ECONOMICS 360
LABOUR ECONOMICS
LECTURE NOTES

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These notes will be updated as the term goes along. Pay attention to the date printed.

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I. Education, Skills, and Wages

I.A Introduction

> *A man educated at the expense of much labour and time to any of those employments which require extraordinary dexterity and skill, may be compared to one of those expensive machines. The work which he learns to perform, it must be expected, over and above the usual wages of common labour, will replace to him the whole expense of his education, with at least the ordinary profits of an equally valuable capital (Smith, Wealth of Nations, page 101.)*

Although the term "human capital" did not appear in print until the early 1960s, it is clear that Smith saw the analogy in 1776. As with its physical counterpart, there are two elements to the "human capital" model of wages and education. First, we develop a framework that makes a link between how much wages rise with response to a rise in education. Second, given the return to education generated by increased wages, when will a person stop educating themselves?

As your parents probably told you the first time you came home from university acting like a know-it-all, learning does not end with graduation, and wages do not just depend on your level of education. The idea of 'on-the-job' training as another form of human capital accumulation . As with classroom education, OTJ is not costless. It takes a worker away from tasks that the employer may want them to perform. Here we build on that insight to understand the pattern of wages across the labour market careers of individuals.

I.B Becker: The Investment Value of Education

Time and energy is put into creating *human capital*, skills embodied in the person through ability and knowledge. Opportunity costs, time, and patience are important factors in making the decision.

Benefits of Education

B.1. Additional earnings over a career

B.2. Utility from education and higher satisfaction from jobs

Costs of investing in human capital through education

C.1. tuition, books, extra expenses not included in tuition

C.2. disutility from schooling

C.3. foregone (delayed) earnings while in school

Simplifying assumptions (BGR 307)

SA.1. Utility and disutility from education cancel out

SA.2. Hours of work (including work in acquiring education) are fixed (but can differ with educational attainment)

SA.3. The income streams associated with different amounts of education are known with certainty

SA.4. Individuals can borrow and lend money at the real interest rate r

I.B.1 Flat earnings profiles and no discounting ($r = 0$)

A one-time decision is made at age 18 ($t = 1$). The decision is to choose either H (stop at high school) or U. The career lasts until age 65 ($T = 48$). U takes 4 (or, in general, E) years, and it costs \$$c$ each year. Let $W_H(t)$ denote the realized earnings at age t if education stops at high school, and $W_U(t)$ the corresponding earnings for university.

$$W_H(t) \equiv w_H \quad \text{for all} t \tag{1}$$

$$W_U(t) \equiv \begin{cases} -c & \text{if } t < 4 \\ w_U & \text{if } t \geq 4 \end{cases} \tag{2}$$

The value of the choices (H, U)

$$PV_H = \sum_{t=1}^{T} W_H(t) = Tw_H \tag{3}$$

$$PV_U = \sum_{t=1}^{T} W_U(t) = Ec + (T-E)w_U. \tag{4}$$

The **decision rule** choose $d \in \{H, U\}$ is simple:

$$d = \begin{cases} U & \text{if } PV_U > PV_H \\ H & \text{if } PV_U \leq PV_H. \end{cases} \tag{5}$$

We can also write this rule using the value of the difference between the two streams on a year-to-year basis:

$$\begin{aligned} D(t) &\equiv PV_U(t) - PV_H(t) \\ PV_D &= \sum_{t=1}^{T} D(t) \end{aligned} \tag{6}$$

So we can also state the investment decision rule as

$$d = \begin{cases} U & \text{if } PV_D > 0 \\ H & \text{if } PV_D \leq 0. \end{cases} \tag{7}$$

I.B.2 Rising wage profiles

Suppose instead that $W_H(t)$ and $W_U(t)$ vary with age. Without discounting the value of the two choices does not simplify, so we are left with the general form:

$$\begin{aligned} PV_H &= \sum_{t=1}^{T} W_H(t) \\ PV_U &= Ec + \sum_{t=E+1}^{T} W_U(t) \\ D(t) &= PV_U(t) - PV_H(t) \end{aligned} \tag{8}$$

Decision rule (7) still applies.

I.B.3 Rising wage profiles, discounting

The interest is used in computing forward values. A \$1 invested today will, in three periods, be worth: $1 \times (1+r) \times (1+r) \times (1+r) = (1+r)^3$. On the other, suppose you will be given \$1 in three periods. How many dollars is that worth today? We need to find the x

such that if it were invested today it would be worth \$1 in three periods. Thus, we wish to solve $x(1+r)^3) = 1$, which leads to $x = [1/(1+r)]^3$. The term

$$\beta \equiv 1/(1+r)$$

is known as the *discount factor* for the interest rate r, and is used to convert future values into present values. For example, one dollar received next year is worth β dollars today. One dollar received in two years is worth β^2 dollars today. Note that for $r=0$ the discount factor is $\beta = 1$. That means that a person does not care when income is received.

Since we described the investment decision by specifying the stream of earnings the person will receive in the future, the discount factor must now enter the value of the two choices.

$$PV_H = \sum_{t=1}^{T} \beta^{t-1} W_H(t)$$

$$PV_U = \sum_{t=1}^{E} \beta^{t-1} c + \sum_{t=E+1}^{T} \beta^{t-1} W_U(t) \tag{9}$$

Decision rule (7) still applies.

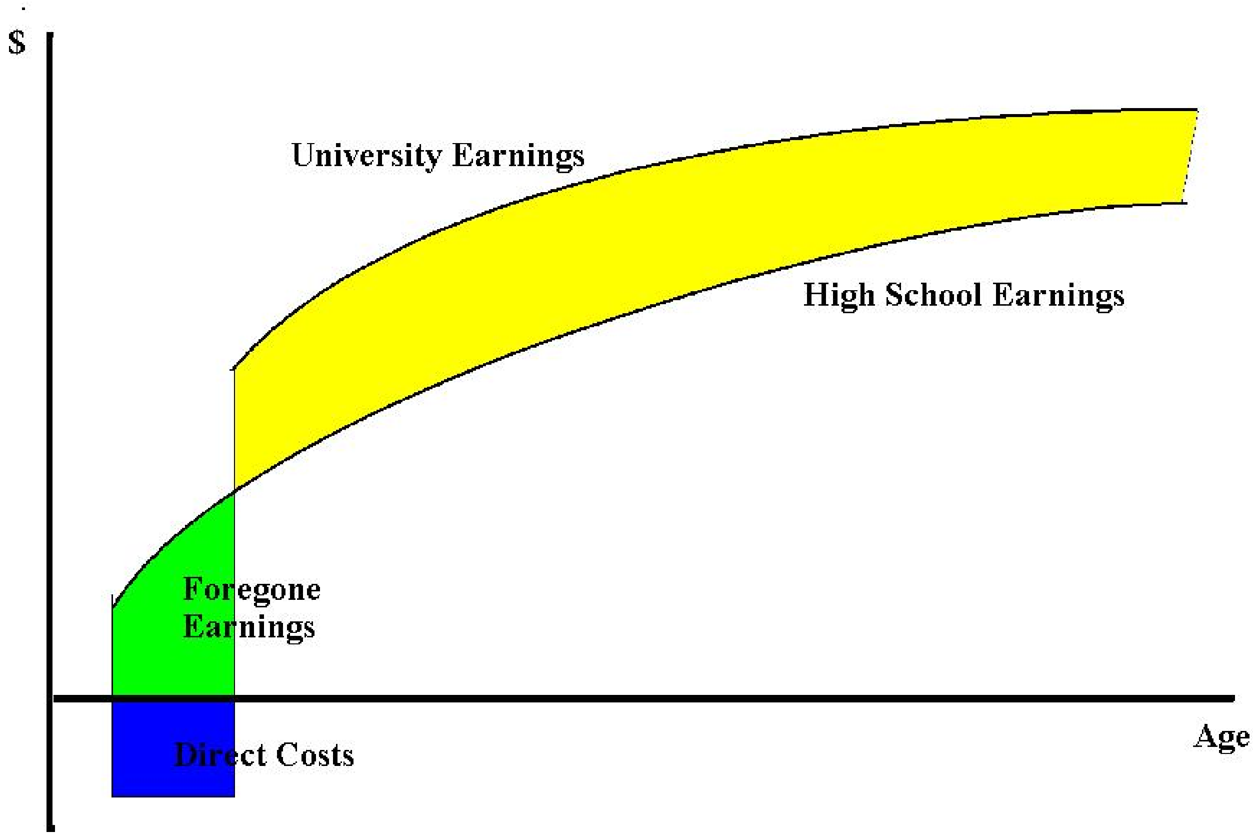
Example 1.Mapping the world into the simple human capital model.

