The OLS estimate of $\beta^\star$ is a consistent estimate of β.

The canonical form for the selection model is

$$\text{observation:} Q = (y, m, x_1, x_2)$$

$$y^\star = x_1\beta_1 + \epsilon_1 \tag{11}$$

$$m^\star = x_2\beta_2 + \epsilon_2 \tag{12}$$

$$m = m^\star > 0 \tag{13}$$

$$y = y^\star m \tag{14}$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \rho\sigma \\ \rho\sigma & 1 \end{pmatrix} \right) \tag{15}$$

When compared to the search model, it is important to note that $m^\star$ is the difference between the offered wage and the reservation wage and its mean and variance have been normalized to 0 and 1, respectively. The full parameter vector is

$$\theta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \sigma \\ \rho \end{pmatrix}. \tag{16}$$

A truncated observation $(m = y = 0)$ occurs with probability $f(0,0) = P(m^\star < 0) = \Phi(-x_2\beta_2)$. The joint probability of a selected observation can be written $f(y,1) = f_y(y)f_m(1|y)$ where

$$\begin{aligned} f_y(y) &= \frac{1}{\sigma}\phi((y - x_1\beta_1)/\sigma) \\ f_m(1|y) &= \int_{-x_2\beta_2}^{\infty} f(\epsilon_2|y)d\epsilon_2 \\ &= 1 - \Phi\left(\frac{-x_2\beta_2 - \rho(y - x_1\beta_1)/\sigma}{\sqrt{1-\rho^2}} \right) \end{aligned}$$

The full log-likelihood of an observation given estimates $\hat{\theta}$ is

$$\begin{aligned} \ln L(\hat{\theta}|Q) =& (1-m)\ln\Phi(-x_2\hat{\beta}_2) \\ &+ m\left[-\ln\hat{\sigma} - \frac{(y - x_1\hat{\beta}_1)^2}{2\hat{\sigma}^2} + \ln\left(1 - \Phi\left(\frac{-x_2\hat{\beta}_2 - \hat{\rho}(y - x_1\hat{\beta}_1)/\sigma}{\sqrt{1-\hat{\rho}^2}} \right) \right) \right] \end{aligned} \tag{17}$$

II.3.c The Unrestricted Search Model