_{T-1} + \beta w_{T-1} \\ V_{T-1}^U &= \alpha + \beta EV_T \end{aligned} \tag{18}$$

The optimal decision: work if $V_{T-1}^W(w_{T-1}) \ge V_{T-1}^U$. This defines the lowest wage offer acceptable at time $T-1$:

$$w_{T-1}^\star = \frac{\alpha + \beta EV_T}{1+\beta} \ge w_T^\star \tag{19}$$

This in turn determines the indirect value of

$$\begin{aligned} V_{T-1}(w_{T-1}) &= \max\{V_{T-1}^U, V_{T-1}^W(w_{T-1})\} \\ &= \max\{\alpha + \beta EV_T, (1+\beta) w_{T-1}\} \\ &= (1+\beta)\max\{w_{T-1}^\star, w_{T-1}\} \end{aligned} \tag{20}$$

V.C General Finite Horizon Decision

The person enters an arbitrary period t unemployed with wage offer w_t in hand. If the job offer is accepted, the job lasts through period T. The person can either take the job or stay at home today and search again tomorrow. Tomorrow's wage offers are drawn from the distribution $F(w)$. If we have solved backwards from $t = T$, then the expected (indirect) value of entering period t+1 unemployed is a value EV_{t+1}. This in turn determines the value of the two choices at period t:

$$\begin{aligned} V_t^W(w_t) &= \sum_{k=t}^{T} \beta^{k-t} w_t = w_t \frac{1-\beta^{T-t+1}}{1-\beta} \\ V_t^U &= \alpha + \beta EV_{t+1} \end{aligned} \tag{21}$$

The optimal decision: work if $V_t^W(w_t) \ge V_t^U$. This defines the lowest wage offer acceptable at time t:

$$w_t^\star = (\alpha + \beta EV_{t+1}) \frac{1-\beta}{1-\beta^{T-t+1}} \ge w_{t+1}^\star \tag{22}$$

The indirect value of a wage offer at time t:

$$\begin{aligned} V_t(w_t) &= \max\{V_t^U, V_t^W(w_t)\} \\ &= \max\{\alpha + \beta EV_{t+1}, V_t^W(w_t)\} \\ &= \frac{1-\beta^{T-t+1}}{1-\beta}\max\{w_t^\star, w_t\} \end{aligned} \tag{23}$$

From this we can compute the value of entering period t unemployed:

$$\begin{aligned} EV_t &= \int_0^\infty V_t(w_t)f(w_t)dw_t \\ &= (\alpha+\beta EV_{t+1})F(w_t^\star) + \int_{w_t^\star}^\infty w_t f(w_t)dw_t \\ &= \frac{1-\beta^{T-t+1}}{1-\beta}\Big(w_t^\star F(w_t^\star) + \int_{w_t^\star}^\infty w_t f(w_t)dw_t\Big) \end{aligned} \tag{24}$$

This allows backward recursion to continue for $t-1, t-2, \dots, 1$. Period T fits into this genernal formula if we define $EV_{T+1} = 0$. With that definition the equations $w_t^\star, V_t, EV_t$ can be used for any period t within a finite decision horizon.

V.D Infinite Horizon Decision

If we let T go to infinity then the decision horizon disappears. Today is just like tomorrow, in the sense that tomorrow is no more closer to T than today. The person enters an arbitrary period ('today') with wage offer w in hand. If the job offer is accepted, the job lasts forever. The person can either take the job or stay at home today and search again next period ('tomorrow'). Tomorrow's wage offers are drawn from the distribution $F(w)$. The value of taking a job offer in any period is the present discounted value of the stream of wages:

$$V^W(w) = \sum_{k=t}^{\infty}\beta^{k-t}w_t = \lim_{T\to\infty} w_t\frac{1-\beta^{T-t+1}}{1-\beta} = \frac{w_t}{1-\beta} \quad \text{for} 0 \le \beta < 1 \tag{25}$$