Suppose you had to forecast the outcome of real changes with this simple model. Match each phenomenon on the left to the parameter of the model on the right that is the parameter you think would be most strongly influenced by the phenomenon, which direction it would change, and how you would guess PV_D might be altered.

A. tuition de-regulation	1. T
B. student loan programs	
C. the "information revolution"	2. $W_U - W_H$
D. reduced income tax rates	
E. lower minimum wages	3. $W_H + c$
F. reduced barriers to work for women with young children	
G. AIDS epidemic in Africa	4. r (or β)

I.C When to stop school

Notation



Notes on Figure 1:

1. The areas under the curves are the correct measures of earnings if the interest rate is zero. Otherwise, future earnings should be *discounted* relative to current earnings.
2. Why might earnings be concave over age?
3. Could income after retirement also depend upon education?
4. Which government policies affect the benefits and costs of acquiring education (both private and public)?

1. $W(t, e)$ yearly earnings at age t with e years of education.
2. $D(a, e) = W(a, e) - W(a, e-1)$ Difference in earnings at age a between e and $e-1$ years of education
3. $PV(e, b)$ Present discounted value of net earnings *after leaving* school with e years instead of $e-1$ years (marginal benefit of education).

$$PV(e, \beta) = \beta D(e+6, e) + \beta^2 D(e+7, e) + ... + \beta^{65-e-5} D(65, e). \tag{10}$$

Note: after e years of schooling started at age 6, the person must be 6+e years old; after e-1 years, the person must be e+4 years old.

4. $MC(e)$ the marginal cost of acquiring e years of education after having acquired $e-1$ years already

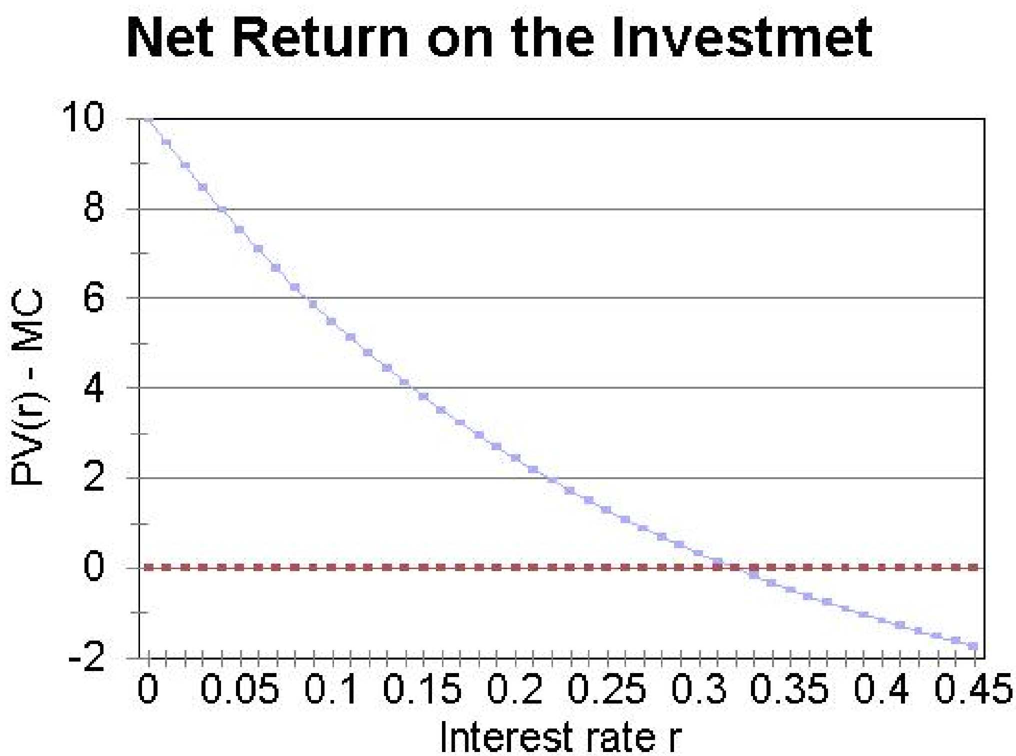
$$MC(e) = c + W(e+4, e-1).$$

5. $I(e)$ The internal rate of return for e years of education. The value $I(e)$ such that $\beta^\star = 1/(1+I(e))$ and

$$PV(e, 1/(1+I(e))) = MC(e).$$

Notes

1. If $PV(e,b) > MC(e)$ then lifetime net earnings are increased by staying in school another year.
2. If we can measure $D(a,e)$ and $MC(e)$, then the decision depends upon the interest rate r that the person uses in the calculation. The greater the value of r the more *impatient* the person is.
3. If the person uses $I(e)$ to calculate $PV(e,b)$ they will be just indifferent to going to another year of school.
4. Given an interest rate r, the value $e^\star$ such that $I(e^\star) = r$ is (under some important conditions) the optimal years of education to earn.
5. The IRR for an investment may not exist, and there may be multiple solutions.
6. As long as costs are paid up front and benefits remain positive during the life of the investment the IRR is a well-defined concept.


Net Return on the Investmet
PV(r) - MC
10
8
6
4
2
0
-2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Interest rate r

Review of the Education Decision.

- Given: $E(=4)$, T, $W_U(t)$, $W_H(t)$, c, r, define:

$$
\begin{aligned}
\beta &= \frac{1}{1+r} \\
D(t) &= \begin{cases} -W_H(t) - c & t \\ W_U(t) - W_H(t) & t > E \end{cases} \\
PV_H &= \sum_{t=1}^{T} \beta^{t-1} W_H(t) \\
PV_U &= \sum_{t=1}^{E} \beta^{t-1} c + \sum_{t=E+1}^{T} \beta^{t-1} W_U(t) \\
{}^{P}V_D(r) &= \sum_{t=1}^{T} \beta^{t-1} D(t)
\end{aligned} \tag{11}
$$

- The optimal investment decision is

$$
d = \begin{cases} U & \text{if } PV_D > 0 \\ H & \text{if } PV_D \le 0. \end{cases}
$$

- An *internal rate of return* for U is a $r^\star$ that solves

$$
PV_D(r^\star) = 0.
$$

- If $D(t)$ is monotonically increasing in t, then $PV_D(r)$ is monotonically decreasing in r, for $r \ge 0$. Under this condition, an equivalent decision rule is:

$$
d^\star = \begin{cases} U & \text{if } r < r^\star \\ H & \text{if } r \ge r^\star. \end{cases}
$$

Example 2.IRR in a three period problem

Suppose $T = 3$, so there are only three ages (1,2,3). A person can spend period 1 in education ($E = 1$). $W_U(t) = 20$ and $W_H(t) = 10$ for all t and $c = 5$. Write out PV_D as a function of β and solve for $\beta^\star$, from which $r^\star = 1/\beta^\star - 1$.

I.D Mincer: The Human Capital Earnings Function

Key Assumptions

KA.1. Competitive Wages. A person is paid a wage equal to their value marginal product (VMP), i.e. how much they add to the revenue of the firm. Under perfect competition for workers (and other assumptions examined later), the wage of a person would be equal to this value.

KA.2. Schooling and on-the-job training. A person's VMP depends on their level of human capital, which is acquired through education and OTJ training.

Simplifying Assumptions

SA.1. Acquired skills last forever (no depreciation)

SA.2. There are constant returns to skill acquisition

SA.3. A person has a fixed career length $(t = 0, 1, 2, ..., T)$

SA.4. Total time working and engaging in OTJ is constant.

The First Period Working

1. p_sThe increase in ln(VMP) for an additional year spent in school.
2. $W_s = W_0 exp(p_s S)$ potential wage rate just after finishing s years of school (called time 0) **if no OTJ training occurs in time 0**
3. k_0 fraction of time spent on the job acquiring skills but not contributing to the firm's product. With competition firms don't pay workers for the time spent acquiring skills. They only pay for $(1 - k_0)$.
4. $w_0 = (1 - k_0)W_s$ observed wage at time 0
5. p_j The increase in human capital per unit of time spent doing OTJ

The Second Period on the Job

1. $W_1 = W_s(1 + pk_0)$ potential wage at time 1 **if no OTJ training occurs in time 1**

2. $k_1$fraction of time spent doing OTJ training in period 1. (We are assuming constant hours on-the-job, but allowing the amount of time working and learning OTJ to vary.)

