An Estimable Dynamic General Equilibrium Model of Work, Schooling and Occupational Choice *

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Abstract

This paper develops and estimates a dynamic general equilibrium overlapping generations model of career decisions. The model is fit to data on life cycle employment, schooling and occupation decisions, and on life cycle labor earnings, within and between cohorts observed in the US between 1968 and 1993.

Based on the estimates of the model, the impact of cohort size on skill prices and, consequently, on career decisions is assessed. Compared to a baseline case in which cohort size increased at a steady (average) rate and the fertility process was stationary, it is found that the male baby bust generations born in the 1930s and 1940s faced higher skill prices (by as much as 2.0%), completed college at a higher rate (by as much as 0.3 percentage points) and worked more over the lifetime (by as much as 0.1 years). In contrast, the males from the baby boom generations born in the 1950s and 1960s faced lower skill prices (by as much as 1.5%), completed college at a lower rate (by as much as 1.0 percentage points) and worked less over the lifetime (by as much as 0.1 years). The impact of cohort size is found to differ across gender due to the differential impact of fertility on the value of home production: unlike males, the female baby bust generations completed college at a lower rate (by as much as 1.5 percentage points) and worked less over the lifetime (by as much as 0.5 years) while the female baby boom generations completed college at a lower rate (by as much as 1.5 percentage points) and worked more over the lifetime (by as much as 0.2 years).

A tuition subsidy experiment is also performed both in a general equilibrium setting in which skill prices are endogenous and in a partial equilibrium setting with fixed skill prices. Based on the estimated model, a 1% increase in tuition is predicted to reduce college enrollment rates by 1.27% in partial equilibrium, consistent with previous estimates, and by 1.05% in the general equilibrium case. Thus, for this policy experiment, the partial equilibrium analysis of schooling choice would not differ significantly from the general equilibrium analysis.

Keywords: career decision, general equilibrium skill price, cohort size, tuition subsidy

JEL Classification: I21, J24, J31

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1 Introduction

This paper develops and estimates a dynamic general equilibrium overlapping generations model of career decisions. The model is fit to data on life cycle employment, schooling, and occupation decisions, and on life cycle labor earnings, within and between cohorts observed in the US between 1968 and 1993. The purpose of this paper is: (1) to extend previous partial equilibrium models of human capital accumulation to a general equilibrium setting; (2) to use the model's estimates to determine the impact of cohort size on human capital investment behavior and labor market outcomes; and (3) to contrast the estimates of a college tuition subsidy on career decisions in partial equilibrium and general equilibrium settings.

Figure 1 shows the actual size of birth cohorts in the US from 1910 to 2000 and a hypothetical cohort trend, the dotted line, over the same period. The fluctuation in actual cohort sizes around this trend is large; the size of the baby boom generation is above the trend-line by as much as 27% (in 1957) and the size of baby bust generation is below by as much as 25% (in 1933). The model permits an evaluation of the impact of cohort size on the skill prices faced by birth cohorts over their lifetimes and, consequently, on their career decisions.

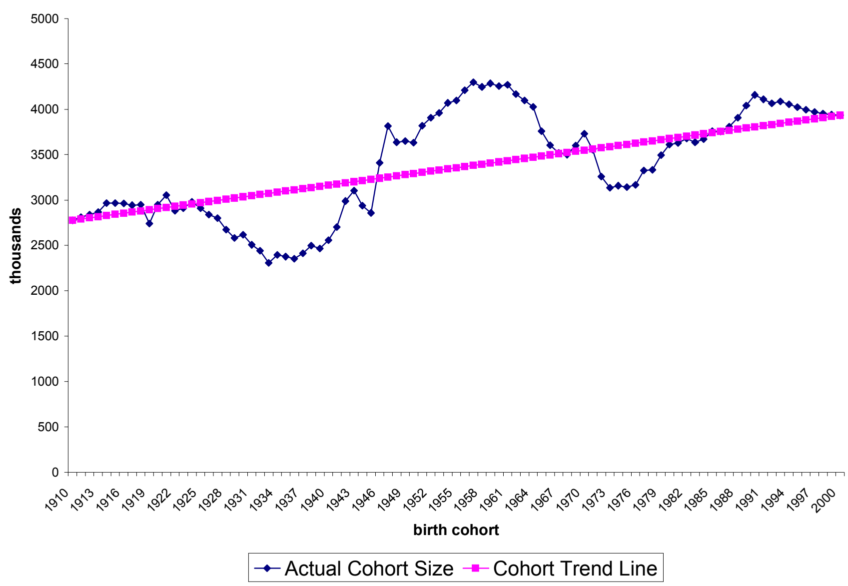

Figure 1: US. birth cohort

Based on the estimates of the model, the impact of cohort size on skill prices and, consequently, on career decisions is assessed. Compared to a baseline case in which cohort size increased at a steady (average) rate and the fertility process was stationary, it is found that the male baby bust generations born in the 1930s and 1940s faced higher skill prices (by as much as 2.0%), completed college at a higher rate (by as much as 0.3 percentage points) and worked more over the lifetime (by as much as 0.1 years). In contrast, the males from the baby boom generations born in the 1950s and 1960s faced lower skill prices (by as much as 1.5%), completed college at a lower rate (by as much as 1.0 percentage points) and worked less over the lifetime (by as much as 0.1 years). The impact of cohort size is found to differ across gender due to the differential impact of fertility on the value of home production: unlike males, the female baby bust generations completed college at a lower rate (by as much as 1.5 percentage points) and worked less over the lifetime (by as much as 0.5 years) while like males, the female baby boom generations completed college at a lower rate (by as much as 1.5 percentage points) but worked more over the lifetime (by as much as 0.2 years).

A tuition subsidy experiment is also performed both in the general equilibrium setting in which skill prices are endogenous and the partial equilibrium setting with fixed skill prices. Based on the estimated model, a 1% increase in college tuition would reduce college enrollment rates by 1.27% in partial equilibrium and by 1.05% in the general equilibrium case. Thus, for this policy experiment, the partial equilibrium effect would not differ significantly from the general equilibrium effect.

In the model developed here, in each year between the ages of 16 and 65, people make a decision based on current and future white- and blue-collar skill prices, their initial skill endowments at age 16 and current payoffs associated with each alternative about whether or not to (1) work in a white-collar occupation, (2) work in a blue-collar occupation, (3) attend school or (4) stay home. Working in a particular occupation provides labor earnings in the current period, which is the product of the occupation's market skill price in that period and the person's skill level in that occupation. Working in either occupation also increases skill levels in future periods through learning by doing, potentially in both occupations. Attending school provides direct consumption value, but entails the payment of a tuition cost in the case of post-secondary education. Schooling also increases skill levels, differentially in the two occupations, and thus future earnings. Staying home, neither working nor attending school, provides utility, with no future payoff. Decisions are made by comparing the sum of current and discounted expected future utilities associated with the four mutually exclusive alternatives and choosing the best one.

Skill prices are determined endogenously in the market. The aggregate supply of white-collar skill in a period is the sum of white-collar skill supplied by individuals who choose the white-collar employment alternative in that period. The aggregate demand for white-collar skill is given by the marginal product of aggregate white-collar skill derived from the aggregate production function. The aggregate supply of and demand for blue-collar skill is similarly given. Aggregate output is produced with homogenous capital and the two aggregate skills according to a constant returns to scale Cobb-Douglas specification. Capital and skill shares are estimated each year from actual data. To obtain the skill supplies, the dynamic programming problem for the decision problem described above is solved assuming that individuals have perfect foresight about future skill prices, but imperfect foresight about future shocks associated with the utility of each of the alternatives. The path of equilibrium skill rental prices for each occupation is determined by equating aggregate skill supplies with aggregate skill demands in each period. Given that cohort size varies over time, equilibrium skill prices are not stationary.

To place the contribution of this paper in context, I provide a brief review of the relevant literature. Ben-Porath (1967) first provided a formal optimization framework to describe the optimal path of investments in human capital. Human capital is produced according to a technology that combines a person's time and market goods. Ben-Porath showed that maximizing wealth would lead optimally to early specialization in schooling and declining human capital investment with age. He interpreted labor earnings as the product of the amount of human capital and its per-unit rental price, which is given by:

$$\log(w_t) = \log(r_t H_t) = \log r_t + \log H_t \tag{1}$$

where r_t is the competitively determined human capital (or skill) rental price at time t and H_t is the amount of human capital possessed by an individual at t. Writing the human capital production function as an exponential in schooling (E) , work experience (X) and a human capital shock (ε) yields a log wage function that takes the conventional form of the Mincer (1974) wage function,

$$\begin{aligned} \log(w_t) &= \log(r_t H_t) = \log r_t + \log H_t \\ &= \log r_t + \alpha_0 + \alpha_1 E_t + \alpha_2 x_t + \alpha_3 x_t^2 + \varepsilon_t \end{aligned} \tag{2}$$

Heckman and Sedlacek (1985) and Willis (1986) extended this interpretation of the wage equation to a multi-dimensional skill framework, in Heckman and Sedlacek to industrial sectors and in Willis to occupations. Both of these papers adopt choice models in the spirit of Roy's (1951) self selection model. Willis and Rosen (1979) and Heckman and Sedlacek (1985) found that individuals exhibit comparative advantages in skills. Keane and Wolpin (1997) extended these models to a dynamic setting taking into account the fact that human capital investments, such as schooling and on-the-job experience, augment future wages. Flinn (1993) first explicitly considered an overlapping generations framework with cohort size variation in which skill prices were determined in equilibrium.

This paper is most closely related to Flinn and to Keane and Wolpin, and I therefore discuss them in more detail. Then, I compare the framework in this paper to that adopted in Heckman, Lochner and Taber (1997), which is the first, and only other, dynamic general equilibrium model of human capital investment that has been estimated.

Flinn developed an equilibrium model of the schooling decision and evaluated the effect of cohort size on schooling and earnings. In his model, people live 5 periods.[1] They can attend school any fraction of the first period, but once they leave school they can not reenter. As soon as they complete schooling, they work until the end of the last period. A cohort is defined to be the number of people born in a 10 year period. By attending school, one's human capital increases with diminishing returns and schooling is the only way to increase human capital. When they work, their earnings are determined by the product of the skill price and the amount of human capital. The skill price is determined endogenously in the market which, in this model, is simply given by the inverse of aggregate human capital supply.[2] Flinn calibrated the model using the actual 10-year-period cohort sizes from 1880 to 2050. He found that a 1% increase in the size of a cohort decreases the average school attainment of that cohort and of preceding cohorts by as much as 0.08% and increases average school attainment of future cohorts by as much as 0.04%.

I extend Flinn's equilibrium schooling choice model in a number of important directions. Individuals not only decide on whether to attend school, but also whether to work and in which

[1]In Flinn, these five periods represented the age intervals 16 to 25, 26 to 35, 36 to 45,46 to 55 and 56 to 65.

[2]The production function is the log of aggregate human capital. Physical capital is not explicitly included in the production function.

occupation. Also, in Flinn, heterogeneity arises only across cohorts; people in one cohort choose the same level of education and thus work the same number of periods and earn the same amount over their lifetime. In the present model, individuals differ in their initial endowments of occupation-specific skill and alternative-specific utilities and are subject to random shocks in each period that induce people within the same cohort to follow different career paths, making different schooling, work and occupational decisions. For the analysis of a cohort size effect, I also pay attention to the fact that the size of a cohort should necessarily be related to the fertility process of its parents: an increase in cohort size affects the career decisions of its own and neighboring cohorts by (1) increasing skill supplies in the future and (2) affecting its parent' fertility. I found that for male cohorts the first effect is more important than the second, but that the opposite is true for female cohorts.

Keane and Wolpin (1997) developed and structurally estimated a discrete-choice dynamic programming model to explain the career decisions of a single young male cohort. In their model, skill prices are assumed to be constant over time, an outcome of the present model if the capital stock changes at the same rate as population, the age distribution of the population is the same over time and initial schooling levels are the same across cohorts. Based on the estimates in their model, Keane and Wolpin evaluated the partial equilibrium effect of a college tuition subsidy on college enrollment rates under fixed skill prices.

I extend their model in two ways. First, skill prices are allowed to change over time as the result of cohort size variation, capital growth and technical change. Second, skill prices are endogenously determined in the model in general equilibrium. The importance of these extensions stems from the fact that, in general, the partial equilibrium effect of a given policy intervention or exogenous change in the economic environment with constant skill prices will differ from the general equilibrium effect with endogenous skill prices. For example, consider evaluating the effect of a college tuition subsidy. The first-order effect is to directly increase the enrollment rate in colleges. However, the second-order effect will be to increase the aggregate skill supply in at least one occupation in the future and, thus, to decrease the equilibrium skill price in that sector. This feedback effect mitigates the incentive to attend college. Therefore, the partial equilibrium effect of a tuition subsidy will exceed the general equilibrium effect. A quantitative assessment of the difference requires empirically implementing the general equilibrium model. As noted above, based on the estimates of the model, the effect of the subsidy based on a partial equilibrium analysis overstates the effect that takes into account general equilibrium feedbacks by only about 10%. Thus, in this case, it turns out that the Keane and Wolpin partial equilibrium estimate is not very misleading.

Heckman, Lochner and Taber (1997) was the first paper to empirically implement a general equilibrium model of human capital investment. In their model, skills are defined by education level: those with high school and those with college. This approach differs importantly from that in Keane and Wolpin, and thus from our model, where skill types are defined by occupation. To further describe their model, at the beginning of the life cycle, people choose the level of education (high school or college) taking into account the schooling-specific skill prices that will prevail over their lifetimes, their ability type and individual shocks to the value of schooling. Schooling choices differ across cohorts, as in the present model, because skill prices vary over time due to changing cohort sizes and technical change. After completing school, individuals work in the sector specified by their education level and make life cycle savings decisions (capital is endogenously determined in the model). In addition, rather than assuming that work experience augments skills through learning by doing, while working individuals decide on the proportion of time to spend on investments in on-the-job skill acquisition. Given on-the-job investment, the wage is given by the product of the skill rental price and the amount of skill devoted to actual production (the proportion of time not devoted to on-the-job investment times the amount of skill). Income is subject to a constant marginal tax rate. There is no option to remain out of the labor force or to reenter school.

Heckman, Lochner and Taber perform a similar comparison of partial and general equilibrium effects of college tuition changes on schooling. Contrary to the finding in this paper, they find that the partial equilibrium effect, which is of similar magnitude to that found in this paper and in the preceding non-structural literature, is almost completely negated by the effect of the aggregate skill-supply induced changes in skill rental prices; the general equilibrium effect is estimated to be close to zero. Potential explanations for the marked difference in this result are discussed in a later section.

The rest of the paper is organized as follows. Section 2 presents the structure of the general equilibrium model. The components of the model include the specification of the aggregate technology and the preferences and constraints, of the individuals in the economy. Equilibrium is defined and it is demonstrated how equilibrium skill prices are determined in the skill market. Section 3 discusses the method adopted for solving the general equilibrium model. Section 4 describes the data used for the estimation and the next section presents and discusses the estimation results. Section 6 and 7 are devoted to the policy experiments of evaluating the effects of cohort size and of a college tuition subsidy. Section 8 discusses how much the increase in female employment between 1968 and 1993 is attributable to the decrease in the fertility rate and the increase in the capital stock. Section 9 concludes this paper.

2 The Model

The economy starts at some initial point in time ($t = 1$). In each period, the labor market consists of overlapping generations with many people aged 16 to 65, with the number of people of a given age reflecting their initial cohort size. At $t = 1$, each cohort's schooling distribution, white-collar work experience distribution and blue-collar work experience distribution are taken as exogenous initial conditions. From $t = 2$ forward, those distributions are endogenously determined as a result of people's optimizing behavior, although each cohort's initial conditions at age 16 are taken as exogenous. The model's description begins with the specification of the aggregate technology.

2.1 Aggregate production function

Aggregate production is given by a Cobb-Douglas, constant returns to scale, production function,

$$Y_t = A_t S_{1t}^{\alpha_{1t}} S_{2t}^{\alpha_{2t}} K_t^{1-\alpha_{1t}-\alpha_{2t}} \tag{3}$$

where Y is output, S_1 is the aggregate amount of white-collar skill used in production, S_2 is the aggregate amount of blue-collar skill used in production and K is the capital stock in the economy.[3] Given that skill units cannot be measured and that the output shares, α_1 and α_2, are allowed to vary over time, the Hicks-neutral technology parameter, A_t, is normalized to unity in all periods. The capital stock is assumed to evolve exogenously. Notice that the above Cobb-Douglas production function specification with changing output shares can accommodate the usual finding that capital is

[3]As Hammermesh (1986) reports, evidence on the degree of substitution in production between white- and blue-collar labor is contradictory, with estimates ranging between the extremes of perfect substitutability to fixed-coefficients.

more complementary to skilled than to unskilled labor (capital-skill complementarity). With capital exogenously increasing over time, capital-skill complementarity is represented by an increasing white-collar skill output share, α_{1t}, and a decreasing blue-collar skill output share, α_{2t}.