The value of not taking an offer is the expected value of staying at home and then entering tomorrow unemployed:

$$V^U = \alpha + \beta EV \tag{26}$$

The term EV has yet to be defined. In the finite horizon case, it could be determined before the optimal decision rule today. But today and tomorrow are the same, so in the infinite horizon problem the optimal decision rule and the indirect value of unemployment must be determined *simultaneously*. Whatever value EV is, the optimal decision rule is still simple: work if $V^W(w) \ge V^U$. This defines the lowest wage offer acceptable at any time:

$$w^\star = (\alpha + \beta EV)(1-\beta) \tag{27}$$

Given a reservation wage $w^\star$ we could determine the value of unemployment:

$$\begin{aligned} V(w) &= \max\{V^U, V^W(w)\} \\ &= \max\{\alpha+\beta EV, V^W(w)\} \\ &= \frac{1}{1-\beta}\max\{w^\star, w\} \end{aligned} \tag{28}$$

And given the reservation wage we can determine the expected value of unemployment:

$$\begin{aligned} EV &= \int_0^\infty V(w)f(w)dw \\ &= \frac{1}{1-\beta}(w^\star F(w^\star) + \int_{w^\star}^\infty wf(w)dw) \end{aligned} \tag{29}$$

The equations for EV and $w^\star$ form a *system of simultaneous equations* that determine optimal decisions. One way to solve this system of equations is to begin with the period T problem and continue backward until $w_t^\star$ converges. This will work because it is straightforward to show that for $0 \leq \beta < 1$

$$\lim_{T\to\infty} w_1^\star = w^\star. \tag{30}$$

That is, as the horizon disappears the reservation wage converges to the infinite horizon reservation wage.

V.E Properties of the Basic Job Search Model

V.E.1 Response of $w^\star$ to α and β

The basic job search model we have set up has a small number of parameters that determine the optimal process of job search. These parameters are the value of a period spent unemployed (α), the discount factor (β), and the distribution of wage offers ($f(w)$).

It is possible to use integration by parts to write an implicit equation for $w^\star$ that does not depend on EV as well. That is, the reservation wage in the infinite horizon model satisfies:

$$w^\star - \alpha = \beta(E(w) - \alpha) + \beta \int_0^{w^\star} F(z)dz. \tag{31}$$

(This is tedious to show and you are not required to know this equation.) Given this result, we can it to see how the reservation wage responds to the parmaeters of the model.

$$\frac{dw^\star}{d\alpha} - \frac{d\alpha}{d\alpha} = -\beta\frac{d\alpha}{d\alpha} + \beta F(w)\frac{dw^\star}{d\alpha} \tag{32}$$

This is an expression for the *total derivative* of the reservation wage with respect to a change in the parameter α. It is analogous to a *comparative static* exercise in micro theory, in the sense that it measures the response of the optimal decision rules to an exogenous parameter. Solve for the total derivative:

$$\frac{dw^\star}{d\alpha} = \frac{1-\beta}{1-\beta F(w^\star)} \in \{0,1\} \tag{33}$$

That is, the reservation wage goes up with the value of unemployment but not dollar-for-dollar. One way to think of this result is the following: an increase in α makes the person richer. Some of this increase in wealth is consumed in longer unemployment spells, but not all of it. If a job is rejected in order to stay unemployed and enjoy the higher value of α, then it may take several periods to get another acceptable offer. We know that $F(w^\star)$ is the probability of rejecting the next job offer as well.

As an exercise, derive $\frac{dw^\star}{d\beta}$.

V.F Exponential Offer Distribution

Let

$$f(w) = \lambda e^{-\lambda w}, \quad w > 0 \tag{34}$$

which implies:

$$\begin{aligned} F(w) &= 1 - e^{-\lambda w}, \quad w > 0 \\ \int_x^\infty w f(w) dw &= (1 - F(x)) [x + \frac{1}{\lambda}]. \end{aligned} \tag{35}$$

So $\Theta = (\alpha, \beta, \lambda)$ defines the basic, infinite-horizon, exponential-offer-distribution, search model. For this distribution,

$$EV = V^U(1 - e^{-\lambda w^\star}) + \frac{1}{1-\beta} \int_{w^\star}^\infty w \lambda e^{-\lambda w} dw \tag{36}$$