3. $w_1 = (1 - k_1)W_1$ Actual wages at time 1

It is generally optimal to set $k_1 < k_0$. Why? This implies $w_1 > w_0$

The General t^{th} period on the Job

1. k_t *fraction of time spent doing OTJ in period t.*

2. $W_t = W_{t-1}(1 + p_j k_t)$ potential wages at any time t **if no OTJ occurs in time t**

3. $w_t = (1 - k_t)W_t$ Actual wages at time t

It is optimal to set $k_t < k_{t-1}$. This implies $w_t > w_{t-1}$. Wages grow with time spent working. For two reasons: (1) an increase in the stock of human capital, which raises wages; (2) a decrease in the amount of investment in further human capital, which raises the amount of productive time spent working.

Note that if x is small number (close to 0) then $ln(1 + x)$ is approximately equal to x

$$\ln(w_t) = \ln(W_t) + \ln(1 - k_t) = \ln(W_0) + p_s S + p_j(k_0 + k_1 + ... + k_{t-1}) + \ln(1 - k_t) \qquad (12)$$

I.D.1 Mincer's Equation

Suppose that training decreases linearly over the labour market career (t=0,1,2,...,T)

$$k_t = k_0(1 - t/T) \qquad (13)$$

After some manipulation this leads to

$$ln(w_t) = ln(W_0) + p_s S + (p_j k_0)t - (p_j k_0/2T)t^2 + ln(1 - k_t) \qquad (14)$$

The natural logarithm of wages with t years of experience depends on years of schooling, t and t^2.

Example 3.Calculus comes in handy

Use calculus to find the point at which wages peak within the career. That is, find the $t^\star$ such that $\ln(w_t)$ attains a maximum.

The coefficients on the variables t and $t^\star$ are not items we can directly measure. But this equation suggests that one can use **multiple regression** to estimate these coefficients from data on wages, schooling, and OTJ experience. For the purposes of regression analysis, this equation is usually re-written in a form called **Mincer's Equation** or the **Human Capital Earnings Function**:

$$ln(w) = \beta_0 + \beta_1 S + \beta_2 X + \beta_3 X^2 + u \qquad (15)$$

where

1. $\beta_0 = ln(W_0)$ Earnings of the person if they have acquired no human capital
2. $\beta_1 = p_s > 0$ Return to Education as a human capital investment.
3. $X = t$ Years of labour market eXperience. In many sources of data actual experience (# of years worked in the past) is not directly measured. It is typical to approximate this with
4. $X = Age - S - 6$ This is usually referred to as **potential labour market experience**.
5. $\beta_2 > 0$ and $\beta_3 < 0$ Coefficients on eXperience and eXperience squared. These depend on the return to OTJ training.
6. u This term captures all other aspects of a persons wage-earning capacity (the residual in a linear regression).

Estimating the returns to schooling using Mincer's equation is simply a *multiple linear regression* problem (see BGR pp. 12-22). It has been carried out thousands of times using data sets from many countries:

Notes

1. These are fairly typical results. Simple estimates on the returns to education suggest a coefficient of around 0.07, although in some countries and in other time period the returns are much different. $\ln(earnings)$ are concave in experience, with the peak somewhere in mid career.
2. Why might the age-profile for women be flatter than for men?
3. At what point in people's career wages peak? How does this compare with the prediction of the model in which training time decreases linearly until retirement?
4. Suppose we estimate this equation for another year (1980, say). Where does inflation come into play? (Hint: think about the $ln(earnings)$ specification.)
5. The value of R^2 can be interpreted as saying that about 13 percent of the variation in ln(wages) is explained by education and potential labour market experience using the Mincer equation. The rest of the variation across people is captured by the error term u.. What other factors do you think would help explain differences in wages across individuals?

I.E Combining Becker and Mincer

NOTES
Secondary (I) : Secondary with respect to elementary schooling
Secondary (II) with respect to incomplete secondary schooling
Collegiate and bachelor's degrees with respect to completed secondary schooling
Master's with respect to bachelor's degrees
PH D with respect to master's degrees

How did Vallaincourt calculate this table?

1. Estimated several linear regression equations using Cdn data
2. Calculated predicted earnings for people by age and education level (and B.A. major)
3. Collected information on tuition costs, costs of education, etc.
4. Used a spreadsheet program (?) to calculate the internal rate of return

II. Self-Selection and Signalling

II.A Roy: Self-selection (B296-299)

Elements

- Two sectors ($i = 1, 2$) with output prices π_i. A person is employed in one or the other sector—people must specialize in production.
- A given person has productivity or skill-level in sector i of S_i. So $S = (S_1, \S_2)$ describes a person's situation. We assume $S_1 \geq 0$ and $S_2 \geq 0$ These are sector-specific skills. They can also be thought of a coming from a set of underlying skills used in both sectors—such as arithmetic, manual dexterity, kissing-up to the boss, etc.—but that are used with different intensity in each sector. For example, manual dexterity is more important than blue-collar jobs, so holding all else constant a person endowed with more dexterity has a greater value of blue-collar skill than someone with less.
- A person earns what they produce. With skill S_i a person would earn $W_i = \pi_i S_i$ if employed in sector i.
- Define $\pi = \frac{\pi_1}{\pi_2}$. Then the set of skill pairs that are indifferent between sector is described by the combinations such that $W_1 = W_2$ or $S_2 = \pi S_1$.

We compare and contrast two selection rules

- Random Assignment (more accurately called independent assignment): a person is assigned to sector i with probability p_i independent of their skills (S_1, S_2). If assigned to i they are paid W_i. Obviously $p_2 = 1 - p_1$.
- Self-selection (or pursuit of comparative advantage): a person chooses or is assigned the sector in which they earn the most: $i^\star(S_1, S_2) = \arg\max_{i=\{1,2\}} \pi_i S_i$ and they are paid $W^\star = \pi_{i^\star} S_{i^\star}$

.

We also compare and contrast three distributions of skills across people. In each case, skill S_1 is uniformly distributed between 0 and some number U across people. Skill S_2 is also uniformly distributed between 0 and the same U. For the statistically inclined student, we say that the *marginal density* of skill is i takes the form

$$f_i(S) = \begin{cases} 1/U & \text{if } 0 \leq S \leq U \\ 0 & \text{otherwise} \end{cases}.$$

This simply means that the skills within sectors is evenly spread over $[0, U]$ in the population. The difference in the three situations is how S_1 and S_2 are correlated across people. That is, how ability in the two sectors move together across individuals in the population. Again, statistically, the different worlds are different assumptions of the *joint density* of skills, denoted $f(S_1, S_2)$.

The three worlds we consider are:

- Life is Unfair (LIU): $S_2 = S_1$ for all people. There is no comparative advantage. A person who is better at sector 1 than someone else is also better than in sector 2. The joint density of skills in this case

  $$f^{LIU}(S_1, S_2) = \begin{cases} 1/U & \text{if } S_2 = S_1 \text{ and } 0 \leq S_1 \leq U. \\ 0 & \text{otherwise.} \end{cases}$$

  The statisticians will realize that the correlation between S_1 and S_2 is +1.

- Every has a special talent (EHT): $S_2 = 1 - S_1$. Now there is nothing but comparative advantage. A person better in sector 1 than someone else is necessarily worse in sector 2. The joint density of skills in this case

  $$f^{EHT}(S_1, S_2) = \begin{cases} 1/U & \text{if } S_2 = 1 - S_1 \text{ and } 0 \leq S_1 \leq U. \\ 0 & \text{otherwise.} \end{cases}$$

  The statisticians will realize that when everyone has a special talent the correlation between S_1 and S_2 is −1.

- Table top talent (TTT): S_2 and S_1 are independently distributed. The independence property is equivalent to assuming the joint density factors into the marginals:

  $$f^{TTT}(S_1, S_2) = f_1(S_1)f_2(S_2) = \begin{cases} 1/U^2 & \text{if } 0 \leq S_1 \leq U \text{ and } 0 \leq S_2 \leq U \\ 0 & \text{otherwise.} \end{cases}$$

Here the correlation is 0.

The Roy model says something about how the allocation of skill and the distribution of skills maps into the distribution of wages (of earnings). We have described six situations that cover the extremes: 2 assignment rules $\times$ 3 skill correlations. (For completeness we could include the very extreme world in which people are assigned to the sector they are least able in.) Let $g_a^c(w)$ denote the distribution of wages under correlation c and assignment rule a. For example, $g_r^{LIU}(w)$ is the distribution of wages in a world where life is unfair and sectors are randomly assigned, and $g_s^{EHT}(W)$ is the distribution when everybody has talent and assignment is based on self-selection.

Wage Distributions under Random Assignment when $\pi_1 = \pi_2 = \pi = 1$.