The aggregate stock of white-collar skill employed at period t, $S_1^s(t)$, is the sum of white-collar skill units over each cohort and each individual in the economy at t.

$$S_1^s(t) = \sum_a \sum_i s_{1i}(a) d_{1i}(a) \tag{4}$$

where $d_{i1}(a)$ is an indicator variable taking on the value of one if an individual who is age a at calendar time t chooses to work in a white-collar occupation, and is zero otherwise. The aggregate stock of blue-collar skill at period t, $S_2^s(t)$ is similarly defined.

The labor market is assumed to be competitive, specifically, the white collar skill price at t, r_{1t}, is given by the marginal product of white-collar skill;

$$r_{1t} = \alpha_{1t} S_{1t}^{\alpha_{1t}-1} S_{2t}^{\alpha_{2t}} K_t^{1-\alpha_{1t}-\alpha_{2t}} = \frac{\alpha_{1t} Y_t}{S_{1t}} \tag{5}$$

The white-collar skill demand function is thus

$$S_1^d(t) = \frac{\alpha_{1t} Y_t}{r_{1t}} \tag{6}$$

The blue-collar skill supply, $S_2^s(t)$, and demand, $S_2^d(t)$, are similarly defined, namely

$$\begin{aligned} S_2^s(t) &= \sum_a \sum_i s_{2i}(a) d_{2i}(a) \\ S_2^d(t) &= \frac{\alpha_{2t} Y_t}{r_{2t}} \end{aligned} \tag{7}$$

where r_{2t} is the blue-collar skill price at t and $d_{2i}(a)$ is an indicator variable of choosing to work in a blue-collar occupation.

2.2 Individual utility maximization

People enter the labor market at age 16 and retire between the ages of 60 and 65. The exact retirement age is taken to be exogenous.[4]

[4] It is assumed that a constant fraction of people retire between ages 60 and 64 and the rest retire at age 65. The fraction of people retiring between 60 and 64 is estimated as a parameter of the model.

At age 16, each agent is endowed with an initial education level and zero years of work experience in both occupations. It is assumed that individuals have perfect foresight about future skill prices. Taking the sequence of skill prices that an individual will face over the lifetime as given, each person maximizes the expected present value of remaining lifetime utility by choosing at each age a one of the four mutually exclusive career alternatives: (1) work in a white-collar occupation, (2) work in a blue-collar occupation, (3) attend school or (4) remain at home.

The problem at age a is,

$$\max_{d_m(a)} E\left[\sum_{\tau=a}^{A} \delta^{\tau-a} u(a) | S(a)\right] \tag{8}$$
$$u(a) = \sum u_m(a)\, d_m(a)$$

where $S(a)$ is the vector of state variables at age a, $u_m(a)$ is the utility from choosing an alternative m at age a ($m = 1, 2, 3, 4$ as ordered above), $d_m(a) = 1$ if career alternative m is chosen at age a, and equals zero otherwise, A is the stochastic retirement age and δ is the discount factor and $u(a) = u_m(a)$ if alternative m is chosen. After retirement, utility is assumed to be zero. The alternative-specific utility functions are specified as follows:

Work in a white-collar occupation ($m = 1$) or a blue-collar occupation ($m = 2$):

$$\begin{aligned} u_{mt}(a) &= r_{mt} s_m(a) + \alpha_{m7} \\ &= r_{mt} \exp(\alpha_{m1} + \alpha_{m2} E(a) + \alpha_{m3} x_1(a) + \alpha_{m4} x_2(a) + \alpha_{m5} x_1^2(a) \\ &\quad + \alpha_{m6} x_2^2(a) + \varepsilon_m) + \alpha_{m7} \end{aligned} \tag{9}$$

where r_{mt} is the equilibrium skill price at calendar time t in occupation m, $s_m(a)$ is the amount of occupation-specific skill possessed at age a, α_{m7} is the non-pecuniary benefit of working in occupation m, E is the education level, x_m is the years of work experience in occupation m and ε_m is an idiosyncratic shock to skill in occupation m in that period. Notice that u_m is a function of calendar time because the skill price, r_m, changes over time. Choosing this alternative provides current earnings and increases work experience in that occupation by one year.

Attend school ($m = 3$):

$$u_{3t}(a) = \alpha_{31} - \alpha_{32} I(d_3(a-1) = 0) - tc_1\; I(E(a) >= 12) - tc_2 I(E(a) >= 16) + \varepsilon_3 \tag{10}$$

where α_{31} is interpreted as the consumption value of attending school, α_{32} is a school reentry cost that arises if the individual did not attend school in the previous period, $(d_3(a-1)=0)$, tc_1 is a college tuition cost, tc_2 is the additional tuition cost for attending graduate school and ε_3 is a random shock attached to the consumption value of schooling. Attending school increases education by one year in the next period, that is $E(a+1) = E(a) + 1$.

Remain home and engage in home production $(m = 4)$:

$$u_{4t}(a) = \alpha_{41} + \alpha_{42} NC(a) + \varepsilon_4 \tag{11}$$

where α_{41} is the consumption value of staying home, $NC(a)$ is the number of preschool children, α_{42} is the increase in the value of home production per preschool child and ε_4 is a random shock attached to the consumption value of staying home. The number of preschool children is assumed to follow a known exogenous stochastic fertility process that depends on an individual's cohort, sex, current age and schooling level.[5]

In order to solve the individual's optimization problem, it is necessary to specify the joint distribution of the four shocks, the two skill shocks and the two preference shocks. They are assumed to be distributed joint normal with mean 0 and given (age and time-invariant) variance-covariance matrix.

2.3 Skill market equilibrium

An equilibrium skill price series $\{r_t^*\}_{t=1}^T = \{r_{1t}^*, r_{2t}^*\}_{t=1}^T$ is a rational expectation equilibrium that satisfies the following two conditions.

1. People make their decisions based on the skill price, $\{r_t^*\}_{t=1}^T$
2. $\{r_t^*\}$ clears the skill market every period.

$$S^s(t) = S^d(t) \text{ for all } t \tag{12}$$

where $S^s(t) = \{S_1^s(t), S_2^s(t)\}$ and $S^d(t) = \{S_1^d(t), S_2^d(t)\}$.

[5]Preschool children are defined to be children under age 6, the age at which children normally start primary school. For simplicity, we require that $NC(a)$ takes a value from 0 to 2. We assume that $NC(a)$ follows a Markovian process from the individual's perspective. The actual number of preschool children an individual will have at any age is thus stochastic. The $NC(a)$ process is estimated directly from the data and employed in solving the model. Since the fertility process also depends on education level, individuals can exercise some control over their fertility by choosing their education level.

Assuming a rational expectations equilibrium requires that individuals have perfect foresight about future rental prices. In order that individuals be able to obtain such a forecast requires knowledge of the future sequences of the capital stock, aggregate technology parameters and cohort sizes. Cohort size, like the fertility process, is exogenously determined outside the model.[6] Appendix A provides a proof of existence both in the case of finite T and in the case where T is infinite.

3 Model solution

We divide the model solution into two parts. The first part demonstrates the solution method used for the individual's decision problem for a given skill price sequence. The second part demonstrates the method for solving for the equilibrium skill price sequence that clears the skill market in every period.

3.1 Solution of the individual career decision problem

The individual's optimization problem is solved recursively from the final age A. Let $S(a)$ be the vector of state variables at age a, variables known at age a that determine the remaining expected present value of lifetime utility. Given the structure of the model, the state space at any age a includes the current and future equilibrium skill rental prices up to age 65, current levels of school attainment, white- and blue-collar work experience, preschool children, white- and blue-collar skill shocks and school attendance and home preference shocks, and lagged school attendance, that is,

$$S(a) = \{E(a), x_1(a), x_2(a), d_3(a-1), NC(a), \varepsilon(a), r(a)\}$$

where $r(a)$ represents the current and future equilibrium skill rental prices up to age A and $\varepsilon(a)$ is a vector of alternative-specific random shocks at age a. Let $V(S(a))$, the value function at age

[6] In the model, both mortality and migration are ignored. Since relative cohort size matters for the analysis, not the absolute level, as long as mortality and migration don't change the relative size of each cohort, the use of birth cohort alone as the measure of cohort size can be justified. The data show that even though the proportion of immigrants in the US population increased dramatically between 1960 and 1995 for those around age 30 from less than 5% to more than 15%, this increase evenly affected wide ranges of age groups at the same time. The proportion of immigrants across ages does not vary much in any one year. For example, in 1960, the proportion of immigrants was least at age 20, 2%, and greatest at age 50, 7%. In 1995, it was least at age 50, 12%, and greatest at age 29, 18%. (between ages 20 and 50).

a given state vector $S(a)$, denote the maximal value at age a over all the possible career decisions given $S(a)$. Similarly, define $V_m(S(a))$ to be the alternative-specific value function at age a, the expected present value of remaining lifetime utility given that alternative m is chosen. Therefore,

$$V\left(S(a)\right) = \max_{m}\left[V_m\left(S\left(a\right)\right)\right] \tag{13}$$

Notice that calculating $V(S(a))$ is equivalent to solving for the optimal sequence of career decisions. The decision rule is to choose the alternative with the highest alternative-specific value function at each age.

To see how the solution proceeds, define $\text{Emax}(S(a))$ to be the expected value of $V(S(a))$ over the stochastic shocks attached to each alternative,

$$\begin{aligned}\text{Emax}(S(a)) &= E\left[V(S(a))\right] \\ &= \int_{-\infty}^{+\infty} \max_{m}\left[V_m(S(a))\right] \mathrm{dF}(\varepsilon)\end{aligned} \tag{14}$$

and $S_m(a+1)$ to be the state vector at age $a+1$ when alternative m is chosen at age a given $S(a)$. Then, with

$$S(a) = \{E, x_1, x_2, d_3(a-1), NC(a), \varepsilon, r(a)\}$$

the $S_m(a+1)$'s are

$$\begin{aligned}S_1(a+1) &= \{E\ ,\ x_1+1,\ x_2\ ,0,\ NC(a+1),\ \varepsilon(a+1), r\left(a+1\right)\} \\ S_2(a+1) &= \{E\ ,\ x_1\ ,\ x_2+1, 0,\ NC(a+1),\ \varepsilon(a+1), r\left(a+1\right)\} \\ S_3(a+1) &= \{E+1,\ x_1\ ,\ x_2\ ,1,\ NC(a+1),\ \varepsilon(a+1), r\left(a+1\right)\} \\ S_4(a+1) &= \{E\ ,\ x_1\ ,\ x_2\ ,0,\ NC(a+1),\ \varepsilon(a+1), r\left(a+1\right)\}\end{aligned}$$

where $\varepsilon(a+1)$ is vector of individual shocks at age $a+1$. Choosing alternative 1 (2), working in a white- (blue-) collar occupation, increases white- (blue-) collar work experience by one year. Choosing alternative 3, attending school, increases school attainment by one year, while choosing alternative 4, remaining at home, leaves the state space unaltered.

From these definitions, it is easily seen that Emax($S(a)$) is recursively determined.

$$\begin{aligned}\text{Emax}(S(a)) &= E\left[V(S(a))\right] \\ &= E\left[\max_m \left(V_m(S(a))\right)\right] \\ &= E\left[\max_m \left(u_m + \delta \text{Emax}\left(S_m(a+1)\right)\right)\right] \end{aligned} \tag{15}$$

Now, in order to solve for a person's decision rule at age 16, we need to know Emax($S(a)$) for all ages from 17 to 65 and at every $S(a)$ that can be reached from $S(16)$. To do that, begin at age 65 and calculate the Emax function for every possible state vector, S(65). Because this is the last period, the alternative-specific value function, $V_m(S(65))$ is simply the current utility for each alternative.

$$V_m(S(65)) = u_m(S\,(65))$$

and

$$\text{Emax}\,(S(65)) = E\left[\max_m \left(u_m\,(S\,(65))\right)\right] \tag{16}$$

After calculating the Emax function at age 65, go back to age 64 and calculate Emax($S(64)$) for every possible $S(64)$,

$$\begin{aligned}&\text{Emax}(S(64)) \\ =\;& E[\max_m\,[V_m\,(S\,(64))]] \\ =\;& E\left[\max_m\,[u_m + \delta\text{Emax}(S_m(65))]\right]\end{aligned} \tag{17}$$

This backward solution for the Emax functions is repeated until age 17. The decision at age 16, then, involves choosing the maximum of $V_m(S(16))$ where

$$\begin{aligned}&V_m\,(S(16)) \\ =\;& u_m\,(16) + \delta\text{Emax}\,(S_m(17))\end{aligned} \tag{18}$$

and $m = 1, 2, 3, 4$.

There are two computational difficulties involved in calculating the sequence of Emax functions. First, calculating the Emax($S(a)$) function for any given value of the state space involves a four dimensional integration with respect to the ε vector. This calculation is performed by Monte

Carlo integration, i.e. for each draw of the ε vector from the joint distribution, $\max_m[V_m(S(a))]$ is obtained and the sample mean is used as a numerical approximation of $\text{Emax}(S(a))$.

The second complication is the "curse of dimensionality" that arises because the number of elements in the state space is too large to be feasible in the context of solving the general equilibrium model and estimating the parameters of the model, both of which are iterative procedures. Recall that the Emax functions must be obtained for every element in the state vector. This computational problem is circumvented by calculating $\text{Emax}(S(a))$ for a subset of state space points and interpolating the non-simulated state space points by regression (see Keane and Wolpin (1994))[7].

For the interpolation, the $\text{Emax}(S(a))$ regressions are specified as second-order polynomial functions in the state variables, namely[8]

$$\begin{aligned}
& \text{Emax}\left(S\left(a\right)\right) \\
= \; & \text{Emax}(E, x_1, x_2, d_3(a-1), NC) \\
= \; & \exp(\beta_0 + \beta_1 E + \beta_2 x_1 + \beta_3 x_2 + \beta_4 d_3(a-1) + \beta_5 NC + \beta_6 r_{1t} + \beta_7 r_{2t} + \text{square and cross terms}) \qquad (19)
\end{aligned}$$

Given the sequence of Emax functions, the choices that individuals make over their life cycles are determined by their initial conditions at age 16 and the sequence of shocks that are drawn. At age 16, an individual receives a set of skill and preference shocks, chooses from among the four alternatives the one with the highest alternative-specific value function and updates the state space given that choice. A new set of draws is received at age 17, the optimal choice is made, the state space is updated and the process is repeated at ages 18, 19, etc. until retirement.

3.2 Solution method for equilibrium skill price $\{r_t^*\}$

Because it is assumed that there are many, possibly an infinite number of, people in the population of the economy, the individual shocks attached to career choices average out. Given the assumption that there are no shocks to aggregate production, there is no uncertainty at the aggregate economy level. Therefore, the equilibrium skill price sequence actually follows a deterministic path, despite the existence of individual stochastic shocks. Each person in each cohort solves the decision problem

[7] In the actual simulation, 300 sate points are drawn for each age and each period for the Emax function interpolation and 200 random shocks are drawn for each Monte Carlo integration of Emax function.

[8] Increasing the number of state points to 1,000 only slightly affects the simulated individual decisions. Increasing the number of Monte Carlo simulation to 500 only slightly affects the Emax function values.

under the deterministic path of skill prices. However, the skill price series each cohort faces is different. Therefore, it is necessary to solve for the optimal career decision rule for each cohort, i.e., for the cohort-specific set of Emax functions. The degree of computational burden is, thus, dramatically greater with varying skill prices compared to a model with constant skill prices (as in Keane and Wolpin (1997)).