We can get a system of two equations that defines the model:

$$EV = \frac{w^\star}{1-\beta} \left(1 - e^{-\lambda w^\star}\right) + \frac{1}{1-\beta} \left[w^\star + \frac{1}{\lambda}\right] e^{-\lambda w^\star} \tag{37}$$

$$w^\star = (1-\beta)[\alpha + \beta EV] \tag{38}$$

As an exercise, derive $\frac{dw^\star}{d\lambda}$. It may simplify things to re-write the model as $\gamma = 1/\lambda$ and solve for $\frac{dw^\star}{d\gamma}$ first. Further, find the expressions for $\frac{dw^\star}{d\alpha}$ and $\frac{dw^\star}{d\beta}$ specific to this problem. If we put the total derivatives in a vector, we have:

$$\frac{dw^\star}{d\Theta} = \begin{pmatrix} \frac{dw^\star}{d\alpha} & \frac{dw^\star}{d\beta} & \frac{dw^\star}{d\lambda} \end{pmatrix} \tag{39}$$

VI. The Search Model and Estimation

VI.A The Search Model Can Produce a Linear Regression

The basic statistical model for earnings paid to individuals is the linear regression model in log-linear form:

$$\ln W = x\beta_x + u \tag{40}$$

Here x is a row vector of observed individual characteristics that are expected to influence earnings; β_x is a column vector of unknown parameters; and the scalar u is the difference between the expected log earnings offer $E[\ln W|x\beta_x] = x\beta_x$ and the actual value. It captures unobserved influences on a person's earnings. With $u \sim N(0, \sigma^2)$, the linear regression model with normally distributed errors is defined by $\Theta^r = (x, \beta_x, \sigma^2)$.

How does (40) relate to the infinite horizon search model (37)-(38) with parameters $\Theta = (\infty, \alpha, \beta, F())$? First, (40) introduces exogenous explanatory variables, x. If we set $\alpha = \beta = 0$, then the reservation wage becomes 0, and the searcher accepts any job offer. If we further set $lnw \sim N(x\beta_x, \sigma^2)$ we see that the regression model is a special case of the job search model, in which log wage offers are normally distributed with a constant variance σ^2 and an individual specific mean $x\beta_x$. That is, $\Theta^r(x, \beta_x, \sigma^2) = \Theta(\infty, 0, 0, N(x\beta_x, \sigma^2)) \equiv \Theta^\star$.

VI.B But It's Unlikely

The OLS estimate of $\hat{\beta}_x$ is $(X'X)^{-1}X'Y$, where X is the matrix of individual observations x and Y is the vector of observations on $\ln W$. Under the classical (Gauss-Markov) assumptions, $E[\hat{\beta}_x] = \beta$ and $Var[\hat{\beta}_x] = \sigma^2(X'X)^{-1}$. Suppose that one estimates (40) using OLS but the data were generated not by $\Theta^\star$ but by the more general model $\Theta = (\infty, \alpha, 0, N(x\beta_x, \sigma^2))$? That is, people still have no foresight but they do have an alternative to working with a value grater than 0.

What would the data look like? Let's start with the case that we get to observe x for a random sample, but that we don't observe earnings for people who rejected the wage offer. Then all observed earnings would be greater than α. The residual u in (40) would no longer be have mean 0 but rather

$$E[u|x] = \sigma E[z|z > (\alpha - x\beta_x)/\sigma], \tag{41}$$

where $z \sim N(0, 1)$. It follows that the error term would be correlated with x. That violates the key assumption of the Gauss-Markov Theorem. Thus, OLS estimates of (40) applied to the static search model ($\beta = 0$) results in biased estimates of β_x:

$$E[\hat{\beta}_x|\alpha > 0] \neq \beta_x.$$

This is an example of a *selection bias* in OLS estimates of a regression equation.

VI.C Instead it Produces a Likelihood Function

VI.C.1 Probit for Working

Under the normality assumption it turns out that

$$E[u|x] = \sigma E[z|z > d] = \sigma \frac{\phi(d)}{1 - \Phi(d)}, \tag{42}$$

where ϕ and Φ are the standard normal density and distribution function, respectively. Further, under search model the chances that a person chooses to work is $1 - F(\alpha) = 1 - \sigma\Phi(\frac{\alpha - x\beta_x}{\sigma})$. We see that the argument is exactly d.