If a person is randomly assigned to sector, then i and skills (S_1, S_2) are independently distributed. The joint density of independent events is the product of the marginal densities. But since skills are sector-specific, only the density in the assigned sector affect wages. And when $\pi = 1$ wages within a sector are the same as skills within the sector. That is, when assigned $i = 1$ we can substitute a W and S_1. Thus,

$$g_r^c(W) = \frac{1}{2} f_1^c(W) + \frac{1}{2} f^c(W) = \begin{cases} 1/U & \text{if } 0 \le W \le U \\ 0 & \text{otherwise} \end{cases}.$$

Since all three skill distributions have uniform skills with sector the wage distribution is also uniformly distributed over $[0, U]$. (This depends on $\pi_1 = \pi_2 = \pi = 1$).

Wage Distributions under Self-selection when $\pi_1 = \pi_2 = \pi = 1$.

Now we can't use formulas that imply that sector assignment and skills are independent of each other. People pursue comparative advantage, meaning that assignment is anything but independent of skills.

- LIU: When $S_2 = S_1$ and $\pi_1 = \pi_2$ sector doesn't matter. Everyone is indifferent between being in $i=1$ and $i=2$. So once again

$$g_s^{LIU}(W) = f_1^{LIU}(W) = f_2^{LIU}(W) = \begin{cases} 1/U & \text{if } 0 \le W \le U \\ 0 & \text{otherwise} \end{cases}.$$

- EHT: At last things get interesting. Now people chose the sector they are best in. Since $S_2 = U - S_1$ they chose based on the maximum of S_1 and $1 - S_1$. Thus people with $S_1 \ge U/2$ earn S_1 and locate in $i = 1$. For $S_1 < U/2$ they earn $S_2 = U - S_1$ and chose $i = 2$. The lowest wage in either sector is $U/2$. Within sectors skills (and thus wages) are uniformly distributed, but now wages are only distributed in the interval $[U/2, U]$.

$$\begin{aligned} g_s^{EHT}(w) &= \begin{cases} f_1^{EHT}(W) + f_2^{EHT}(W) & \text{if } U/2 \le W \le U \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 2/U & \text{if } U/2 \le W \le U \\ 0 & \text{otherwise} \end{cases}. \end{aligned}$$

  Pursuit of comparative advantage makes the distribution of earnings more equal than the distribution of talent.

- TTT: It's not obvious how to show this without deriving the cumulative density of wages:

$$\begin{aligned} G_s^{TTT}(W) &= Prob(Wage \le W) \\ &= Prob(\max S_1, S_2 \le W) \\ &= Prob(S_1 \le W \text{ AND } S_2 \le W) \\ &= Prob(S_1 \le W)Prob(S_2 \le W) = \frac{W}{U^2} \end{aligned}.$$

  for $0 \le W \le U$. The second-to-last step is where the independence of the table-top distribution comes into play (joint probability of independent events is the product of the marginal probabilities). If you follow that, then just take the derivative of $G_s^{TTT}(W)$ to get the density of wages:

$$g_s^{TTT}(w) = \frac{\partial G_s^{TTT}(W)}{\partial W} = 2W/U^2.$$

  Now the wage distribution doesn't look like the marginal skill distributions at all. It turns out that wages are less equal than under random assignment, but we leave it to Econ 361 to define the measures of inequality that lead to this result.

Wage Distributions under Self-selection when $\pi_1 = \pi < \pi_2 = 1$.

Now we look at how a difference in relative prices leads to different wage distributions under self-selection.

- LIU: When $S_2 = S_1$ and $\pi < 1$ sector matters. Everyone migrates to sector 2. But it has no affect on wages, because skills in sector 2 are uniformly distributed. So $g_s^{LIU}(W) = 1/U$ still.

- EHT: With equal prices there is one type of person indifferent between sector, those with $S_1 = S_2 = U - S_1 = U/2$. When $\pi < 1$ the indifference point is $\pi S_1 = S_2 = U - S_1$, so $S_1 = U/(1+\pi) > 1/2$. That person earns $\pi U/(1+\pi)$ in either sector, so the lower bound on wages is $\pi U/(1+\pi) < U/2$. Spreading out from that point there are equal uniform people at each wage. However, it takes a little statistics to work out that the density is $(1+\pi)/(\pi U)$. (Try working at $G_s^{EHT}(W)$ and then take its derivative.) The maximum wage in sector 1 is now πU, while the maximum wage in sector 2 is still U. So in the range $[\pi U, U]$ the density falls back to $1/U$. Thus for $\pi < 1$ we get

$$g_s^{LIU}(W) = \begin{cases} \frac{(1+\pi)}{\pi U} & \text{if } \frac{\pi U}{1+\pi} \le W \le \pi U \\ 1/U & \text{if } \pi U < W \le U. \end{cases}$$

  As π falls the wage distribution approaches the uniform case, which means it *increases* inequality when there is no absolute advantage.

- TTT:

$$\begin{aligned} G_s^{TTT}(W) = Prob(Wage \le W) &= Prob(\max \pi S_1, S_2 \le W) \\ &= Prob(\pi S_1 \le W \text{ AND } S_2 \le W) \\ &= Prob(S_1 \le W/\pi)Prob(S_2 \le W) \\ &= \begin{cases} \frac{W^2}{\pi U^2} & \text{if } 0 \le W \le \pi U \\ \frac{W}{U} & \text{if } \pi U < W \le U \end{cases} \end{aligned}$$

  Taking the derivative:

$$g_s^{TTT}(w) = \frac{\partial G_s^{TTT}(W)}{\partial W} = \begin{cases} \frac{2W}{\pi U^2}. & \text{if } 0 \le W \le \pi U \\ \frac{1}{U} & \text{if } \pi U < W \le U \end{cases}$$

As π falls the wage distribution approaches the uniform case, which means it *decreases* inequality.

The conclusion: relative prices, assignment mechanisms, and skill distributions are all tangled up together. The same distribution of wages may be associated with quite different responses to price changes.

II.B Self-selection bias in the HCEF (B238-240)

II.C Griliches: Ability Bias Estimating Returns to Education

Correcting Ability Bias The early attempts to correct for ability bias used measures of ability, such as scores on IQ or aptitude tests in the regression as well:

$$ln(w) = \beta_0 + \beta_1 S + \beta_2 X + \beta_3 X^2 + \beta_4 AbilityMeasure + u \tag{16}$$

The typical result was that estimates of this kind of equation lead to *lower* estimated values β_1 because schooling (S) was partly picking up the effect of ability when ability was omitted from the regression equation. There are some serious statistical problems with these types of estimates. In particular, if the ability measure has a lot of noise in it, then the estimates will be biased by that as well.

But for many years the consensus was that β_1 is slightly over-estimated using linear regression.

The next attempts were to try to control for ability in some other way. For example, consider two people (person A and person B) who we somehow know ahead of time have the same ability, call it F (for reasons explained below). This term F must capture all aspects of ability that help determine education levels that also explain wages. There can other post-education factors that help explain wages as well (such as luck finding a job). Now write Mincer's equation out for those two people:

$$ln(w_A) = \beta_0 + \beta_1 S_A + \beta_2 X_A + \beta_3 X_A^2 + F + e_A \tag{17}$$

$$ln(w_B) = \beta_0 + \beta_1 S_B + \beta_2 X_B + \beta_3 X_B^2 + F + e_B \quad (18)$$

The e terms in both equations represent elements of wages that are not correlated with levels of education. Taking the difference in these two equations:

$$ln(w_A) - ln(w_B) = \beta_1(S_A - S_B) + \beta_2(X_A - X_B) + \beta_3(X_A^2 - X_B^2) + (e_A - e_B) \quad (19)$$

In other words, the difference in earnings between person A and B have been purged of the ability effect. The difference in their education levels still has the same coefficient β_1. So we can estimate this equation based on the differences in our two people. However, it must be the case that these two people have different amounts of schooling. Otherwise, schooling is 0 and it wouldn't matter what value takes β_1. (In econometric jargon, β_1 would not be *identified*.)

However, this method introduces a new problem: that the difference between self-reported schooling measures amplifies any errors in these variables. That is, people often don't report the amount of education accurately. When taking the difference between the schooling variables the variance of the measurement error increases, which leads to a downward bias in the estimates. Thus certain researchers have carried out analyses that β_1 is *underestimated* by standard regression analysis.

Strategies for controlling for F

1. Run a controlled random experiment on educational attainment.
2. Analyze a change in government policy that leads people to choose different levels of education for 'random' reasons.
3. Mandatory attendance laws and season of birth (Angrist and Krueger QJE 1991).
4. Study geographic access to universities (Card 1995).
5. Siblings and Twins

II.C.1 Selected Studies of Returns to Education for Twins

These results can be questioned on several grounds. But the primary problem is that the difference approach requires the difference in schooling levels $(S_A - S_B)$ across twins be *uncorrelated* with the difference in their errors $(e_A - e_B)$.