It is possible to solve for optimal career decisions for each cohort and, consequently, for the occupation-specific supply of skills for each cohort at each age, given any skill price sequence, $\{r_t\} = \{r_{1t}, r_{2t}\}$, on and off the equilibrium path. To do that, I simulate N people for each cohort and calculate the white- and blue-collar individual skill supplies, $s_{mi}(t,a)d_{mi}(a)$ for $i = 1, ..., N$ and $m = 1, 2$. Since cohort size differs by cohort, to calculate the aggregate white- and blue-collar skill supply, $S_m^s(t)$, we give each person a weight proportional to his cohort size, $C(t,a)$. So the aggregate white- and blue-collar skill supply at t given skill price series, $\{r_t, r_{t+1}, \ldots\}$ is

$$S_m^s(t) = \sum_a \sum_i C(t,a) \frac{s_{mi}(t,a)d_{mi}(a)}{N} \tag{20}$$

To obtain the rational expectations equilibrium, we need to find r such that

$$S^s(t) = S^d(t) \text{ for all } t$$

The method we use to obtain the rational expectations equilibrium, $\{r_t^*\}$, is to start from a constant skill price and update it until the skill price sequence converges. In the first iteration, we solve the individual career decision problem at period t assuming that future skill prices will be the same as current ones, that is, we start from a constant skill price sequence, namely.

$$\{r_t\}_{t=1}^T : r_1 = r_2 = ... = r_t = r_{t+1} = r_{T-1} = r_T$$

and calculate the aggregate skill supply, $S^s(1)$, and skill demand, $S^d(1)$ respectively. The equilibrium at $t = 1$, r_1, is determined so that the skill market at $t = 1$ clears.

$$S^s(1) = S^d(1)$$

and we denote this equilibrium by $r_t^{(1)}$, where the superscript (1) denotes the iteration number. We go to the next period, $t = 2$, and keep the restriction of the constant skill price from that period on.

$$r_2 = r_3 = r_4 = \ldots$$

As above, $r_2^{(1)}$ is determined in the same way to satisfy

$$S^s(2) = S^d(2)$$

Notice that in general, $r_2^{(1)}$ is not the same as $r_1^{(1)}$, which violates the expectation of the constant skill price. We continue this process to the final period, T, to obtain the first iteration equilibrium, $\{r_t^{(1)}\}_{t=1}^T$. Notice that $\{r_t^{(1)}\}$ is an equilibrium; however, it is not the rational expectations equilibrium, because $\{r_t^{(1)}\}$ is not constant over time as was assumed in solving for the two aggregate skill supplies. The rational expectation equilibrium, $\{r_t^*\}$ is one which clears the skill market and, moreover, the one that people use to make their career decisions. Therefore, finding the rational expectations equilibrium, $\{r_t^*\}$ is equivalent to finding the fixed point skill price sequence in the model.

When we obtain the first iteration equilibrium, $\{r^{(1)}\}$, we go to the second iteration. Now we assume the ratio of skill price series is the one obtained from the first iteration;

$$\frac{r_{t+1}}{r_t} = \frac{r_{t+1}^{(1)}}{r_t^{(1)}} \text{ , } \forall t \tag{21}$$

where $\{r_t^{(1)}\}$ is obtained from the first iteration.[9] This relationship yields a sequence of skill rental prices that can be written solely in terms of r_1. The value of r_1 is determined by equating skill supply and demand in period 1. Then a new r_2 is calculated by writing all of the skill rental prices from $t = 2$ on as a function of r_2 and the new value of r_2 is determined by equating skill supply and demand in period 2. This procedure is repeated to the final period T and yields a second iteration for the sequence of skill prices that clears the skill market in each period, denoted by $\{r^{(2)}\}$. This process is continued, obtaining repeated iterations of the skill price sequences, $\{r^{(3)}\}, \{r^{(4)}\}, \ldots$, until a sequence $\{r^{(n)}\}$ is close enough to $\{r^{(n-1)}\}$ under some criterion. The converged skill price sequence $\{r^*\}$ is the rational expectations equilibrium, where people's expectation about future skill prices is realized as the equilibrium skill price sequence.[10]

[9] In other words, we restrict the differences in $\{\log r_t\}$ to be the same as $\left\{\log r_t^{(1)}\right\}$.

[10] Simulations based on wide range of parameter values show that this method is very quick to find the rational expectations equilibrium in a robust way.

4 Data

To estimate the model, ideally we would need longitudinal data on individuals from many overlapping generations where employment, schooling and occupational decisions and labor earnings were reported from age 16 until retirement. In that case, all of the state variables and choices could be calculated from histories of school attendance and occupational-specific employment and earnings. Based on this data, it would be possible to calculate the likelihood of observing the life cycle choices and labor earnings of individuals in the sample data. Then, the parameters could be estimated by maximizing the sample likelihood. Unfortunately, such extensive data do not exist.

The analysis in this paper is based on 26 years of the March Current Population Survey (CPS), from 1968 to 1993.[11] The CPS is a repeated cross-sectional sample of about 60,000 US households. Schooling and employment information is obtained about all household members. Although the CPS has a short panel feature, it is primarily useful as a single year cross-section. It is not possible to use the CPS to calculate the state variables necessary to calculate the likelihood for the choices made at the individual level. However, one can combine the set of CPS's to calculate the choice distributions for a large number of cohorts over significant portions of their lifetimes. Indeed, the data from the CPS spans individuals who are between the ages of 16 and 65 from cohorts born as early as 1903 and as late as 1977.

The model is estimated by matching sample moments in the series of CPS's related to career choices classified by individual characteristics, age, cohort, sex, education level and whether a preschool child is present in the household, to the moments that are predicted through simulation by the model. Estimation is based on the simulated method of moments (SMM). The parameters of the model are estimated to minimize the weighted average distance between the sample moments and the simulated moments. The exact moments used in the estimation procedure are: the proportion who work in the white-collar (blue-collar) occupation, the proportion attending school, the proportion at home, the proportion with completed schooling classified into four education categories, the mean and standard deviation of white-collar log earnings, the mean and standard

[11] The March CPS is actually available prior to 1968 and after 1993. However, between 1964 and 1967, the CPS asked "How much did you receive in wages or salary?" and after 1967, "How much did you receive in wages or salary before any deductions?" It seems that it recorded after-tax earnings between 1964 and 1967, and before-tax earnings after 1967. After 1993, the variable indicating whether a person currently attended school was dropped from the March CPS. For these reasons, we don't make use of the data before 1968 and after 1993.

Table 1: Aggregate Moments

aggregate moment	number of conditional moments
white-collar employment rate	26×50×2×4×2
blue-collar employment rate	26×50×2×4×2
attending school rate	26×50×2×4×2
staying home rate	26×50×2×4×2
mean white-collar labor earnings	26×50×2×4
mean blue-collar labor earnings	26×50×2×4
sd. white-collar labor earnings	26×50×2×4
sd. blue-collar labor earnings	26×50×2×4
one-period occupation transition rate	2×26×50×2
schooling distribution	26×50×2×4

1. The first four choice moments are conditioned on year (26), age (50), sex (2), education level (4) and whether one has a preschool child or not (2).
2. one-period occupation transition rate (blue-collar to white-collar & white-collar to blue-collar) is conditioned on year (26),age (50) and sex(2)
3. The rest are conditioned on year (26), age (50), sex (2) and education level (4).

deviation of blue-collar log earnings, the one-period transition rate between occupations (the fraction choosing to work in a white- (blue-) collar occupation at t conditional on choosing to work in a blue- (white-) collar occupation at $t-1$).[12] Education is divided into 4 categories: less than high school (0 - 11 years of education), high school graduate (12 years), some college (13 - 15 years) and college graduate (16 years or more). The distribution of completed schooling is conditioned on cohort, age and sex. Table 1 provides a complete list of the moments used in estimation.

The model assumes that schooling and employment choices are mutually exclusive. However, within a year individuals may be engaged in several of these activities. For example, some people attend school and work at the same time and some people work only half of the year but stay home for the rest. To assign each person to one of the four mutually exclusive and exhaustive alternatives in each year, the following hierarchical rule is used:

1. An individual is assumed to have attended school in the year if they reported that schooling was their major activity.[13]

[12] The CPS provides information about the occupation that one worked in for both the previous and current year of the survey.

[13] The survey question asks "What were you doing most of last week?"
1. Working
2. With a job but not at work
3. Looking for work

2. The work alternative is assigned to those not in school, who reported that they worked at least 39 weeks and at least 20 hours per week in the previous year. The assignment is for the year prior to the survey year. When the individual is working, an occupational assignment either to white collar or blue collar is then made.[14]

3. Individuals are classified as having been at home if they were neither working nor in school.

4.1 Descriptive statistics

Figure 2 shows the choice distribution by age averaged over the period from 1968 to 1993 for males and figure 3 for females. Although these tables do not represent the choice distribution of any particular cohort, the overall age pattern does hold for given cohorts. Figures 4 and 5 present the choice distribution for two cohorts: the cohort born in 1928 and the cohort born in 1952. The 1928 cohort was 16 to 43 years old and the 1952 cohort was 40 to 65 years old over the 1968 to 1993 period. As seen in the figures, the propensity to work increases steadily until the mid 40s and decreases after that for males. Female employment, however, increases until around age 25, then slightly decreases until the mid-30's, coinciding with an increase in the fertility rate, and increases until the mid 40s and then, like males, declines. Blue-collar employment increases a few years before white-collar employment increases, reflecting the relationship between schooling and occupational choice. School attendance is highly concentrated early in the life cycle. At age 16, the average (over the period) attendance rate is almost 90% for both males and females. By age 18, the normal high school graduation age, the attendance rate falls to 60% for males and to 55% for females, and by age 22, the normal college graduation age, it is less than 20%. After age 30, very few people attend school.

Figures 6 and 7 show the choice distribution (averaged over all ages) by year. The main changes between 1968 and 1993 were the decrease in blue-collar employment among males, from 44% to 34%, the increase in female white-collar employment, from 22% to 37%,[15] and the small increase

4. Keeping house
5. Going to school
6. Unable to work
7. Retired
8. Other(Specify)

[14] White-collar workers are professional, managerial, technical, sales and clerical workers and blue-collar workers are craftsmen, operatives, laborers, service workers and farm workers.

[15] This is also well represented in figure 5 where only 24% of the 1928 female cohort worked in white-collar occupation

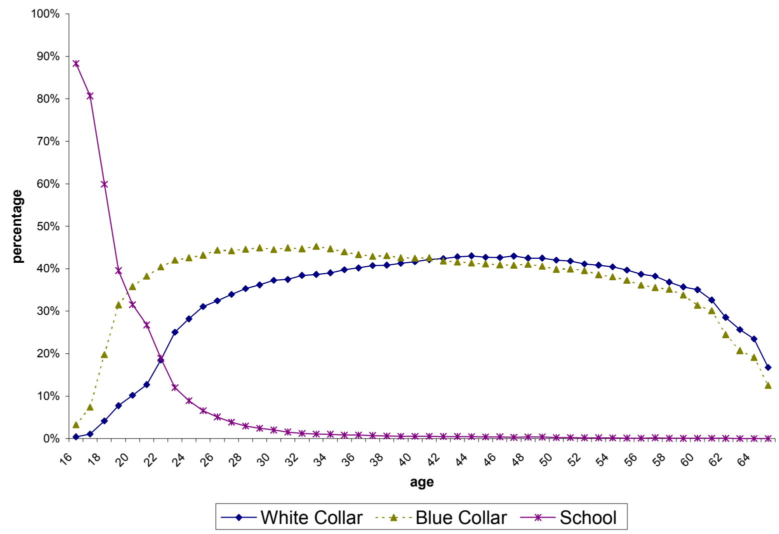

Figure 2: Male Career Decision by Age (Average)

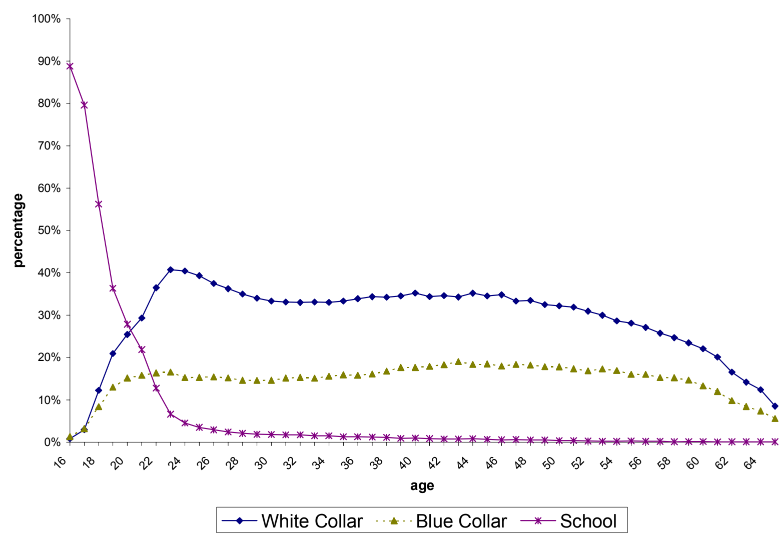

Figure 3: Female Career Decision by Age (Average)

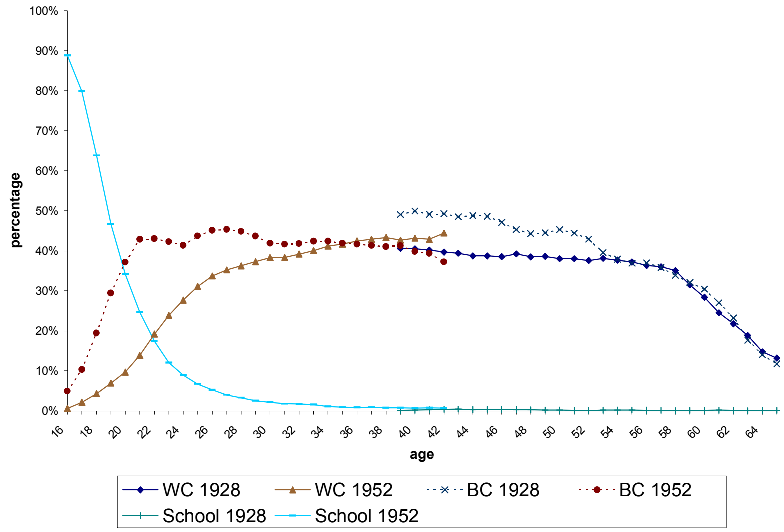

Figure 4: Male Career Decision by Age (2 cohorts)

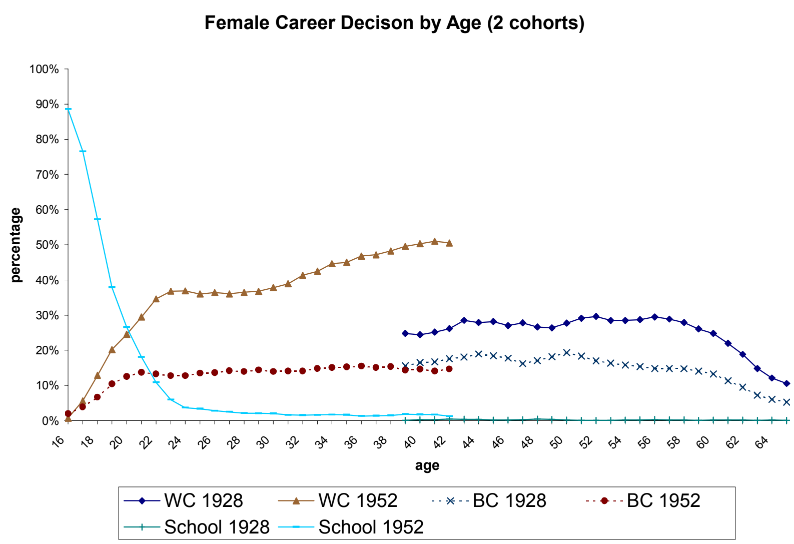

Figure 5: Female Career Decision by Age (2 cohorts)

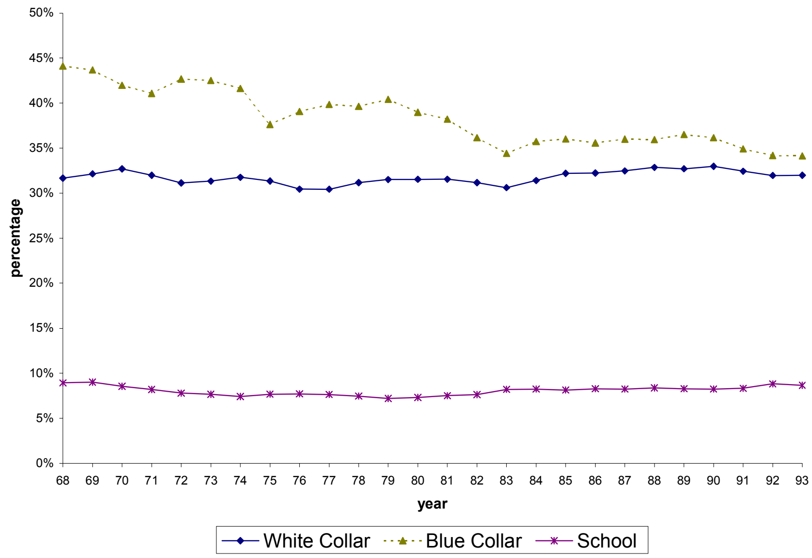

Figure 6: Male Career Decision by Year (Average)

in school attendance rates among females, from 7% to 9%. Male white-collar employment and female blue-collar employment were stable. Overall, market participation decreased for males and increased for females over the period.