Let's begin again with a model that says that the probability a person is employed is $1 - \Phi(-x\gamma_x)$ and not employed $\Phi(-x\gamma_x)$. Code a new variable m to equal 1 for people working and $m = 0$ for those not working (created from the earnings data: $m = W > 0$). Then the probability of a given observation is

$$Prob(m|x, \gamma_x) = [1 - \Phi(-x\gamma_x)]^m [\Phi(-x\gamma_x)]^{1-m}. \tag{43}$$

The likelihood of the observation is this probability given the data:

$$L(\gamma_x|m, x) = Prob(m|x, \gamma_x) \tag{44}$$

The log-likelihood of a random sample of searchers:

$$\ln L(\gamma_x) = \sum_{i=1}^{N} \ln L(\gamma_x|m_i, x_i). \tag{45}$$

This function is globally concave in γ_x. The parameter vector that maximizes the sample likelihood are called *maximum likelihood estimates*:

$$\hat{\gamma}_x^{MLE} \equiv \arg\max_{\gamma_x} \ln L(\gamma_x). \tag{46}$$

MLE estimates of γ_x are consistent. The search model says that $\gamma_x = \beta_x/\sigma$ except for the constant term, which would be $\gamma_x[1] = (\beta_x[1] - \alpha)/\sigma$. The parameter σ of the search model is not separately identified from the likelihood function for m alone.

We can generalize. Suppose that rather than having a common opportunity cost α, it is

$$\alpha = z\beta_z$$

where z is a vector of observed characteristics of the person (which may or may not overlap with x). Now the search model is $\Theta = (\infty, z\beta_z, 0, N(x\beta_x, \sigma^2))$. The working proability becomes $z\beta_z - x\beta_x$ and the *reduced-form* probability for m is

$$Prob(m|x, z, \beta_x, \beta_z) = [1 - \Phi(z\beta_z - x\beta_x)]^m [\Phi(z\beta_z - x\beta_x)]^{1-m}. \tag{47}$$

Based on this probability, we can't distinguish the homo- and heterogeneous opportunity cost search models, except by sorting the demographic variables into x and z *a priori*.

VI.C.2 Aside: The Heckman-Lee Two-Step Procedure

Given consistent estimates of the model's probability of working, we could correct the earnings regression for selection bias. First, compute an auxillary random variable

$$\hat{\lambda}([zx]) = \frac{\phi\left(z\hat{\beta}_z - x\hat{\beta}_x\right)}{1 - \Phi(\left(z\hat{\beta}_z - x\hat{\beta}_x\right))} \tag{48}$$

Then run the second-stage regress

$$\ln W = x\beta^\star_x + \gamma_\lambda \hat{\lambda}([zx]) + u_\lambda. \tag{49}$$

The OLS estimate of $\beta^\star$ is a consistent estimate of β.

The canonical form for the selection model is

$$\text{observation:} Q = (y, m, x_1, x_2)$$

$$y^\star = x_1\beta_1 + \epsilon_1 \tag{50}$$

$$m^\star = x_2\beta_2 + \epsilon_2 \tag{51}$$

$$m = m^\star > 0 \tag{52}$$

$$y = y^\star m \tag{53}$$

$$\begin{pmatrix}\epsilon_1\\ \epsilon_2\end{pmatrix} \sim N\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma^2 & \rho\sigma\\ \rho\sigma & 1\end{pmatrix}\right) \tag{54}$$

When compared to the search model, it is important to note that $m^\star$ is the difference between the offered wage and the reservation wage and its mean and variance have been normalized to 0 and 1, respectively. The full parameter vector is

$$\theta = \begin{pmatrix}\beta_1\\ \beta_2\\ \sigma\\ \rho\end{pmatrix}. \tag{55}$$

A truncated observation ($m = y = 0$) occurs with probability $f(0,0) = P(m^\star < 0) = \Phi(-x_2\beta_2)$. The joint probability of a selected observation can be written $f(y,1) = f_y(y)f_m(1|y)$ where

$$f_y(y) = \frac{1}{\sigma}\phi((y - x_1\beta_1)/\sigma)$$

$$f_m(1|y) = \int_{-x_2\beta_2}^{\infty} f(\epsilon_2|y)d\epsilon_2$$

$$= 1 - \Phi\left(\frac{-x_2\beta_2 - \rho(y - x_1\beta_1)/\sigma}{\sqrt{1-\rho^2}}\right)$$