That is, the difference across twins in schooling must be "random" or due to "errors in selecting the optimal level of education." If the difference in schooling levels is not random, but also based on real differences in the ability of the twins, then these estimates are open to the same kinds of ability bias as ordinary data.

II.D Spence: Education as a signal (pp. 312-315)

In the HC model, spending time in school is thought of as an investment in a form of capital, skills learned while in school. This interpretation, however, is not the only one. Another possibility is that time spent in school does not **add** to a person's skills, but merely **signals** those skills to prospective employers.

Key Assumptions

1. People differ *intrinsically* in their productivity while working.
2. Employers cannot directly observe a worker's productivity
3. Productivity is related to the cost/difficulty person has in acquiring education.

Simplifying Assumptions

1. Two types of workers:

   low ability (type L) and high ability (type H).
2. The proportion of type H in the population is q.
3. Type H have marginal productivity P^h; type L have P^l
4. Marginal cost of acquiring another year of education is constant: c^h, c^l
5. Firms compete for workers, leading to zero economic profit in equilibrium.

II.D.1 Equilibrium in a signalling model

Equilibrium Conditions

1. Employers form **beliefs**

   about the relationship between (unobserved) productivity and (observed) educational attainment.

2. They pay workers according those beliefs (making wages depend on education).
3. The beliefs of employers are fulfilled by the choices of people acting in the own best interests.

Two Types of Equilibria

1. Case 1: No signalling (*pooling equilibrium*).

   If workers cannot or do not signal their type and firms cannot learn it from watching the person work, then everyone will be paid the same wage. The zero-profit condition requires then that firms pay workers their average productivity:

$$W^P = qP^h + (1-q)P^l \tag{20}$$

2. Case 2: Signalling (*separating equilibrium*).

   Beliefs of employers:

$$\begin{cases} MP = P^l & \text{if } e < e^\star \\ MP = P^h & \text{if } e \geq e^\star \end{cases} \tag{21}$$

   Wage Offers

$$W(e) = \begin{cases} P^l & \text{if } e < e^\star \\ P^h & \text{if } e \geq e^\star \end{cases} \tag{22}$$

   In equilibrium, $e^\star$ must be such that high quality workers find in worthwhile acquiring $e^\star$:

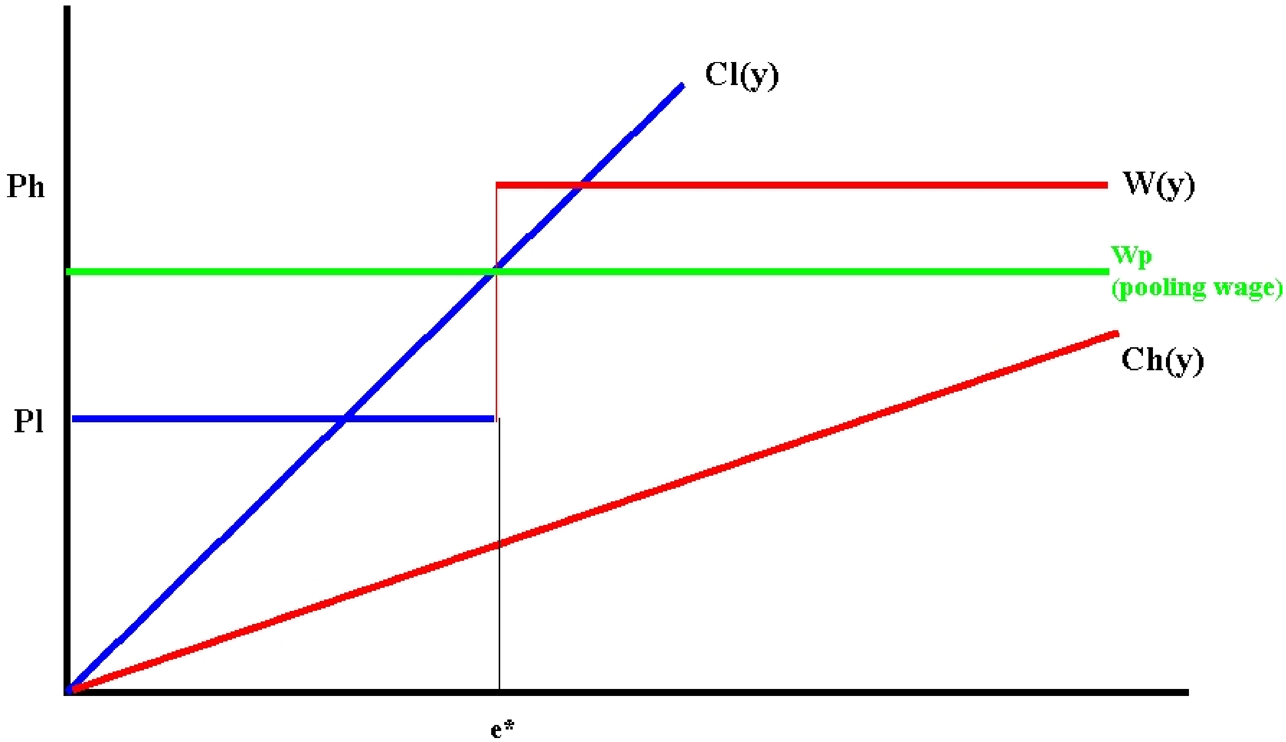
$$P^h - P^l \geq c^h e^\star \tag{23}$$

   low quality workers do not find it worthwhile.

$$P^h - P^l < c^l e^\star \tag{24}$$

   Bounds on the equilibrium signal:

$$(P^h - P^l)/c^l < e^\star \leq (P^h - P^l)/c^h. \tag{25}$$



Notes about signalling equilibrium

1. Imagine two worlds (economies), one in the signalling equilibrium and one in the pooling equilibrium. Who gains from signalling? Who loses?
2. How good an explanation is signalling for the earnings difference across education groups at older ages?
3. Are there ways to distinguish between signalling and human capital explanations?
4. Are there other situations in the labour market where signalling may play a role?

Example 1.Who prefers signalling?

Find an expression for the $e^\star$ such that the H workers are just indifferent between the signalling equilibrium and the pooling equilibrium. **Answer:** Utility for type H in the pooling equilibrium equals

$$W^P - c^h 0 = qP^h + (1-q)P^l \qquad (26)$$

Utility for type H in the separating equilibrium equals

$$P^h - c^h e^\star \tag{27}$$

Setting these equal we find a value

$$e^{\star\star} = (1-q)(P^h - P^l)/c^h \tag{28}$$

If the education requirement is above $e^{\star\star}$ then type H people are worse off with signalling than without. Otherwise, they are better off. Notice that q enter $e^{\star\star}$ but not the bounds on equilibrium values of $e^\star$. As q increases $e^{\star\star}$ falls. Interpretation?

II.D.2 Evidence for the Signalling Hypothesis

For the most part, signalling and human capital make similar empirical predictions, but they have very different implications for policy. For example, costly increases in degree requirements would, under the human capital model, improve the skills of graduates.

Strategies

1. Find cases when no signal is being sent (Wolpin 1977)
2. Find cases where signalling and human capital make *opposite*predictions
3. Solve a model with both signalling and human capital

Bedard (1998) identifies such a situation: with signalling easier access to university should actually **increase** the proportion of students who drop out of high school (holding all else constant). She finds consistent support for this effect, which suggests that high school graduation has a signalling element.

Notes

1. Shaded areas are correct costs and benefits only assuming an interest rate of 0 (discount factor of 1).
2. With specific training are there cost/benefit sharing arrangements that give one side or the other an incentive renege on the agreement?

3. What other factors might determine how the costs and benefits are shared?
4. How does the presence of firm-specific skills alter the Human Capital Earnings Function?

III. Job Search And Unemployment

III.A Introduction

None of the earlier work accounted for the fact that at all times a sizeable fraction of the labour force is not currently working, but rather is "actively seeking work." In a world of perfect information and market clearing there is no explanation for unemployment of this type. Here we introduce a model for why workers seeking employment do not necessarily find it right away. It is based on the assumption that there are differences across jobs that can only be discovered by sampling from jobs. In this simple model, the only difference across jobs is the wage or earnings they pay. These differences in earnings may reflect the fact that a given worker is a better match for some jobs than others, and this match is reflected in productivity and the wage the firm offers the worker.