Figuress 8 and 9 show the average real (1983 dollars) labor earnings over the same period by education level and separately by occupation. Earnings are computed as a full-time equivalent; for those who worked during the year it is calculated by multiplying the average hourly wage times 2080 hours (52 weeks $\times$ 40 hours per week). Variation in earnings therefore reflects only variation in the hourly wage. The blue-collar wage remained roughly constant over the period for both sexes as did the male white-collar wage. However, the female white-collar wage increased between 1968 and 1993. Indeed, while the average male wage in blue-collar occupations was higher than the female wage in white-collar occupations by about 20% in 1968, they were basically the same by 1990. With respect to education, as has been noted by others, the male high school wage (high school graduate or lower) decreased and the college wage (some college plus college graduates) was relatively constant. For females, the high school wage was constant and the college wage increased. Thus, the relative college - high school wage rose for both sexes between 1968 and 1993. As the result of these changes, the average male high school wage which was about 15% higher than the

at the age of 40 while as much as 50% of the 1952 female cohort worked in white-collar occupation at the same age.

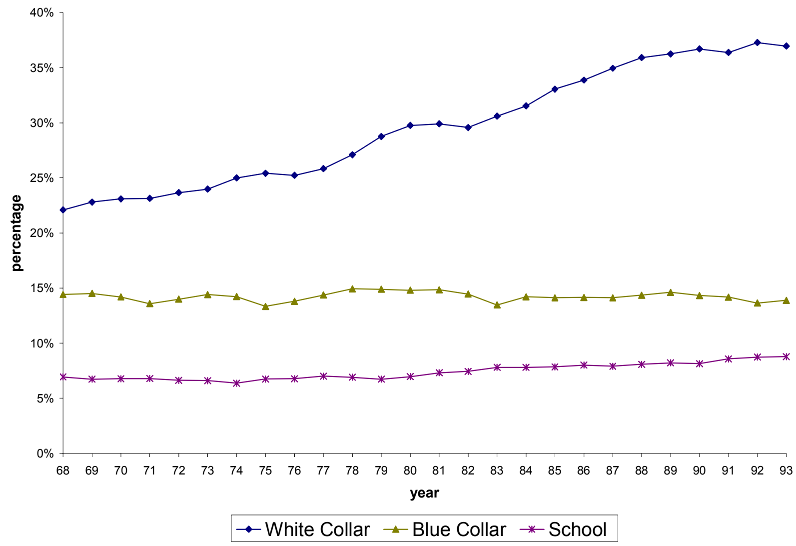

Figure 7: Female Career Decision by Year (Average)

female college wage in 1968, was about 15% lower by 1993.

Those who work in white-collar occupations consistently have completed more years of education than those with blue-collar occupations both for men and women, as seen in Table 2. For example, in 1991 about 72% of male and 57% of female white-collar workers completed at least one year of college, whereas only about 25% of male and 20% of female blue-collar workers had as much education. However, since 1968, the proportion of both white-collar and blue-collar workers with some college increased. In 1968, only one-half of males and one-third of females working in a white-collar occupation had some college, with the corresponding figures for blue-collar workers being less than 10%.

4.2 Exogenous variables

- **skill and capital shares**

  Given the Cobb-Douglas specification of the production function, production parameters can be directly calculated from output shares in each year (recall that the Hicks-neutral technology parameter is normalized to one). Capital's shares between 1968 and 1993 are calculated as

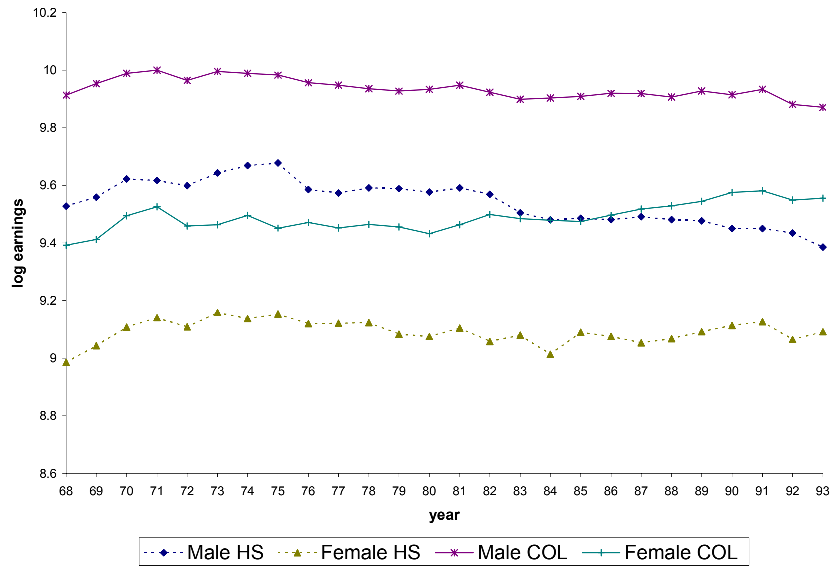

Figure 8: Average Log Earnings by Year (by Education Level)

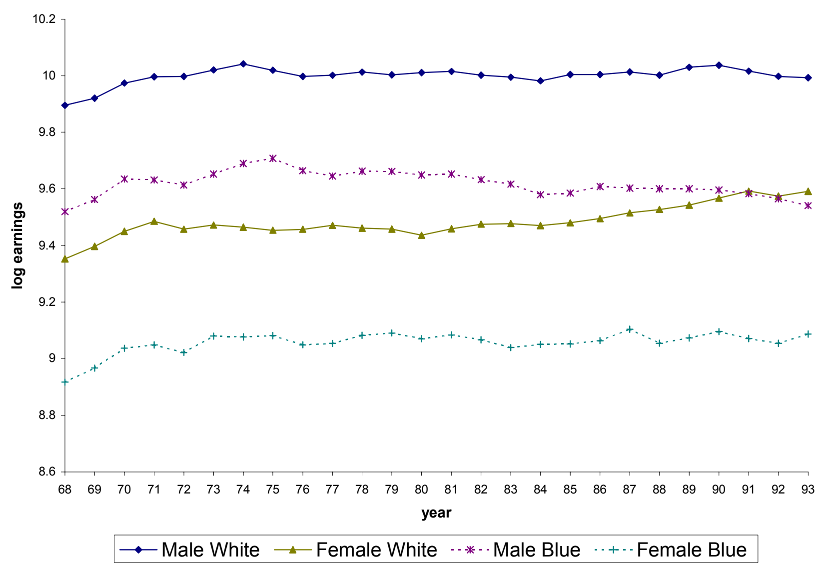

Figure 9: Average Log Earnings by Year (by Occupation)

Table 2: Average years of education by occupation

year	average years of education				At least 1 year of college			
	Male		Female		Male		Female	
	W.C.	B.C.	W.C.	B.C.	W.C.	B.C.	W.C.	B.C.
68	13.4	10.1	12.7	9.8	0.51	0.09	0.33	0.06
69	13.4	10.2	12.7	9.9	0.51	0.10	0.33	0.05
70	13.5	10.2	12.8	10.0	0.53	0.09	0.34	0.05
71	13.6	10.4	12.9	10.1	0.55	0.10	0.36	0.06
72	13.8	10.5	12.9	10.3	0.57	0.11	0.36	0.07
73	13.9	10.6	13.0	10.5	0.58	0.13	0.38	0.08
74	14.0	10.8	13.1	10.5	0.60	0.14	0.41	0.09
75	14.1	11.0	13.1	10.6	0.62	0.16	0.41	0.10
76	14.1	10.9	13.2	10.6	0.63	0.16	0.42	0.10
77	14.2	11.0	13.2	10.7	0.65	0.17	0.43	0.12
78	14.3	11.1	13.2	10.8	0.66	0.18	0.44	0.12
79	14.4	11.2	13.3	10.8	0.67	0.18	0.45	0.13
80	14.4	11.3	13.4	11.1	0.67	0.19	0.47	0.15
81	14.4	11.3	13.4	11.1	0.67	0.20	0.48	0.15
82	14.5	11.3	13.4	11.1	0.68	0.20	0.49	0.15
83	14.6	11.5	13.5	11.2	0.70	0.22	0.51	0.15
84	14.6	11.5	13.6	11.3	0.70	0.22	0.52	0.16
85	14.6	11.6	13.6	11.3	0.70	0.23	0.53	0.16
86	14.6	11.6	13.7	11.4	0.70	0.23	0.54	0.17
87	14.7	11.7	13.7	11.4	0.71	0.23	0.54	0.19
88	14.7	11.6	13.7	11.4	0.71	0.23	0.55	0.19
89	14.7	11.7	13.8	11.5	0.72	0.24	0.55	0.19
90	14.7	11.7	13.8	11.5	0.72	0.25	0.56	0.19
91	14.7	11.7	13.8	11.5	0.72	0.25	0.57	0.20

W.C. : white-collar occupation

B.C. : blue-collar occupation

residual shares; namely using data on 'total compensation of employees' and GDP,[16]

$$
\begin{aligned}
\text{capital share} &= 1 - \text{skill share} \\
&= 1 - \frac{\text{total compensation of employees}}{\text{GDP}}
\end{aligned}
\tag{22}
$$

The white-collar (blue-collar) skill share is calculated by aggregating individual earnings for the occupation from the CPS namely

$$
\begin{aligned}
\text{white-collar skill share} &= (1 - \text{capital share}) \\
&\quad \times \frac{\text{aggregate total compensation of white-collar employees}}{\text{aggregate total compensation of employees}}
\end{aligned}
\tag{23}
$$

Figure 10 shows that between 1968 and 1993, the white-collar skill share has been increasing, and the blue-collar skill share has been decreasing. The capital share has been roughly constant.

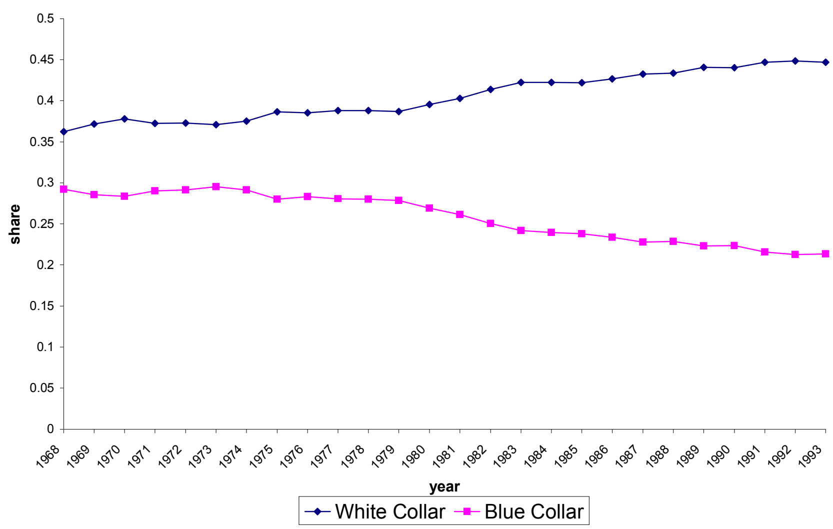

Figure 10: Output Share Parameters in the Production Function

- **initial schooling and work experience at age 16**

The age 16 decision depends on whether the individual attended school in the previous year and on the level of schooling attained by age 16. The proportion of age 16 people who

[16]source: Council of Economic Advisors, "Economic Report of the President", February 1998

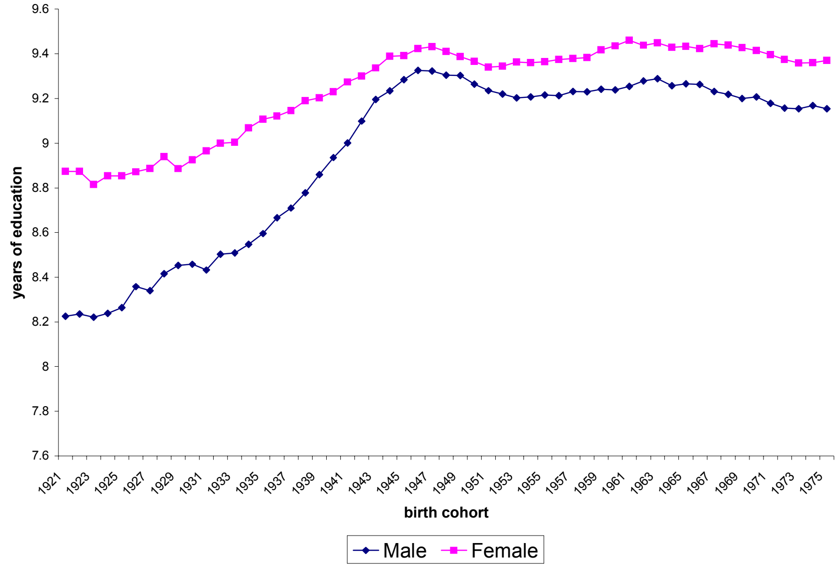

Figure 11: Completed years of education at age 16 by cohort

attended school the previous year was calculated up to 1940 conditional on each schooling level.[17]

The distribution of initial education level at age 16 is obtained from the CPS between 1964 and 1991 and from the US census of various years before 1964.[18] Figure 11 shows average years of education at age 16 for various cohorts. Until the 1950 birth cohort, the average education level attained at age 16 steadily increased, after which it decreased slightly.

[17]It was assumed that those who were 16 in 1940 had the same schooling distribution at age 15 in 1939 as those who were 15 in 1940. Those who were 16 before 1940 were given the same distribution of having attended school in the prior year as those who were 16 in 1940 conditional on age 16 initial schooling level. Additionally, initial work experiences, at age 16, are assumed to be zero in both occupations.

[18]The distribution of initial education level at age 16 after 1991 is assumed to be the same as in 1991. The following procedure is used for years before 1964.

1. Education levels at age 16 in 1960, 1950 and 1940 are retrieved from the corresponding Census.

2. For years between 1960 and 1964, we use the mixture of the two distributions of the 1960 and 1964 initial education levels by assuming linear weights between the two. For example, for the distribution of 1961, we draw 1/4 from the 1964 distribution and 3/4 from the 1960 distribution. Similarly, for years between 1940 to 1950, and 1950 to 1960.

3. For years before 1940, because there is no data containing the age 16 education level distribution, we use the education level of people older than 16 in the 1940 Census. To deflate the education level of older people back to the age 16 level, we assume that the education level at age 16 in 1940 stochastically dominates those before 1940 in the first order sense. For example, to get the education level at age 16 in 1930, we draw 1000 number of observations each from age 16 and 26 in the 1940 Census, sort each of them from the smallest to the largest and keep the smaller of the matched pair. In this way, we keep the lower education level of the earlier cohort and remove the higher education level accumulated after age 16. To get the initial education level at age 16 in 1929, we draw the same number of observations from age 16 and 27 in the 1940 Census, and do the same.

- **cohort size**

  Cohort size is obtained from Vital Statistics of the United States and from US Census Bureau reports. Cohort sizes between 1909 and 1999 are the actual sizes and cohort sizes after 2000 are based on Census Bureau forecasts.[19] Figure 1 shows the fluctuations in cohort size in the US between 1910 and 2000.