The full log-likelihood of an observation given estimates $\hat{\theta}$ is

$$\ln L(\hat{\theta}|Q) = (1-m)\ln\Phi(-x_2\hat{\beta}_2) + m\left[-\ln\hat{\sigma} - \frac{(y - x_1\hat{\beta}_1)^2}{2\hat{\sigma}^2} + \ln\left(1 - \Phi\left(\frac{-x_2\hat{\beta}_2 - \hat{\rho}(y - x_1\hat{\beta}_1)/\sigma}{\sqrt{1-\hat{\rho}^2}}\right)\right)\right] \tag{56}$$

VI.C.3 The Unrestricted Search Model

What happens when $\beta \neq 0$? Now the reservation wage $w^\star$ is influenced by both opportunity costs and the wage offer distribution. Estimating the reduced-form probit requires iteration. Since we are not assuming that offers follow the log-normal distribution the iteration includes the evaluation of an improper integral in (29). It is important to note that the searcher cares about the level of wages, so $f(w) \neq \phi(w)$ in (29). Instead, $f(w)$ is the density of the log-normal distribution.

If we ignore an important technical point, the likelihood function for a single observation is:

$$Prob(m, \ln W; \Theta) = \left[\Phi\left((w^\star(\Theta) - \beta_x x)/\sigma\right)\right]^{1-m} \left[\phi\left((\ln W - \beta_x x)/\sigma\right)/\sigma\right]^m. \tag{57}$$

The demographic variables x and z do not enter this expression symmetrically. The z variables only shift the reservation wage (the lower-bound) of observed data, whereas variables in x also shifts the distribution of accepted earnings above the reservation wage.

VII. Signaling

VII.A Spence: Education as a Signal

In the HC model, spending time in school is thought of as an investment in a form of capital, skills learned while in school. This interpretation, however, is not the only one. Another possibility is that time spent in school does not **add** to a person's skills, but merely **signals** those skills to prospective employers.

Framework

Key Assumptions

- People differ *intrinsically* in their productivity while working.
- Employers cannot directly observe a worker's productivity
- Productivity is related to the cost/difficulty person has in acquiring education.

Simplifying Assumptions

- Two types of workers:

  low ability (type L) and high ability (type H).
- The proportion of type H in the population is q.
- Type H have marginal productivity P^h; type L have P^l
- Marginal cost of acquiring another year of education is constant: c^h, c^l
- Firms compete for workers, leading to zero economic profit in equilibrium.

Equilibrium Conditions

1. Employers form **beliefs** about the relationship between (unobserved) productivity and (observed) educational attainment.
2. They pay workers according those beliefs (making wages depend on education).
3. The beliefs of employers are fulfilled by the choices of people acting in the own best interests.

Two Types of Equilibria

1. Case 1: No signalling (*pooling equilibrium*). If workers cannot or do not signal their type and firms cannot learn it from watching the person work, then everyone will be paid the same wage. The zero-profit condition requires then that firms pay workers their average producitivity:

$$W^P = qP^h + (1-q)P^l \tag{58}$$

2. Case 2: Signalling (*separating equilibrium*). Beliefs of employers:

$$\begin{cases} MP = P^l & \text{if } e < e^\star \\ MP = P^h & \text{if } e \geq e^\star \end{cases} \tag{59}$$

Wage Offers

$$W(e) = \begin{cases} P^l & \text{if } e < e^\star \\ P^h & \text{if } e \geq e^\star \end{cases} \tag{60}$$

In equilibrium, $e^\star$ must be such that high quality workers find in worthwhile acquiring $e^\star$:

$$P^h - P^l \geq c^h e^\star \tag{61}$$

low quality workers do not find it worthwhile.

$$P^h - P^l < c^l e^\star \tag{62}$$

Bounds on the equilibrium signal:

$$(P^h - P^l)/c^l < e^\star \leq (P^h - P^l)/c^h. \tag{63}$$

Notes

- Imagine two worlds (economies), one in the signalling equilibrium and one in the pooling equilibrium. Who gains from signalling? Who loses?
- How good an explanation is signalling for the earnings difference across education groups at older ages?
- Are there ways to distinguish between signalling and human capital explanations?
- Are there other situations in the labour market where signalling may play a role?