III.B Stigler / McCall: Basic Job Search Model

Basic Assumptions

1. a simple *work*or *not work (be unemployed)* decision
2. a decision to take a job is permanent
3. a finite number of periods of life (relaxed later on)
4. a constant value of not working
5. a known and constant distribution of wage offers

III.B.1 One Period (T) Decision

Start with the problem of a person starting out unemployed for a single period T. The person values staying at home at some value c. The person has a wage offer in hand, w_T. The choices are to either stay unemployed (U) or work (W) at the wage offer. The values of the two choices are:

$$V_T^U = c$$
$$V_T^W(w_T) = w_T. \tag{29}$$

The optimal choice (or *optimal decision rule*) is: work if $V_T^W(w_T) \geq V_T^U$, or if $w_T \geq c$. This defines the lowest wage offer, the *reservation wage*, that will be accepted by the person in period T:

$$V_T^U = V^W(w_T^\star).$$

Or simply, $w_T^\star = c$. The person will work if offered more than their opportunity cost of c. Given the optimal decision rule, we can determine the *indirect value* of entering period T unemployed with the offer w_T in hand:

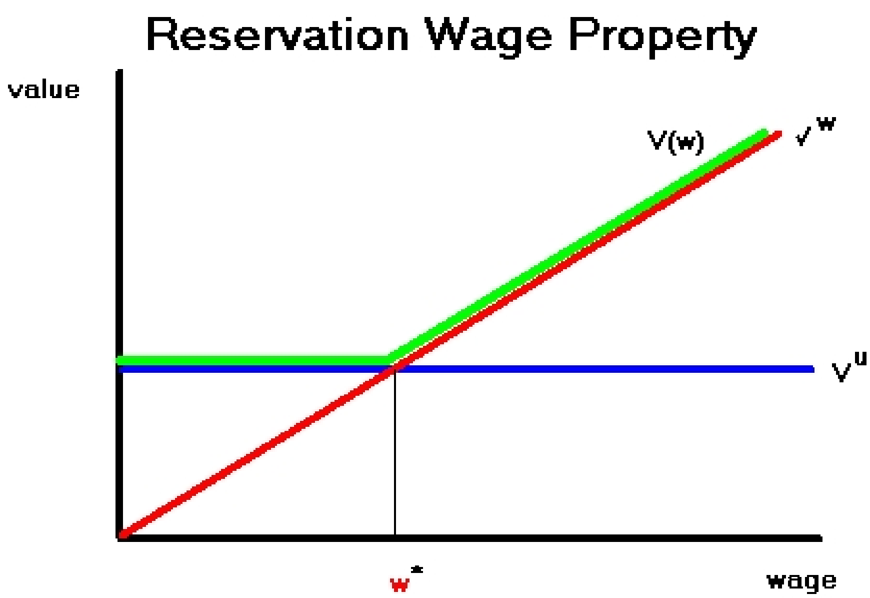
$$\begin{aligned} V_T(w_T) &= \max\{V_T^U, V_T^W(w_T)\} \\ &= \max\{c, w_T\} \\ &= \max\{w_T^\star, w_T\} \end{aligned}$$

III.B.2 Two Period (T and T-1) Decision

Now suppose the person enters the period before T unemployed, with wage offer w_{T-1} in hand. If the job offer is accepted, the job lasts for $T-1$ and T. He can either take the job or stay at home today and search again tomorrow according to the optimal rule described above. The current offer w_{T-1} disappears and at the beginning of T a new offer will appear drawn from the distribution $F(w)$. That is,

$$F(w) = Prob(\text{wage offer} < w)$$
$$f(w) = F'(w) = \text{density of wage offers}$$

Figure 3.



Income tomorrow is discounted by the rate β, $0 \leq \beta < 1$. An unemployed person makes a decision in period T-1 to accept the job offer or not to maximize discounted expected income.

⋄ The values of the parameters $(T, c, \beta, F())$ define the *basic job search model.*

Given the distribution of wage offers expected in time T, we can write down the expected income of arriving in period T unemployed:

$$\begin{aligned} EV_T &= \int_0^\infty V_T(w_T) f(w_T) dw_T \\ &= \int_0^\infty \max\{w_T^\star, w_T\} f(w_T) dw_T \\ &= \int_0^{w_T^\star} w^\star f(w_T) dw_T + \int_{w_T^\star}^\infty w_T f(w_T) dw_T \\ &= w^\star \int_0^{w_T^\star} f(w_T) dw_T + \int_{w_T^\star}^\infty w_T f(w_T) dw_T \\ &= F(w_T^\star) w_T^\star + (1 - F(w_T^\star)) E[w | w > w^\star] \\ &= F(c)c + (1 - F(c)) E[w | w > c] \end{aligned}$$

The last step is very particular to the period T problem. The other steps are close to the general expressions. The second to last step comes from the usual definition of a conditional expectation: $E[x|Y] = \int xf(x|Y)dx$. And $f(x|x > z)$ is 0 if $x < z$ and otherwise is $f(x)/(1-F(z))$. This means that the value of entering a period unemployed is a weighted average of the value of the reservation wage and the value of the average *accepted* wage.

The value of the two choices at T-1 are then

$$V^W_{T-1}(w_{T-1}) = w_{T-1} + \beta w_{T-1}$$

$$V^U_{T-1} = c + \beta EV_T$$

The optimal decision: work if $V^W_{T-1}(w_{T-1}) \geq V^U_{T-1}$. This defines the lowest wage offer acceptable at time $T-1$:

$$w^\star_{T-1} = \frac{c + \beta EV_T}{1+\beta} \geq \frac{w^\star_T + \beta w^\star_T}{1+\beta} = w^\star_T.$$

The inequality holds because $c = w^\star_T$ and $EV_T \geq w^\star_T$. Thus, the reservation wage in T-1 is higher than it will be in period T. This in turn determines the indirect value of

$$\begin{aligned} V_{T-1}(w_{T-1}) &= \max\{V^U_{T-1}, V^W_{T-1}(w_{T-1})\} \\ &= \max\{c + \beta EV_T, (1+\beta)w_{T-1}\} \\ &= (1+\beta)\max\{w^\star_{T-1}, w_{T-1}\} \end{aligned}$$

III.B.3 General Finite Horizon Problem

The person enters an arbitrary period t unemployed with wage offer w_t in hand. If the job offer is accepted, the job lasts through period T. The person can either take the job or stay at home today and search again tomorrow. Tomorrow's wage offers are drawn from the distribution $F(w)$. If we have solved backwards from T, then the expected (indirect) value of entering period t+1 unemployed is a computed value EV_{t+1}. This in turn determines the value of the two choices at period t:

$$V^W_t(w_t) = \sum_{k=t}^{T} \beta^{k-t} w_t = w_t \frac{1-\beta^{T-t+1}}{1-\beta}$$

$$V^U_t = c + \beta EV_{t+1}$$

The optimal decision: work if $V_t^W(w_t) \geq V_t^U$. This defines the lowest wage offer acceptable at time t:

$$w_t^\star = (c + \beta EV_{t+1})\frac{1-\beta}{1-\beta^{T-t+1}} \geq w_{t+1}^\star$$

The indirect value of a wage offer at time t:

$$\begin{aligned} V_t(w_t) &= \max\{V_t^U, V_t^W(w_t)\} \\ &= \max\{c + \beta EV_{t+1}, V_t^W(w_t)\} \\ &= \frac{1-\beta^{T-t+1}}{1-\beta}\max\{w_t^\star, w_t\} \end{aligned}$$

From this we can compute the value of entering period t unemployed:

$$\begin{aligned} EV_t &= \int_0^\infty V_t(w_t)f(w_t)dw_t \\ &= (c+\beta EV_{t+1})F(w_t^\star) + \int_{w_t^\star}^\infty w_t f(w_t)dw_t \\ &= \frac{1-\beta^{T-t+1}}{1-\beta}\left(w_t^\star F(w_t^\star) + \int_{w_t^\star}^\infty w_t f(w_t)dw_t\right) \end{aligned}$$

This allows the backward recursion to continue for $t-1, t-2, \ldots, 1$. Period T fits into this general formula if we define $EV_{T+1} = 0$. With that definition the equations $w_t^\star, V_t, EV_t$ can be used for any period t within a finite decision horizon.

Example 1.Let

$$\begin{aligned} \beta &= .5 \\ c &= 10 \\ f(w) &= \begin{cases} \frac{1}{100} & \text{if } 0 \leq w \leq 100 \\ 0 & \text{otherwise.} \end{cases} \\ F(w) &= \frac{w}{100} \end{aligned}$$

This leads to $w_T^\star = 10$. Note also that:

$$\int_x^{100} wf(w)dw = \frac{100^2 - x^2}{2(100)}.$$

Work out $EV_T, w^\star_{T-1}$.