- **capital stock**

  We use a series collected by the Bureau of Economic Analysis for the actual capital stock for the years between 1925 and 1997. According to that data , as shown in Figure 12, there had been little growth in the capital stock between 1927 and the end of World War II., but since then the growth in the capital stock has been at a fairly constant 3% rate per annum. Measures of the capital stock prior to 1927 and after 1997 are based on extrapolations.[20]

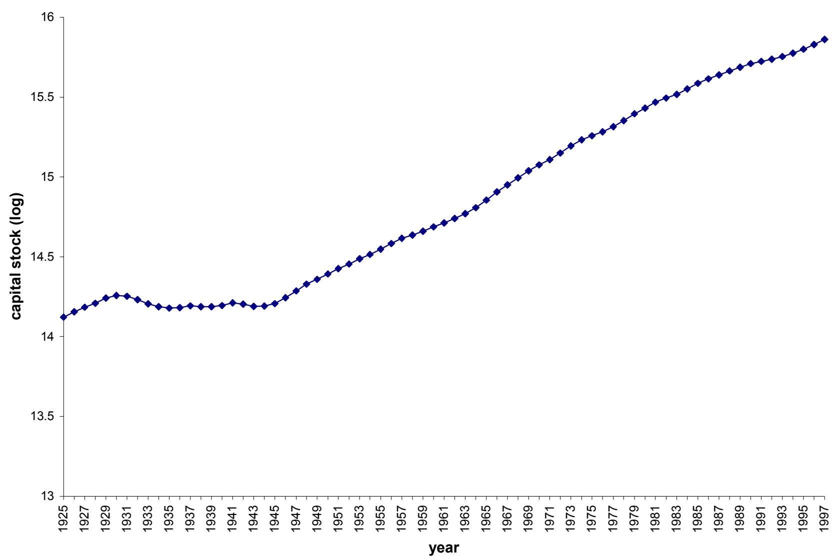

Figure 12: log capital stock (1925 - 1997)

- **The Markov process of number of preschool children**

[19]Approximate cohort sizes from 1800 to 1908 are obtained from cohort sizes between 0 and 4 years reported every ten years from 1800 through 1910.

[20]The capital stock series before 1925 and after 1997 are extrapolated using the growth rates of 1925 and 1997 respectively. For example, we assume that capital growth rates before 1925 and after 1997 are the same as in 1925 and 1995 respectively. The career decisions in the estimation period is not sensitive to this calculation of the out-of-sample capital stock series.

We use the CPS from 1968 to 1995 and various years of the US Census to estimate the process generating the number of preschool children, which is assumed to differ by cohort, age, sex and the four educational categories.[21] Figure 13 shows the average number of preschool children by age and sex between 1961 and 1995. It peaks at age 28 for females and at age 30 for males. Figure 14 shows that the fertility rate decreased rapidly between 1960 and 1980, and since then has continued to decline at a slower rate.

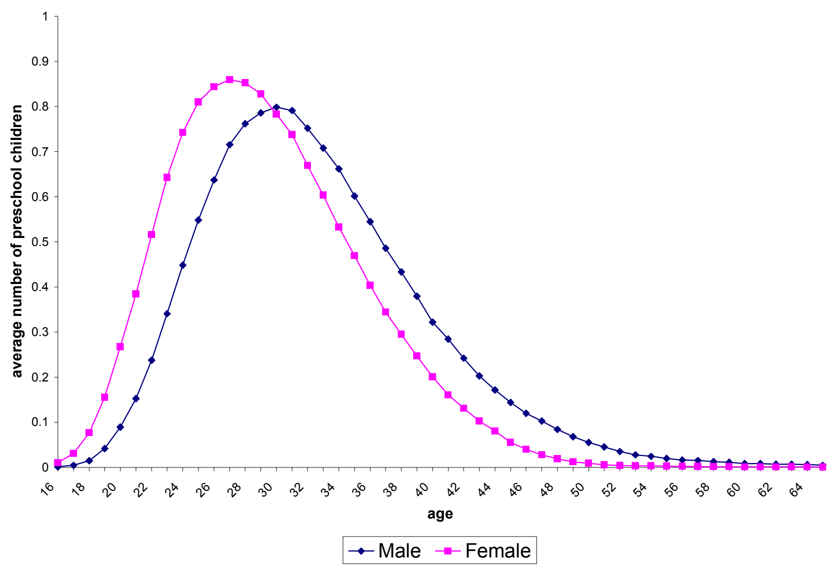

Figure 13: Average number of preschool children by age (1961 - 1995)

Table 3 summarizes the exogenous variables employed in the model.

[21]We estimate the process for preschool children between 1968 and 1995 based on the number of family members between 0 and 6 in the corresponding CPS data. For example, to calculate the transition probability from 0 to 1 preschool children in 1968, we count the number of people who had zero children between 0 and 5 years old in 1967 and calculate the proportion of people with one child between 0 and 5 years old in 1968. We repeat this calculation in each year between 1968 and 1995. To obtain the 1967 transition rate, because the 1967 CPS has data only on family members older than 14, we decrease each individual's age by one from the 1968 CPS and obtain the 1967 transition rate, as if the data represented the family structure in 1967. We repeat this calculation in each year and construct the transition rates between 1961 and 1968 from the 1968 CPS, those between 1951 and 1960 from the 1960 Census, those between 1941 and 1950 from the 1950 Census, those between 1931 and 1940 from the 1940 Census, those between 1901 and 1910 from the 1910 Census, those between 1871 and 1880 from the 1880 Census. The transition rates after 1995 are assumed to be the same as that of 1995 and those before 1871 the same as 1871. Between 1911 and 1930 and between 1881 and 1900, the process is approximated by linearly interpolating the 1910 and the 1931 process and the 1880 and the 1901 process respectively.

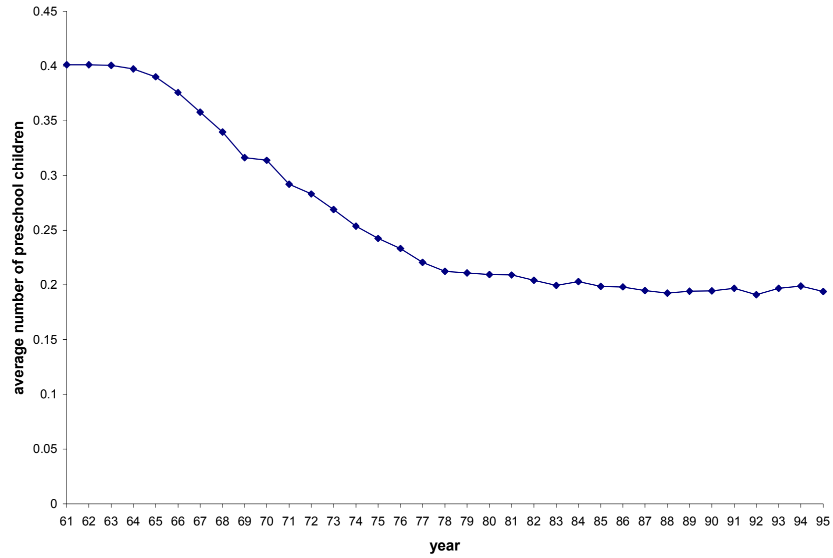

Figure 14: Decrease in fertility rate over time

Table 3: Exogenous variables

variable	source	coverage years
education level at age 16	CPS & Census	1865 - 1991
cohort size	Census Bureau	1800 - 2050
capital stock	Bureau of Economic Analysis	1925 - 1997
preschool children	CPS & Census	1871 - 1995

5 Estimation Results

We estimate the model by minimizing the weighted squared deviation between the actual moments obtained from the data and the corresponding simulated moments from the model. For example, the difference between the proportion of individuals with no pre-school children who were age 30 in 1968 and who were working in a white-collar occupation observed in the data and simulated from the model is calculated. We sum up the distances over all the conditional moments with proper weights and search for the set of parameters that minimizes this sum.[22]

To obtain the simulated moments between 1968 and 1993, we start the economy at 1865, where each cohort is given a distribution of schooling and work experiences in both occupations and a distribution of the number of preschool children.[23] Each person in each cohort alive in 1865 chooses the optimal career decision based on current and future skill prices. The aggregate skill supply in each occupation is calculated by summing individual skill supplies weighted by the cohort sizes (the 1800 cohort through the 1849 cohort) and skill prices are calculated from marginal products given the two aggregate skill supplies, the capital stock in 1865, and the output shares. In the next year, in 1866, the 1850 cohort enters the economy at the age of 16 with the initial schooling distribution obtained above and each cohort again solves their problem. The economy evolves until the year 2065. We record each year's skill prices for the two occupations to use for the next iteration and solve the model again from 1865 to 2065. When the skill prices converge by the iterative method specified above, we collect the simulated moments corresponding to the actual moments above between 1968 and 1993.

[22]Our estimation method is different from the standard GMM estimation. In standard GMM estimation, the same observations are repeatedly used for each different moment condition. But in our case each moment is obtained from different observations. For example, the aggregate choice moments (e.g. the school attendance rate) are obtained from all the individuals in the data, whereas the labor earnings moment is obtained only from those who choose to work. For this reason, the optimal weighting matrix (off-diagonal elements) can not be calculated and we treat each conditional moment as if it is a single observation. We assume that within one group of moments (e.g. white-collar employment rate) the variance covariance matrix is a scalar times the identity matrix, and across groups of moments (e.g. between white-collar employment rate & earnings), we allow for heteroskedasticity and non-zero correlation. The error structure here, therefore, resembles that of the SUR model. We can not estimate the model by maximum likelihood using individual data in the CPS, because the appropriate state variables, e.g. work experience, are not observed.

[23]We give the distribution of schooling and average work experiences and the distribution of preschool children for each age in 1940 to each cohort in 1865. This initial distributions of schooling and work experiences do not affect the career decisions in our estimation period at all. We could instead have given 0 years of schooling and 0 years of work experiences to each cohort and obtained the same career decision moments after 1968.

Table 4: Estimation Results - working utility

parameter		white-collar skill		blue-collar skill	
constant	male I	12.19	(.00099)	13.28	(.00144)
	male II	11.71	(.00096)	13.50	(.00096)
	female I	11.73	(.00098)	12.81	(.00128)
	female II	11.22	(.00117)	12.94	(.00098)
education (E)		.079	(.000088)	.048	(.000099)
w.c. work experience (x_1)		.094	(.000095)	.021	(.000142)
w.c. work exp. square $\left(x_1^2\right)$		-.0029	(.0000046)	-.0027	(.000012)
b.c. work experience (x_2)		.022	(.000164)	.096	(.00010)
b.c. work exp. square $\left(x_2^2\right)$		-.007	(.000020)	-.0026	(.0000044)
skill sd.		.46	(.00069)	.48	(.00069)
non-pecuniary	male	328.2	(11.8)		
benefit	female	-145.4	(9.5)		
skill correlation (σ_{12})		.009	(.0013)		

Standard errors are in parenthesis.

Blue-collar non-pecuniary benefit is normalized to be zero.

Table 5: Estimation Results - schooling utility

parameter		male		female	
constant	type I	18,687.7	(11.4)	14,521.2	(11.5)
	type II	10,906.9	(15.1)	9,035.0	(19.0)
college tuitoin		7,465.4	(8.6)		
graduate school tuition		25,799.2	(43.0)		
reentry cost		33,688.6	(47.3)	28,106.5	(67.8)
age 16 - 17		9,166.7	(21.4)	4,306.5	(19.2)
schooling shock sd.		5,470.0	(12.8)	5,350.9	(11.8)

5.1 Parameter Estimates

This section reports the estimation results, provides an interpretation of the estimated parameters and discusses the fit of the model.

The optimization problem describing career decisions pertains to that of a single individual. Individuals of different cohorts, because their lifetimes overlap different calendar periods, face different skill rental price paths, different distributions of initial schooling and different fertility processes. However, individuals within a cohort are ex-ante identical (except for their actual initial schooling). Although the CPS data is not longitudinal, one of the moments used in the estimation,

Table 6: Estimation Results - staying home utility

parameter		male		female	
constant	type I	13,239.8	(13.1)	11,472.9	(7.9)
	type II	8,374.5	(13.3)	8,613.7	(9.9)
preschool children (NC)		992.5	(17.0)	4,187.4	(12.5)
staying home shock sd.		9,391.1	(14.2)	3,445.3	(9.4)

Table 7: Estimation Results - discount factor, retirement rate and type proportion

parameter		male		female	
discount factor		.93	(.00010)	.95	(.000107)
retirement rate	type I	.074	(.00059)	.18	(.00041)
	type II	.13	(.00044)	.21	(.00058)
type I proportion		.47	(.00051)	.66	(.00096)

Retirement rate is the constant retirement rate between ages 60 and 64.

the one-period transition rate between white- and blue-collar employment, captures the degree to which there is some permanence in occupational decisions. As in most recent studies (e.g., Keane and Wolpin (1997)) that are based on longitudinal data, it has proven necessary to allow for additional heterogeneity in order to capture the degree to which there is permanence in these decisions. In implementing the model, it is assumed that in each cohort there are two types of people (both for men and women), treated as unobservable, who differ in their skill endowments, their consumption values of schooling and home, and in their retirement rates.

Tables 4 through 7 report the parameter estimates. The estimation results indicate the existence of significant heterogeneity among the two types. For both males and females, one type is better at the white-collar occupation and the other type at the blue-collar occupation, indicating comparative advantage. The white-collar skill constant (the age 16 endowment) is higher for type I's than for type II's, but the blue-collar endowment is lower for type I's than for type II's. There are also differences among the types in the utilities attached to the non-work alternatives. Type I's (both sexes) have a substantially higher consumption value for school attendance and also a higher home value. In addition, type I's are more likely to retire at age 65, rather than at an age between 60 and 64, than are type II's.

The education effect on skill, assumed to be the same for men and women, indicates that white-collar skill (and, thus, earnings) increase by 7.9% for each additional year of education and blue-collar skill by 4.8%. Also assumed to be the same for men and women, the first year of work experience increases skill by 9.4% for the white-collar occupation and by 9.4% for the blue-collar occupation. Peak earnings is reached at 16.2 (18.4) years of white- (blue-) collar work experience. The first year of white-collar (blue-collar) work experience increases blue-collar (white-collar) skill by 2.1% (2.2%).

Rather than use actual data on post-secondary tuition costs, both the cost of college and of graduate school are estimated as parameters. Crude data that averages college tuition costs over college attendees shows that real tuition costs have increased over the 1968-1993 period.[24] However, these increasing costs do not accurately reflect the accessibility to post-secondary education through increased access to two-year colleges and to subsidized loans and other forms of financial aid. The cost of attending college (tuition plus other school-related expenditures) is estimated to be 7,465 dollars and graduate school tuition, 25,799 dollars.[25]

School attendance, as noted, is heavily concentrated between the ages of 16 and 25. Although the finite horizon by itself leads to a declining attendance pattern with age (as in Ben-Porath (1967)), with *iid* shocks to the consumption value of schooling, the model generates too many transitions in and out of school. The reentry cost was introduced into the model to be able to fit the school attendance pattern in the data. It can be rationalized as measuring the increased cost of schooling associated with the depreciation of knowledge that occurs while not attending school. The reentry cost is estimated to be quite large, about $34,000 for men and $28,000 for women.[26] The sex difference reflects the fact that reentry does occur more frequently for women. The utility attached to remaining home increases by about $1,000 for men and $4,200 for women with each preschool child. The higher value for women helps explain the decrease in female labor force participation during the high fertility ages (around 25 to 30).[27] Finally, the discount factor

[24]source: US Department of Education, "Digest for Education Statistics", National Center for education statistics, Office of Educational Research and Improvement, 1997

[25]In Keane and Wolpin (1997), the estimated cost of attending college is 4,168 dollars and the estimated additional cost of attending graduate school is 7,030 dollars.

[26]Keane and Wolpin's (1997) estimate based on NLSY79 data for men is $20,030 to reenter high school and $8,709 to reenter college.

[27]Note that in addition to the reentry cost, there is additional positive utility of school attendance specific to ages

Table 8: Goodness of Fit: R^2 statistic

moment	male	female
white-collar employment rate	0.88	0.80
blue-collar employment rate	0.89	0.85
school attendance rate	0.96	0.96
white-collar log labor earnings	0.82	0.69
blue-collar log labor earnings	0.80	0.46

is estimated to be almost the same for men and women, 0.93 and 0.95 respectively.

5.2 Model Fit to Data

Table 8 shows the goodness of fit of the estimated model in terms of R^2. This measure of fit demonstrates the degree to which the actual variation in the white-collar employment rate, the blue-collar employment rate, the school attendance rate and the average log labor earnings, classified by cohort, age, sex, education level and whether there is a preschool child, is explained by the model. This R^2 measure should be interpreted similarly to the same measure in a linear regression. The model captures the changes in career decisions and labor earnings over time and across cohorts reasonably well.[28]

The fit of the model to the career decision patterns previously described is depicted in a set of figures provided in the appendix. As seen there, generally the model matches the data in almost all dimensions quite well.

6 Policy Experiment I: Cohort size effect

In this section, the effect of cohort size changes on career decisions and earnings over various cohorts is evaluated. To do this, the model is simulated under the baseline case in which cohort size hypothetically increases at a steady rate (see Figure 1) and the preschool children process is stationary (consistent with the cohort trend line, see Figure 15) and the result is compared to the actual data. The baseline, or cohort trend line, was constructed by removing the major cohort fluctuations; a straight line was fit between the 1910 and 2000 cohort sizes. The steady state

16 and 17.

[28]However, the fit of female labor earnings is less good than the other moments.

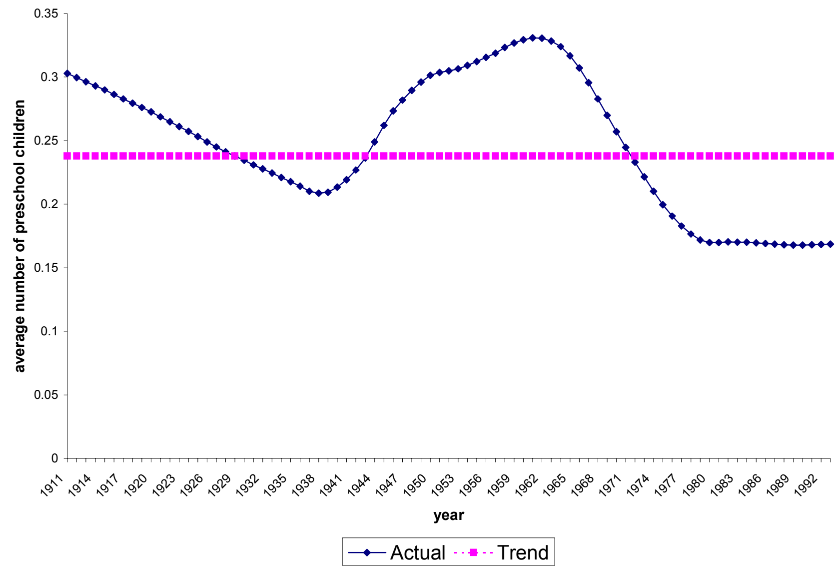

Figure 15: Average number of preschool Chilren (Actual vs. Trendline)

preschool children process was constructed by pooling 1910 through 1990 datasets together using the same number of observations and calculating the preschool children process conditioned on age and sex only.[29] Thus, the cohort constructed in this way maintains the stable trend from 1850. Relative to this trend line, actual cohort sizes reflect the periodic occurrences of baby booms and busts, e.g., baby bust during WWII and the baby boom that followed and that peaked in the early 1960's.

Figure 16 shows the predicted equilibrium log skill price over the period 1968 to 1993 based on actual cohort sizes. We see that the white-collar skill price has increased over time, whereas the blue-collar skill price has decreased since 1974, which explains the earnings pattern by occupation during the period. Figure 17 shows the difference between the occupation-specific log skill prices based on actual cohort sizes and on the cohort trend line. The graph shows the skill price differences that arise between the actual cohort size and the hypothetical cohort size. As is apparent, skill prices based on actual cohort sizes are higher than those based on the cohort trend line until 1985. However, after 1985, this relationship is reversed. Skill prices that result from actual cohort sizes

[29] Notice that the actual preschool children process was conditioned also on year (cohort). To explain the details of the steady state preschool children process, I draw the same number of observations for each age from the 1910, 1940, 1950 and 1960 Census datasets and 1970, 1980 and 1990 CPS datasets and pool them together so that it represents the average family structure in the US between 1910 and 2000 and construct the steady state preschool children process. Since we don't have data for 1920 and 1930, I substitute 1910 and 1940 data respectively.

are lower than those from the cohort trend line because by 1985 most of the baby boom generation has entered the labor market and the economy was mainly dominated by them.

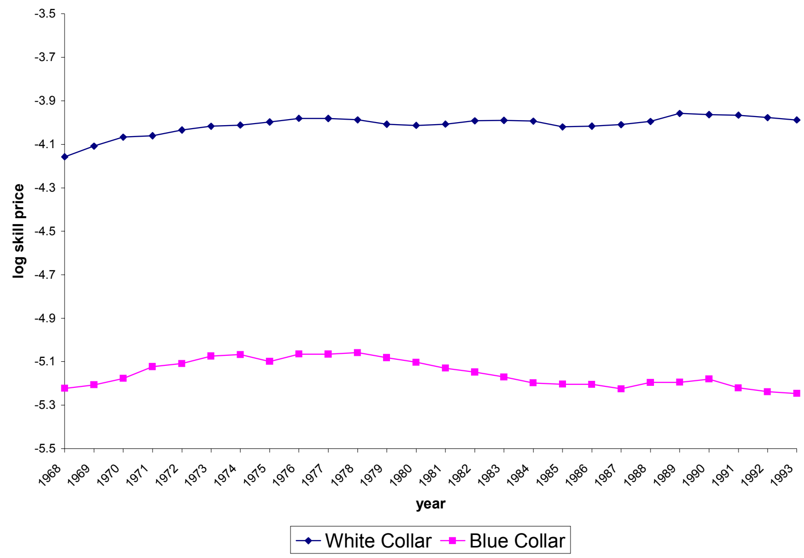

Figure 16: Equilibrium Skill Price by Year

The results of this comparison are as follows. First, the labor market was affected by the new baby bust generation born in the 1930s and 1940s. Skill prices were higher due to the decreased skill supply, and this lower skill price resulted in the male baby bust cohorts working more and earning more, and also attending school more in order to take advantage of higher skill prices. Completed education and lifetime years of work experience were, concomitantly, also higher for this generation. Notice that the relative white/blue-collar skill price is essentially invariant to cohort size, probably due to the fact that type I's and II's are fairly evenly represented in the population. However, the impact of increased skill prices on the female baby bust generation was offset by the increase in fertility rates, since they were the mothers of the baby boom generation born in the 1950s and 1960s. As a result, the female baby bust generations chose to stay home more and, consequently, attended school less and worked less in both occupations despite the increase in skill prices, whereas males from the baby bust generations were hardly affected by the increased fertility.

On the other hand, when the labor market was dominated by the baby boom generation, the opposite happened. Skill prices were lower in those periods and as a result, the male baby boom generation worked less, earned less and attended school less compared to the cohort trend line.

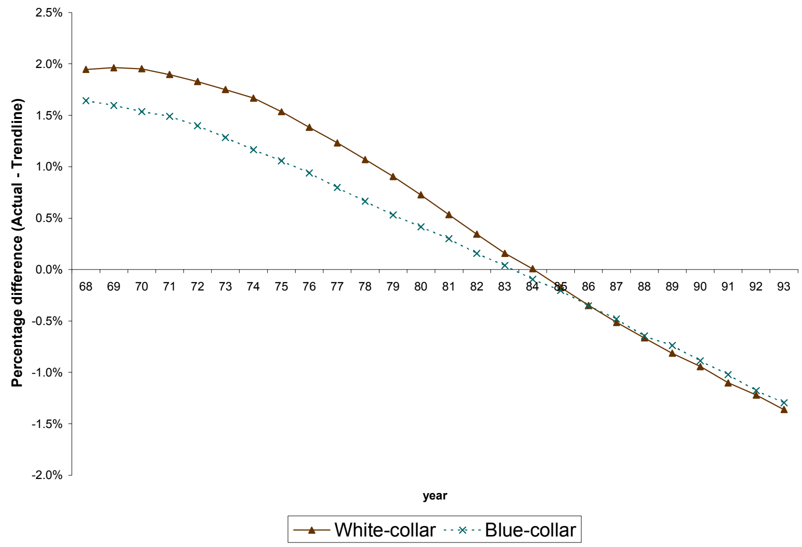

Figure 17: Skill Price Differential

Completed education and the lifetime years of work experience were, therefore, lower for them. However, because the baby boom generation had a lower fertility rate in the 1980s and 1990s, the effect of lower skill prices on female schooling and working decisions were offset to some degree by the decrease in fertility rates.

Figure 18 shows the difference in the college completion rate at age 30 between the actual cohort size and the cohort trend line. Males (females) from the baby bust generation completed college by as much as 0.3 (1.5) percentage points more (less) and males (females) from the baby boom generation completed college by as much as 1.0 (1.5) percentage points less relative to the cohort trend line. Figure 19 shows the difference in completed work experience at age 65 by cohort between the actual cohort and the cohort trend line. The male baby bust generation worked more compared to the cohort trend line in both occupations, whereas the baby boom generation worked less and the opposite for females due to fertility changes. In terms of total years of completed work experience (white-collar plus blue-collar), males (females) from the baby bust generation worked by as much as 0.1 (0.5) years more (less) and males (females) from the baby boom generation worked by as much as 0.1 (0.2) years less (more).

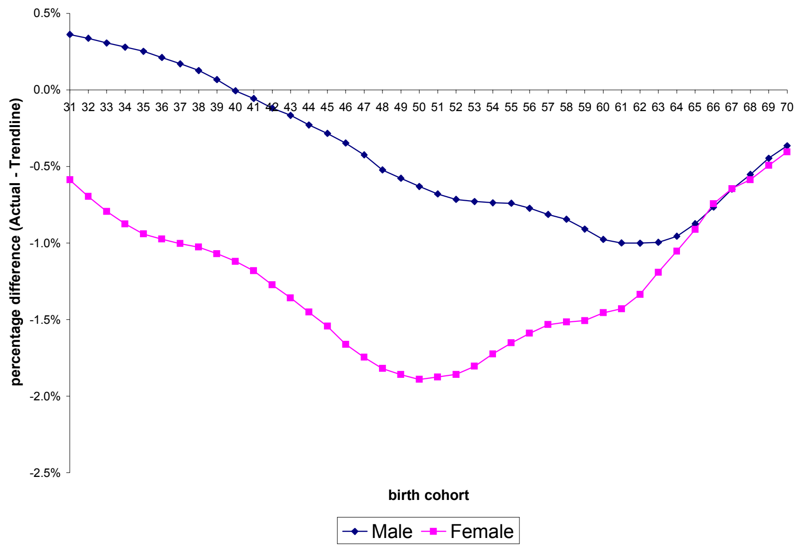

Figure 18: College Completion Rate Difference by Cohort

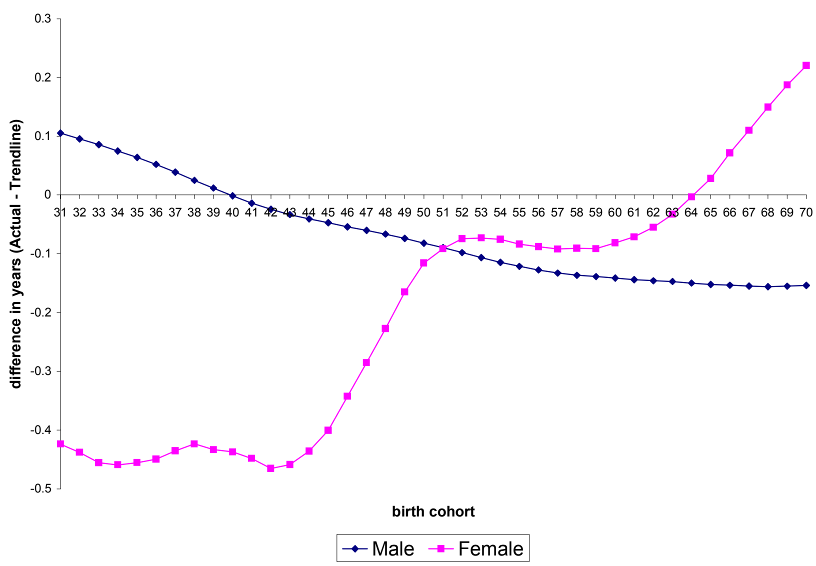

Figure 19: Years of Work difference by Cohort

Table 9: Tuition subsidy (increase) effect

experiment	sex	predicted value	PE value	GE value	PE effect	GE effect
1	Male	27.17%	26.86%	26.88%	-1.12%	-1.05%
	Female	27.46%	27.01%	27.05%	-1.66%	-1.52%
2	Male	17.78%	17.54%	17.55%	-1.34%	-1.27%
	Female	17.35%	17.01%	17.02%	-1.95%	-1.86%
3	Male	11.98	12.20	12.17	0.57	0.50
	Female	12.22	12.56	12.53	0.88	0.82

PE effect = Partial Equilibrium effect

GE effect = General Equilibrium effect

PE effect and GE effect in experiment 1 & 2 are in terms of percentage increase.

Experiment 1: a 100 dollar tuition increase (in 1995 dollar) on the college enrollment rate (age 18 and 19)

Experiment 2: a 100 dollar tuition increase (in 1995 dollar) on the college enrollment rate (age 18 through 24)

Experiment 3: a 50% tuition reduction on completed years of education by age 30.

7 Policy Experiment II: The Effect of a Tuition Change

The evaluation of large-scale policy interventions based on partial equilibrium analyses that do not account for general equilibrium effects may be misleading. In this section, the effect of a change in college tuition costs on the career decisions and labor earnings of the birth cohorts between 1958 and 1965 is evaluated under both partial equilibrium and general equilibrium assumptions. The partial equilibrium effect of a tuition change alters career decisions assuming that skill prices do not change. The general equilibrium effect of a tuition change alters career decisions allowing skill prices to change. For example, the partial equilibrium effect of a tuition subsidy is to increase the incentive to obtain more schooling, and with this increased human capital, to work more over the lifetime. However, this increased schooling, and subsequent increased work experience, increases the aggregate skill supply, which has the effect of decreasing skill prices. As a feedback effect, the initial incentive to attend school will be reduced.

The following three experiments are evaluated: (i) the effect of a 100 dollar tuition increase on the college enrollment rates between the age of 18 and 19; (ii) the effect of a 100 dollar tuition increase on college enrollment rates between the ages of 18 and 24; (iii) the effect of a 50% tuition reduction on completed years of education by age 30.

The results of these experiments are given in Table 9. The partial equilibrium effect of a $100 increase in college tuition is predicted to reduce the college enrollment rate for 18-19 year olds in these cohorts by 1.12% for males and 1.66% for females. Widening the age span to 18 to 24 leads to slightly larger reductions. These effects are consistent with what has been found in the literature (see Keane and Wolpin (1999, forthcoming)). The third experiment shows that the partial equilibrium effect of a 50% tuition subsidy would increase completed schooling by more than half a year for men and by almost a full year for women. In all of the experiments, the general equilibrium effect is smaller than partial equilibrium effect as a result of skill price adjustments. However, the magnitude of differences between general and partial equilibrium effects is small, usually less than 10%.

As discussed in the introduction, in the paper by Heckman, Lochner and Taber, general equilibrium effects were found to be close to zero. Partial equilibrium effects are of a similar magnitude as in table 15. Although it is not possible to completely reconcile the results in their paper and those presented here given the complexity of both models and the major modeling differences that exist, the following insights are potentially useful. Recall that in the present model, people have both white- and blue-collar skills at any point in time, and can choose either occupation irrespective of their education level. A policy intervention, such as a tuition subsidy, that increases schooling will increase the aggregate skill supply more in the occupation whose skill production is most augmented by additional schooling, in this case in the white-collar occupation. However, because individuals are still subject to random shocks in both their amounts of white- and blue-collar skill, only a proportion of those who obtain more schooling and who previously chose the blue-collar occupation will now choose the white-collar occupation. Given that there is learning by doing, the initial occupation decision will have some permanence. The larger are the variances in skill shocks, the smaller will be the increase in the relative aggregate supply of white- to blue-collar skill, and the smaller will be the fall in the relative white-collar skill rental price. In contrast, in the Heckman, Lochner and Taber framework, everyone who chooses additional schooling must enter the sector that uses (only) more educated labor. Thus, a given partial equilibrium increase in educated labor will translate into a similar magnitude increase in the aggregate supply of sector-specific skill.

The second feature that may be of particular significance in this regard is the treatment of individual heterogeneity. In the Heckman, Lochner and Taber paper, people are assumed to be identical after controlling for observable ability (measured by the AFQT score) and educational attainment. In the present model, heterogeneity is treated as unmeasured (as in Heckman and Singer (1984)) and arises in skill endowments and in consumption values of schooling and home. This unobservable heterogeneity captures differences in many dimensions, the quality of schooling, family background, innate abilities, etc. The degree of heterogeneity critically affects the extent to which the tuition subsidy affects the relative amounts of white- to blue-collar aggregate skill. Individuals who have relatively more blue-collar skill and who are induced to obtain more schooling by the subsidy will not necessarily enter the white-collar occupation, even though the additional schooling augmented their relative white- to blue-collar skill level. Some of the difference in the results could be due to the differential degrees of heterogeneity allowed for in the two papers.