III.B.4 Infinite Horizon Problem (T=∞)

If we let T go to infinity then the decision horizon disappears. Today is just like tomorrow, in the sense that tomorrow is no more closer to T than today. The person enters an arbitrary period ('today') with wage offer w in hand. If the job offer is accepted, the job lasts forever. The person can either take the job or stay at home today and search again next period ('tomorrow'). Tomorrow's wage offers are drawn from the distribution $F(w)$. The value of taking a job offer in any period is the present discounted value of the stream of wages:

$$V^W(w) = \sum_{k=t}^{\infty} \beta^{k-t} w_t = \lim_{T\to\infty} w_t \frac{1-\beta^{T-t+1}}{1-\beta} = \frac{w_t}{1-\beta} \quad \text{for} 0 \leq \beta < 1$$

The value of not taking an offer is the expected value of staying at home and then entering tomorrow unemployed:

$$V^U = c + \beta EV.$$

Notice we haven't defined the term EV yet. In the finite horizon case, it could be determined before the optimal decision rule today. But today and tomorrow are the same, so in the infinite horizon problem the optimal decision rule and the indirect value of unemployment must be determined *simultaneously*. Whatever value EV is, the optimal decision rule is still simple: work if $V^W(w) \geq V^U$. This defines the lowest wage offer acceptable at any time:

$$w^\star = (c + \beta EV)(1-\beta)$$

Given a reservation wage $w^\star$ we could determine the value of unemployment:

$$\begin{aligned} V(w) &= \max\{V^U, V^W(w)\} \\ &= \max\{c + \beta EV, V^W(w)\} \\ &= \frac{\max\{w^\star, w\}}{1-\beta} \end{aligned}$$

And given the reservation wage we can determine the expected value of unemployment:

$$
\begin{aligned}
EV &= \int_0^\infty V(w)f(w)dw \\
&= \frac{w^\star F(w^\star) + \int_{w^\star}^\infty wf(w)dw}{1-\beta} \\
&= \frac{F(w^\star)w^\star + (1-F(w^\star))E[w|w>w^\star]}{1-\beta}.
\end{aligned}
$$

The equations for EV and $w^\star$ form a *system of simultaneous equations* that determine optimal decisions. One way to solve this system of equations is to begin with the period T problem and continue backward until $w_t^\star$ converges. This will work because it is possible to show that for $0 \leq \beta < 1$

$$\lim_{T\to\infty} w_1^\star = w^\star$$

That is, as the horizon disappears the reservation wage converges to the infinite horizon reservation wage.

Example 2.

Using the values for Problem 1, derive the system of equations that determine EV and $w^\star$ in the infinite horizon. Solve the equations for the value of $w^\star$ (which involves solving a quadratic equation).

Two Wage Distributions that aren't so hard

1. Uniform wages:

$$f(w) = \begin{cases} 1/M & \text{if } 0 \leq w \leq M \\ 0 & \text{otherwise.} \end{cases}$$

$$F(w) = \begin{cases} 0 & \text{if } w < 0 \\ w/M & \text{if } 0 \leq w \leq M \\ 1 & \text{if } w > M \end{cases}$$

$$E[w] = \int_0^M wf(w) = w^2/(2M)\Big|_0^M = M/2.$$

for $0 < w^\star < M$,

$$E[w|w > w^\star] = \int_0^M wf(w|w > w^\star) = w^2/(2(M - w^\star))\Big|_0^M$$
$$= (M^2 - {w^\star}^2)/(2(M - w^\star)) = \frac{(M - w^\star)(M + w^\star)}{2(M - w^\star)}$$
$$= (M + w^\star)/2.$$

$$EV = \frac{1}{1-\beta}\left(F(w^\star)w^\star + (1 - F(w^\star))E[w|w > w^\star]\right)$$
$$= \frac{1}{1-\beta}\left({w^\star}^2/M + (1 - w^\star/M)(M + w^\star)/2\right)$$
$$= \frac{1}{1-\beta}\left({w^\star}^2/(2M) + M/2\right)$$

2. Exponential Wages (above a minimum wage m):

$$f(w) = \begin{cases} \gamma e^{-\gamma(w-m)} & \text{if } w \geq m \\ 0 & \text{otherwise.} \end{cases}$$
$$F(w) = \begin{cases} 0 & \text{if } w < m \\ 1 - e^{-\gamma(w-m)} & \text{otherwise} \end{cases}$$
$$E[w] = \int_m^\infty f(w) = m + 1/\gamma.$$

For $w^\star \geq m$:

$$E[w|w > w^\star] = \int_0^\infty wf(w|w > w^\star) = w^\star + 1/\gamma$$

$$EV = \frac{1}{1-\beta}\left(F(w^\star)w^\star + (1 - F(w^\star)\,E\left[w|w > w^\star\right]\right)$$
$$= \frac{1}{1-\beta}\left(w^\star\left[1 - e^{-\gamma(w^\star - m)}\right] + e^{-\gamma(w^\star - m)}\left(w^\star + 1/\gamma\right)\right)$$
$$= \frac{1}{1-\beta}\left(w^\star + \frac{e^{-\gamma(w^\star - m)}}{\gamma}\right)$$

III.C Wolpin: Empirical Content of the Basic Job Search Model

III.C.1 Duration of Unemployment Spell

Outcome of the basic model: a constant reservation wage

$$w^\star = w^\star(\infty, c, \beta, F()).$$

This is not usually observed in data unless the person is asked a hypothetical question ("What is the lowest wage you would accept?"). What is usually observed? Outcomes of the search process, such as the duration of unemployment spell and the *accepted* wage.

Given that I am unemployed this period, the probability that a job is rejected is:

$$Prob(\text{wage offer} < w^\star) = F(w^\star).$$

And the probability that a job is accepted:

$$\lambda \equiv 1 - F(w^\star).$$

λ is the *conditional probability* of taking a job in any period, conditional upon being unemployed that period. It is the theoretical *hazard rate* out of unemployment. It answers the question: "If I come into a period unemployed what is the probability that I am employed next period?" It is a conditional probability because of the condition of "If I come into a period unemployed."

A person stays unemployed t periods if they reject $t-1$ job offers and then accept the t^{th} offer. The probability of $t-1$ offers in a row is $(1-\lambda)^{t-1}$. The *unconditional probability* that I leave unemployment in week t of my unemployment spell is therefore:

$$p(t) = \text{probabiliy of } t-1 \text{ rejections followed by an acceptacne}$$

$$(1-\lambda)^{t-1}\lambda.$$

This answers the question: "At the start of the unemployment spell, what is the probability that I will be unemployed for exactly t periods?" Thus, $p(t)$ is the probability function for unemployment duration or unemployment spell lengths. The cumulative distribution of spell lengths is:

$$P(t) = \sum_{k=1}^{t} p(k)$$

and the *survivor function* is just the complement:

$$S(t) = 1 - P(t-1).$$

The survivor function answers the question: "At the start of the unemployment spell, what is the probability that I will be unemployed for t or more periods?" Any one of the three functions p, S, or h characterizes the distribution of durations completely.

For the infinite horizon job search model these probabilities are fairly simple:

$$P(t) = \lambda \sum_{k=1}^{t} (1-\lambda)^k = \frac{\lambda(1-(1-\lambda)^t)}{1-(1-\lambda)} = 1-(1-\lambda)^t$$

$$S(t) = (1-\lambda)^{t-1}$$

$$h(t) = p(t)/S(t) = \lambda$$

Furthermore the expected duration of unemployment spell can be shown to equal

$$E(t) = \sum kp(k) = \frac{1}{\lambda}.$$

In the finite horizon case the hazard rate is a function of how long the person has been unemployed:

$$\lambda_t = 1 - F(w_t^\star).$$

Now the probability that a spell of unemployment lasts exactly t periods is the probability that the person rejected offers in the previous $k = 1, 2, \ldots, t-1$ periods, all with different conditional probabilities λ_t. So we must multiply all the probabilities of rejecting previous offers together.

$$p(t) = (1-\lambda_1)(1-\lambda_2)\ldots(1-\lambda_{t-1})\lambda_t = \lambda_t \prod_{k=1}^{t-1}(1-\lambda_k)$$

Notice then that in the finite horizon case the optimal decisions must be solved backwards in time, but the probabilities are calculated forward in time. This makes the use of a spreadsheet very helpful for solving simple versions of the search model, because spreadsheets allow you to make calculations depend on other calculations.