8 Policy Experiment III: Explaining the Increase in Female Employment

Perhaps, the most noticeable change in the career paths between 1968 and 1993 is the increase in female employment, especially in white-collar occupations. During this period, overall female employment increased 39%, from 36.5% in 1968 to 50.8% in 1993. This increase in female employment is solely attributable to an increase in white-collar employment from 22% in 1968 to 37% in 1993; female blue-collar employment was 14% both in 1968 and 1993. According to the model, the increase in female employment is potentially attributable to changes in all of the exogenous processes; the aggregate technology, cohort size, the number of preschool children, the capital stock and the initial schooling distribution at age 16. In this section, we evaluate the role that two of these factors played in the change in female employment over this period, the change in the number of pre-school children and the increase in the capital stock.

First, simulating the model under the assumption that the process generating the number of preschool children after 1960 was the same as that of 1960, it appears that female employment would have been 5 percentage points lower in the 1990s if the high fertility had persisted after 1960. Thus, the decrease in the fertility rate since 1960 captures about 36% of the increase in female employment. On the other hand, male earnings would have been higher by 1 percent in the same period due to the decreased female skill supply. Although the change in fertility would account for a significant part of the employment increase, the increase in the capital stock by itself would have increased female employment even more. Based on a simulation where the capital stock is assumed to be constant after 1968, it is found that the actual increase in the capital stock captures about 77% of the increase in the female employment.

9 Conclusion

In this paper a dynamic general equilibrium model of career decisions was estimated. The model was shown to be able to fit data on the schooling, employment and occupational choices, and wages, of individuals represented in the Current Population Surveys from 1968 to 1993. The model was used to estimate the effects of cohort size on these decisions as well as to understand the changes that have occurred in these outcomes over cohorts and through time. The model was also used to assess the extent to which college tuition policy changes affect these behaviors and to compare the estimates that are obtained from a partial equilibrium vs. a general equilibrium analysis. Finally, the model was used to assess the reasons behind the increase in female labor force participation over this period.

Based on the estimates of the model, the impact of cohort size on skill prices and, consequently, on career decisions was assessed. Compared to a baseline case in which cohort size increased at a steady (average) rate and the fertility process was stationary, it is found that the male baby bust generations born in the 1930s and 1940s faced higher skill prices (by as much as 2.0%), completed college at a higher rate (by as much as 0.3 percentage points) and worked more over the lifetime (by as much as 0.1 years). In contrast, the males from the baby boom generations born in the 1950s and 1960s faced lower skill prices (by as much as 1.5%), completed college at a lower rate (by as much as 1.0 percentage points) and worked less over the lifetime (by as much as 0.1 years). The impact of cohort size is found to differ across gender due to the differential impact of fertility on the value of home production: unlike males, the female baby bust generations completed college at a lower rate (by as much as 1.5 percentage points) and worked less over the lifetime (by as much as 0.5 years) while the female baby boom generations completed college at a lower rate (by as much as 1.5 percentage points) and worked more over the lifetime (by as much as 0.2 years).

With respect to the sensitivity of schooling decisions to changes in college tuition, the model predicts that a 1% increase in tuition would reduce college enrollment rates by 1.27% in partial equilibrium with fixed skill prices, consistent with previous estimates, and by 1.05% in the general equilibrium case where the skill prices are endogenous. Thus, for this policy experiment, the partial equilibrium analysis of schooling choice does not differ much from the general equilibrium analysis. Finally, with respect to the increase in female employment, it is found that by itself, 36% of female employment increase can be explained by the decrease in fertility over the period. As a comparison, 77% of female employment increase can be explained by the increase in capital stock over this period.

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[18] Willis, Robert J. "Wage Determinants: A Survey and Reinterpretation of Human Capital Earnings Functions." In Handbook of Labor Economics, vol. 1, edited by Orley Ashenfelter and Richard Layard. Amsterdam: North-Holland, 1986.

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A Existence of equilibrium skill price

In this appendix, we prove that the equilibrium skill price exists. We prove the existence of equilibrium skill price with one skill price. Existence of equilibrium skill prices with multidimensional skill types can be similarly derived.

Now we assume that there is a continuum of people in each cohort, where each cohort is represented by a real line $[0, C(t,a)]$ and $C(t,a)$ is the cohort size at age a at period t. Each individual i is a point in that real line. individual skill supply function is

$$s_{it}(a; r_t, r_{t+1}, ..., r_{t+A-a}) = \exp\left\{\alpha_0 + \alpha_1 E + \alpha_2 x + \alpha_3 x^2 + \varepsilon\right\}(d_{1it}(a) = 1) \tag{24}$$

Notice that individual skill supply is a function of skill price series, $(r_t, r_{t+1}, ..., r_{t+A-a})$ where r_{t+A-a} is the skill price at the last age A.

Aggregate skill supply is the sum (integral) of individual skill supply.

$$\begin{aligned} S_t^s &= \sum_{a=16}^{A} \int_0^{C(t,a)} s_{it}(a; r_t, r_{t+1}, ..., r_{t+A-a}) \mathrm{d}i \\ &= \sum_{a=16}^{A} \int_0^{C(t,a)} \exp\left\{\alpha_0 + \alpha_1 E + \alpha_2 x + \alpha_3 x^2 + \varepsilon\right\}(d_{1it}(a) = 1) \mathrm{d}i \end{aligned} \tag{25}$$

Notice that S_t^s is a function of a skill price series $(r_1, r_2, ..., r_{t+A-16})$ since r_{t+A-16} is the skill price that age 16 people at t will face at the last age A. Notice also that aggregate skill supply, unlike individual skill supply, depends on the past history of skill price, since the distribution of state variables at t evolves from the initial period distribution of state variables at $t = 1$.

Skill demand at t is calculated by the marginal product of skill at t

$$r_t = \alpha \left(S_t^d\right)^{\alpha-1} K_t^{1-\alpha} \tag{26}$$

Therefore, the equilibrium skill price r^* is one that satisfies

$$r_t = \alpha \left(S_t^s\left(r_1, r_2, ..., r_{t+A-16}\right)\right)^{\alpha-1} K_t^{1-\alpha} \text{ for all } t \tag{27}$$

We write this as

$$r_t = F(r_1, r_2, ..., r_{t+A-16}) \text{ for all } t \tag{28}$$

First, we will prove such an equilibrium exist for $t = 1, ..., T$, given $(r_{T+1}, r_{T+2}, ..., r_{T+A-16})$ and then by increasing T to infinity we show that the equilibrium skill price exists even in an infinite horizon. The proof is not direct, so we proceed in several steps.

1. Step 1.

   We fix $(r_{T+1}, ..., r_{T+A-16})$ and the equilibrium $\{r_t\}_{t=1}^{T} \in \mathbb{R}_+^T \backslash \{0\}$ satisfies

$$\begin{aligned} r_t &= \alpha S_t^{s\alpha}(r_1, r_2, ..., r_{t+A-16}) K_t^{1-\alpha} \\ &= F_t(r_1, r_2, ..., r_{t+A-16}) \\ &\quad \text{for all } t \end{aligned} \tag{29}$$

   Notice that we restrict $r_t > 0$ for all t since $r_t \leq 0$ can not be an equilibrium.

2. Step 2.

   Given $\{r_1, ..., r_T\}$, the aggregate skill supply, S_t^s is uniquely determined.

   Proof. For each individual i at t,

$$Pr\{V_m(S(a)) = V_{n \neq m}(S(a)) \text{ for some } a\} = 0$$

   for $n, m = 1, 2, 3$

   This means that at an individual level, individual career decision is uniquely determined (given ε) with probability 1. Even though individual skill supply is stochastic (since ε is stochastic *ex ante*), since there is a continuum of people, S_t^s is still deterministic. (Law of Large numbers) ■

3. Step 3.

   Irrespective of $\{r_1, ..., r_T\}$, S_t^s is absolutely bounded above and below.

$$0 < \varepsilon < S_t^s < \infty$$

   Proof. $S_t^s < \infty$ since

$$S_t^s \leq \sum_{a=16}^{A} \int_0^{C(t,a)} \max_{E,x} \exp\left\{\alpha_0 + \alpha_1 E + \alpha_2 x + \alpha_3 x^2 + \varepsilon\right\} \mathrm{d}i < \infty$$

which simply means that aggregate skill supply is bounded by the total skill endowment in the economy. Notice that even though individual skill has an infinite support to infinity, due to law of large numbers, aggregate skill supply is bounded.

$S_t^s > \varepsilon > 0$ since at least there is a positive fraction of people of age A at t that choose to work even if the skill price, r_t, is near 0. We see this since

$$R_{1t}(A) > 0$$

for all people and

$$Pr\left\{R_{2t}(A) < 0 \text{ and } R_{3_t}(A) < 0\right\} > \varepsilon > 0$$

This means that some positive measure of people at age A will choose to work since their other two alternatives are even worse, irrespective of $\{r_1, ..., r_T\}$. This doesn't generally apply to younger people, since the fraction of people who choose to work might converge to 0 as $\{r_{t+1}, ..., r_T\}$ changes. (Think of some cases where r_{t+1} is increasing to infinity and the work experience square coefficient is so big below zero, so they want to save the choice of working for the next period at any cost. Notice that age A people are not affected by this, since they leave the labor market before $t + 1$.)[30] ∎

As a result, F_t is also absolutely bounded irrespective of $\{r_t\}$. This means that $\exists M_t, m_t$ such that

$$0 < m_t < F_t < M_t$$

where m_t and M_t is not a function of $\{r_t\}$.

[30] This is the easiest way to show that some people, whatever the skill price series is, are willing to work at each t in our model. We used the fact that there should be some people at age A whose schooling reward and staying home reward is negative, therefore, who want to work at any rate.

In case where schooling reward and staying home reward is bounded above zero, still we can show that some people should work. To show this, we restrict the skill price series in the beginning

$$\{r_t\}_{t=1}^T > \varepsilon > 0 \text{ for some } \varepsilon$$

rather than

$$\{r_t\}_{t=1}^T > 0$$

ε is chosen so that

$$\varepsilon = \alpha \bar{S}^{\alpha-1}\left(\min_t K_t\right)^{1-\alpha} = \bar{F}$$

where $\bar{S}$ is the greatest possible aggregate skill endowment at any time $t = 1, ..., T$, so that $\bar{S}^{\alpha-1}$ is minimized. This way we start from skill price series above ε and the resulting marginal product of skill is still greater than ε. In this case, those people who work at age A whatever the skill price is, do so not because schooling and staying home reward is too negative, but because their working reward is too high.

1. Step 4.

   S_t^s is a continuous function of $\{r_t\}_{t=1}^{T}$.[31]

   Proof. For an individual i,

   $$\lim_{\{r_t'\}\to\{r_t\}} \Pr\left\{d_{it}(a,\left\{r_t'\right\}) = d_{it}(a,\{r_t\})\right\}$$

   This means that by changing r_t for a very small amount, the fraction of people who change their career decisions is also very small. Since most people keep their original career decisions, aggregate skill supply, S_t^s, does not change much. Therefore, S_t^s is a continuous function of $\{r_t\}_{t=1}^{T}$. As a result, F_t is also continuous in $\{r_t\}_{t=1}^{T}$, since F_t is continuous in S_t^s. ■

2. Step 5.

   Given $\{r_{-t}\} = \{r_1, r_2, ..., r_{t-1}, r_{t+1}, ..., r_T\}$, $\exists r_t > 0$[32] such that

   $$r_t = F_t(r_t, r_{-t}) \text{ for all } t$$

   This means that given $\{r_{-t}\}$, we can find r_t such that skill market clears at t. (conditional equilibrium)

   Proof. To see this, note that from step 4, F_t is continuous in r_t and from step 3, $F_t(r_t, r_{-t})$ is absolutely bounded. The left hand side of the above equation r_t is an increasing function from 0 to infinity, and the right hand side, $F_t(r_t, r_{-t})$ is always bounded in $\mathbb{R}_+$[33] for all r_t and continuous in r_t, therefore, r_t and $F_t(r_t, r_{-t})$ must cross each other at some point given $\{r_{-t}\}$. (not necessarily unique) $r_t > 0$ since $F_t(r_t, r_{-t}) > m_t > 0$ ■

   This means that we can always solve for conditional equilibrium r_t^* given r_{-t} and write

   $$r_t^* = r_t(r_{-t})$$

3. Step 6.

   >From step 5, we can write

   $$r_1^* = r_1(r_2, r_3, ..., r_T)$$

[31] We can show that S_t^s is a strictly increasing function of $\{r_t\}_{t=1}^{T}$. But we don't need this.

[32] This is unique since S_t^s is strictly increasing in r_t.
When schooling and staying home is bounded above zero, we replace this by "$\exists r_t \geq 0$" .

[33] When schooling reward and staying home reward is bounded above 0, we replace this by "$F_t(r_t, r_{-t})$ is always bounded above ε".

and

$$
\begin{aligned}
r_2^* &= r_2(r_1, r_3, r_4, ..., r_T) \\
&= r_2(r_1(r_2, ..., r_T), r_3, r_4, ..., r_T) \\
&= r_2(r_3, r_4, ..., r_T)
\end{aligned}
$$

by induction, we have

$$
\begin{aligned}
r_3^* &= r_3(r_4, ..., r_T) \\
r_4^* &= r_4(r_5, ..., r_T) \\
&\ldots \\
r_{T-1}^* &= r_{T-1}(r_T)
\end{aligned}
$$

So we first solve r_T^* which satisfies

$$
r_T = r_T(r_1(r_T), r_2(r_T), ..., r_{T-1}(r_T)) \tag{30}
$$

and recover all $\{r_t^*\}_{t=1}^{T-1}$.

4. Step 7.

Step 1 through step 6 completes the proof given $\{r_{T+1}, r_{T+2}, ..., r_{T+A-16}\}$. We now increase T to infinity, then we obtain the equilibrium skill price series $\{t_t\}_{t=1}^{\infty}$.

This completes the proof.

We need following conditions to have an equilibrium.

1. We need continuum of people to have continuity of aggregate skill function in skill price.

2. We need a finite horizon of life. If people live forever, there might not be an equilibrium at all.

3. Aggregate skill supply should be bounded given t. This means that the distribution of the skill function should have a finite variance.[34]

[34]Our exponential function follows a log normal distribution with finite variance.

B Model Fit

This section shows the model fit between the actual data and the estimated model.

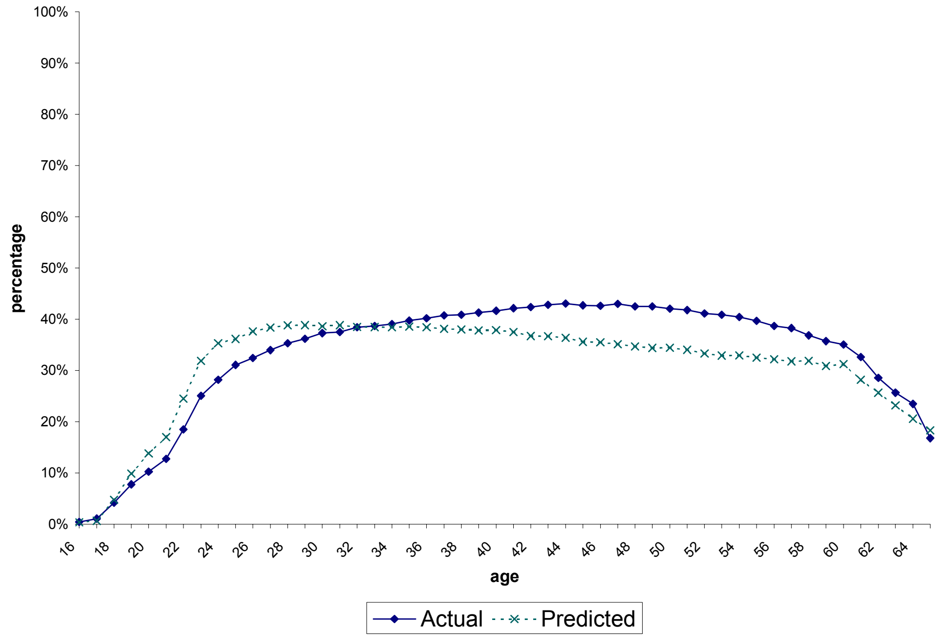

Figure 20: Male White-collar Employment rate by Age

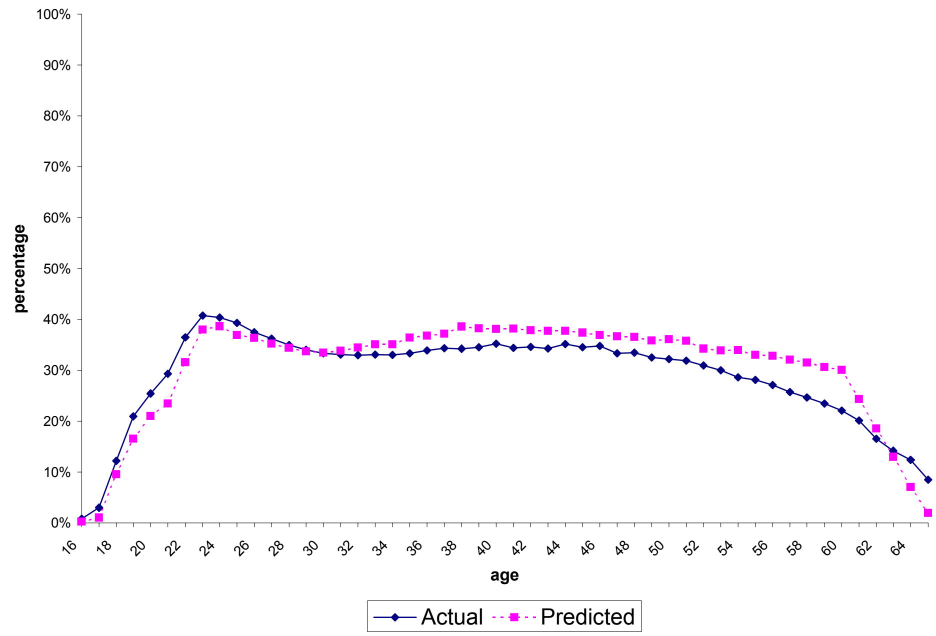

Figure 21: Female White-collar Employment rate by Age

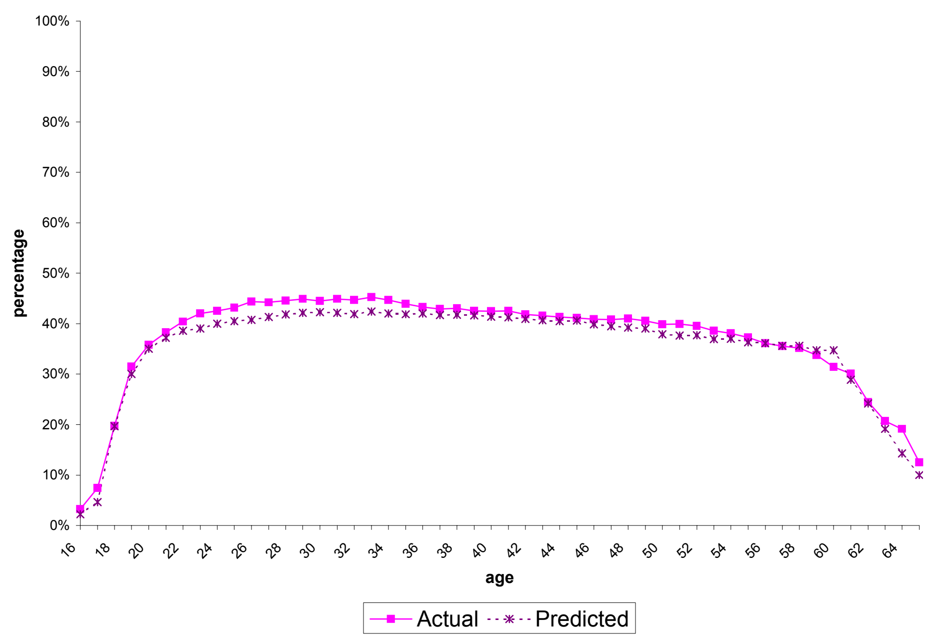

Figure 22: Male Blue-collar Employment Rate by Age

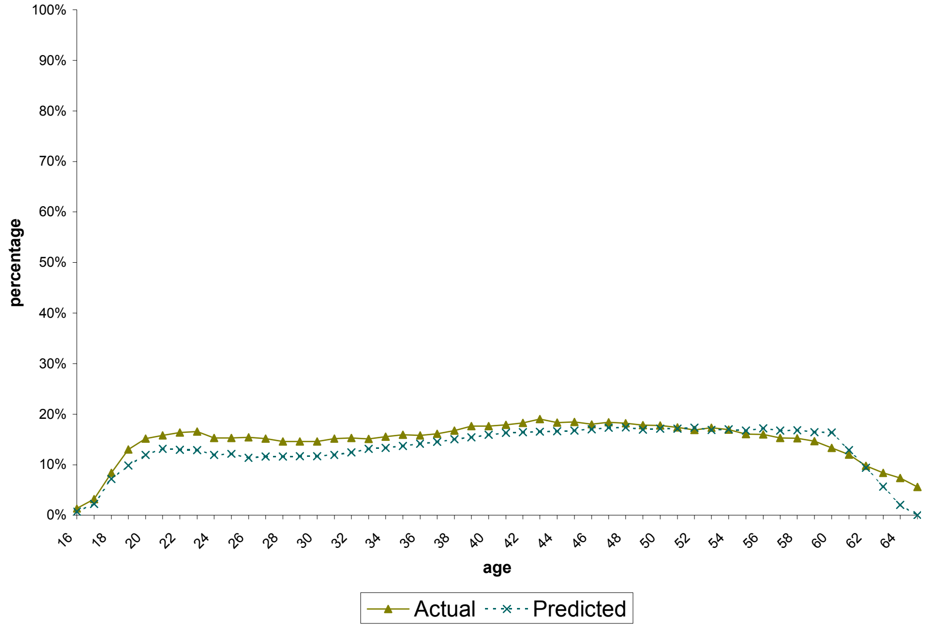

Figure 23: Female Blue-collar Employment Rate by Age

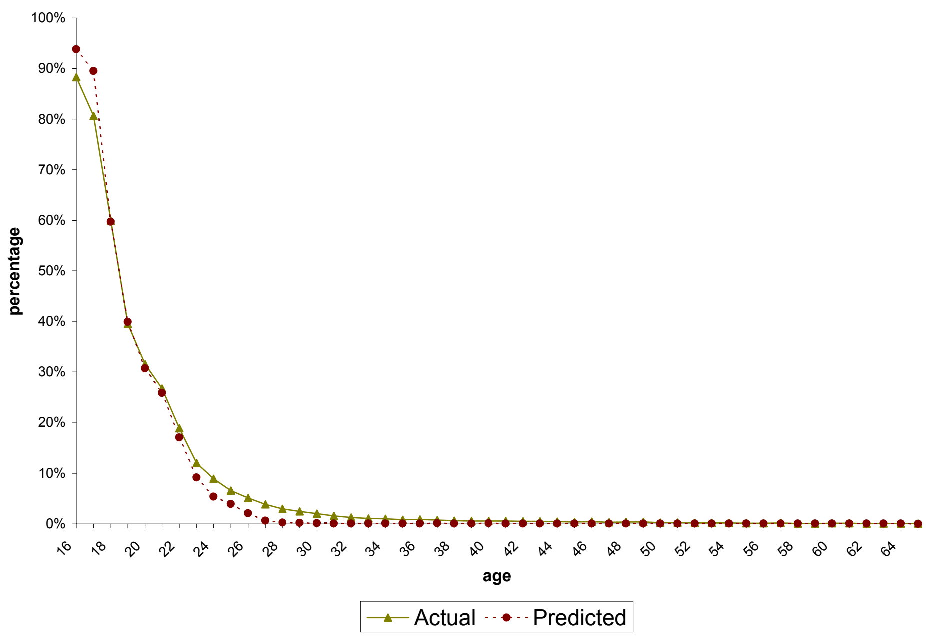

Figure 24: Male School Attendance Rate by Age

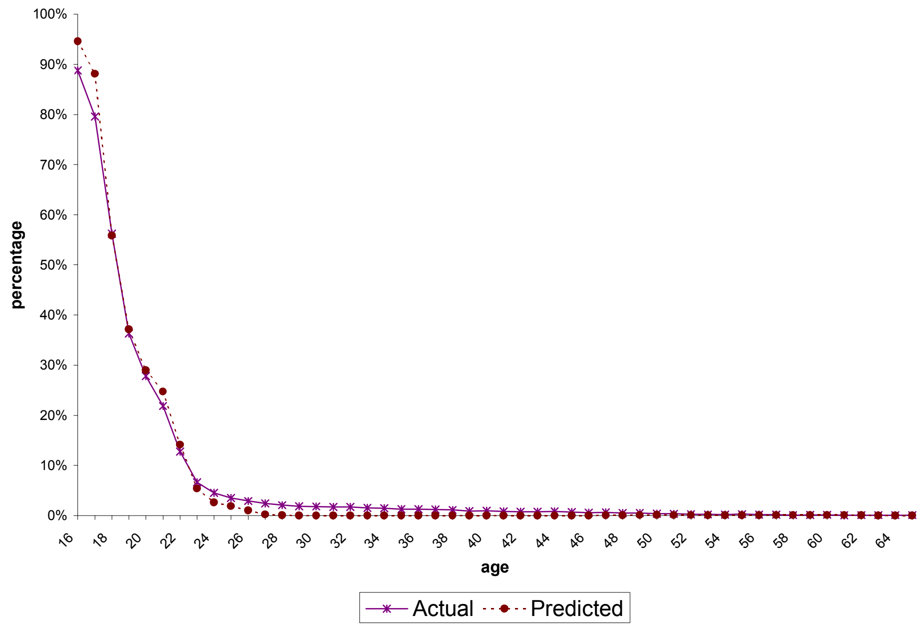

Figure 25: Female School Attendance Rate by Age

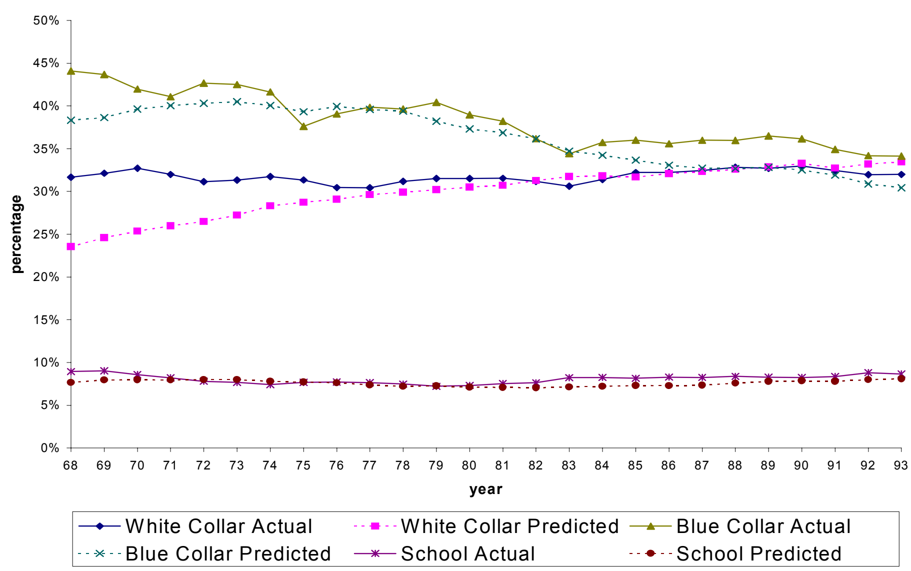

Figure 26: Male Career Decision Rate by Year

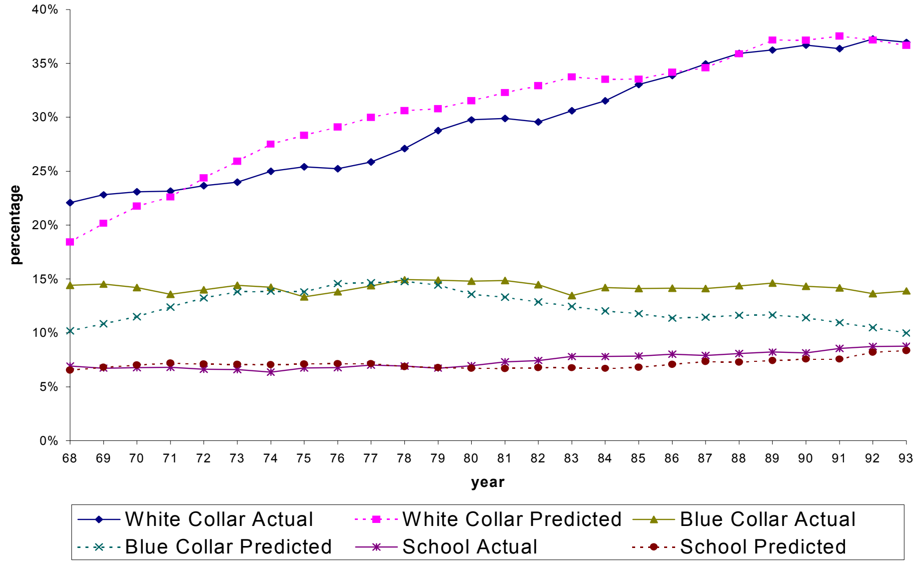

Figure 27: Female Career Decison Rate by Year

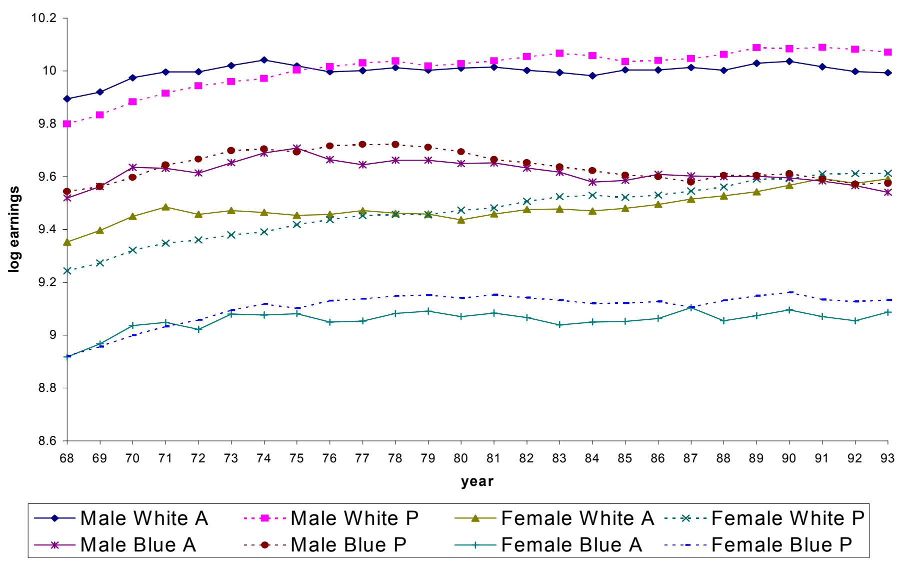

Figure 28: Earnings by Year and Occupation (A=Actual, P=Predicted)

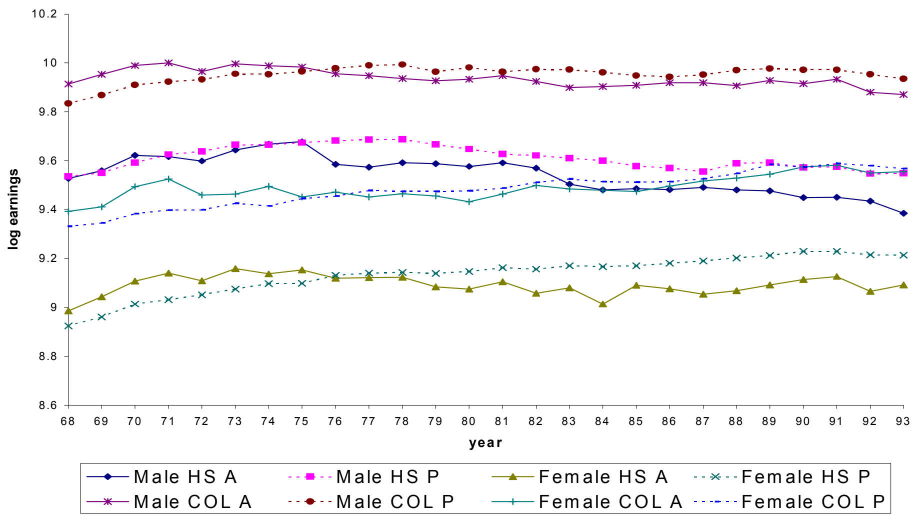

Figure 29: Earnings by Year and Education Level (A=Actual, P=Predicted, HS=High School, COL=College)