III.C.2 Distribution of accepted wages

Observed accepted wages are those for which $w > w^\star$. Accepted wages are different than offered wages. In particular, the distribution of accepted wages equals

$$F^a(w) = \begin{cases} \frac{F(w)-F(w^\star)}{1-F(w^\star)} & \text{if } w \geq w^\star \\ 0 & \text{else} \end{cases}$$

The average accepted wage is

$$E^a(w) = \frac{1}{1-F(w^\star)} \int_{w^\star}^{\infty} w f(w) dw > E(w)$$

Optimal job search *truncates* the wage offer distribution.

III.D Properties of the Basic Job Search Model

III.D.1 Response of $w^\star$ to c and β

The basic job search model we have set up has a small number of parameters that determine the optimal process of job search. These parameters are the value of a period spent unemployed (c), the discount factor (β), and the distribution of wage offers ($f(w)$).

It is possible to use integration by parts to write an implicit equation for $w^\star$ that does not depend on EV as well. That is, the reservation wage in the infinite horizon model satisfies:

$$w^\star - c = \beta(E(w) - c) + \beta \int_0^{w^\star} F(z) dz.$$

(This is tedious to show and you are not required to know this equation.) Given this result, we can it to see how the reservation wage responds to the parameters of the model.

$$\frac{dw^\star}{dc} - \frac{dc}{dc} = -\beta\frac{dc}{dc} + F(w)\frac{dw^\star}{dc}$$

This is an expression for the *total derivative* of the reservation wage with respect to a change in the parameter c. It is analogous to a *comparative static* exercise in micro theory, in the sense that it measures the response of the optimal decision rules to an exogenous parameter. Solve for the total derivative:

$$\frac{dw^\star}{dc} = \frac{1-\beta}{1-\beta F(w^\star)} \in \{0, 1\}$$

That is, the reservation wage goes up with the value of unemployment but not dollar-for-dollar. One way to think of this result is the following: an increase in c makes the person richer. Some of this increase in wealth is consumed in longer unemployment spells, but not all of it. If a job is rejected in order to stay unemployed and enjoy the higher value of c, then it may take several periods to get another acceptable offer. We know that $F(w^\star)$ is the probability of rejecting the next job offer as well.

We can also calculate the *elasticity* of the reservation wage:

$$\frac{d\ln w^\star}{d\ln c} = \frac{1-\beta}{1-\beta F(w^\star)}\frac{c}{w^\star} < 1$$

Problem 4

Derive $\frac{dw^\star}{d\beta}$.

III.D.2 Response of $w^\star$ to $F(w)$

It is more complicated to derive how reservation wages respond to changes in the distribution of wages. First, we must manipulate the equations for $w^\star$ and EV to get

$$w^\star - c = \frac{\beta}{1-\beta}\int_{w^\star}^{\infty}(z-w^\star)f(z)dz \qquad (*)$$

This equation actually has a nice interpretation. Recall that the reservation wage is that wage which makes the person indifferent between their two options (W and U). So in some sense the benefit and costs of accepting the reservation wage must be equal. But in what sense? Well, the left-hand side of the equation is the one period benefit from accepting $w^\star$. Instead of receiving c the person gets $w^\star \geq c$ this period. What is the cost? The cost of taking $w^\star$ is that starting tomorrow (hence the β) and in all future periods (hence the $1/1-\beta$) the person has $w^\star$ instead of some better wage. The integral part of the right hand side of the equation is therefore the expected cost of having wage $w^\star$ tomorrow for sure instead of a better wage tomorrow.

Now consider two different types of changes to the wage distribution.

a. A translation of $F(w)$, or a change in its mean

Suppose that the mean wage offer changes. For example, it changes from $E_0[w]$ to $E_1[w]$ in the figure below. How does the reservation wage respond?

If we let μ denote $E[w]$, then it turns out that the equation(*) implies

$$\begin{aligned} w^\star(\mu) - c &= \beta(E_F(w(\mu)) - c) + \beta \int_0^{w^\star(\mu)} G(z)dz \\ &= \beta(E_0(w) + \mu - c) + \beta \int_0^{w^\star(\mu)} F(z-\mu)dz \end{aligned}$$

This allows us to derive the response

$$\begin{aligned} \frac{dw^\star}{d\mu} &= \beta + \beta F(w_\mu)\frac{dw}{d\mu} - \beta \int_0^{w^\star(\mu)} f(z-\mu)dz \\ &= \frac{\beta(1 - F(w^\star - \mu))}{1 - \beta F(w^\star - \mu)} \\ &\in \{0, 1\}. \end{aligned}$$

This shows that a shift in the wage offer distribution is like a change in c. Higher mean wage offers increase the reservation wage but not so that the reservation wage is in the same point in the distribution before. That means that a higher mean wage offer leads to a lower probability of rejecting offers and a shorter expected duration of unemployment.

Problem 6

Modify distribution to be

$$f(w) = \begin{cases} \frac{1}{u-l} & \textbf{if } l < w < u \\ 0 & \textbf{else} \end{cases}$$

Determine what changes in u and l constitute a translation of the distribution

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b. Mean Preserving Spread

Now what happens when wage offers become more spread out, in the sense of an increased variance in wages holding constant the mean?

Unlike changes in the mean wage or changing the value of unemployment, the effect is not very obvious. On the one hand, high wage offers are more likely with higher variance, so perhaps it is better to raise the reservation wage in order to have a better chance at receiving those offers. On the other hand, higher variance also means that low wage offers are more likely as well. Perhaps that put a downward pressure on reservation wages. Let r denote the variance. Technically, r must refer to the mean-preserving spread of the distribution. For distributions that have some skewness this may not be the variance itself. Beginning with (*) it is possible to show that

$$\frac{dw^\star(r)}{dr} = \frac{\beta \int_0^{w^\star(r)} \frac{dF(z,r)}{dr} dz}{1 - \beta F(w^\star, r)} > 0$$

That is, more risk or spread in wage offers unambiguously leads to *greater* reservation wages. Search truncates the increased probability of low draws, leaving higher probability of good wages.

Problem 6

1. Modify the infinite horizon problem, by allowing that with probability ϕ the person gets an offer next period. If no offer is received then the person will be unemployed tomorrow.

2. Modify the infinite horizon problem by assuming that a job offer arrives every period while unemployed (as before), but while employed job offers arrive with probability ϕ. Show that if $\phi = 1$ then the reservation wage equals c.

III.E Employment Insurance

- coverage and funding: who holds jobs that have UI and what do they pay into the system
- eligibility: among those covered, who qualifies to receive benefits
- benefits: among those who are eligible, what is the level of benefits and how long do they last.

III.E.1 Coverage and Funding

To eligible for UI/EI you must pay into the system. That is, you must have held an insured job in the past and somehow lost it. Approximately 92 percent of the labour force pays UI/EI premiums. UI/EI taxes are taken out of the insured portion of your earnings at a rate that has gone up and down several times recently.

III.E.2 Eligibility

Besides losing an insured job a person must meet several requirements to receive any benefits. First, they must have build up a minimum amount of work hours (previously weeks) in order to qualify. The minimum is between 420 and 700 hours (12 to 20 weeks at 35 hours per week). The minimum depends upon the local unemployment rate. The higher the unemployment rate the lower the minimum number of hours. Furthermore, people who have received EI in the recent past must accumulate more hours than those who haven't been on EI recently.

People who quit jobs (without 'just cause') are no longer eligible to receive EI.

People can also work a small number of hours and continue to receive EI benefits.

Furthermore, a person on EI must nominally be available to work and actively seeking it.

Figure on EI Eligibility (Source: Paper W-98-35E,Applied Research Branch Strategic Policy HRDC.)

III.E.3 Benefits

The benefit level is based on the previous wage. Currently a person on EI receives about 55 percent of their last wage up to the insured maximum. This *replacement* rate is lower for repeat users of EI.

Given that a person just meets the entrance requirement in hours they are eligible for a number of weeks of EI. They can earn additional weeks of EI with additional hours of work built up. The maximum number of potential weeks of EI is about 48 weeks.

III.E.4 UI and Job Search

Suppose that the infinite horizon job search problem we discussed before pertained to a case in which there were no EI benefits.

$$V_t^U = \begin{cases} c + b + \beta EV_{t+1} & \text{if } t \leq T \\ c + \beta EV & \text{if } t > T \end{cases}$$

$$V_t^W(w) = w/(1-\beta), \text{ for all } t$$

$$EV_t = EV, \text{ for } t > T$$

$$EV_t = \frac{1}{1-\beta}\left(w_t^\star + e^{-\gamma\left(w_t^\star - m\right)}/\gamma\right), \text{ for } 1 \leq t \leq T$$