Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2009/10

Practice Questions for the Midterm Exam
Due: Never

1 Endogenous growth with transitional dynamics

Use the following definitions: s = (exogenous) savings rate, δ = depreciation rate of capital, n = population growth rate, $L = e^{nt}$ = labor supply, $k = \frac{K}{L}$ and $y = \frac{Y}{L}$. Assume the following CES production function:

$$Y = A\left\{\eta K^{\sigma} + (1-\eta)\, L^{\sigma}\right\}^{\frac{1}{\sigma}}$$

where $0 < \eta < 1$ and $\sigma < 1$.

The resource constraint for the closed economy is $Y = C + \delta K + \dot{K}$, where C is consumption, and $\dot{K} = \frac{dK}{dt}$. Since the saving rate is exogenous, we can rewrite the resource constraint as a saving relation: $S = sY = Y - C = \delta K + \dot{K}$

(a) Calculate the elasticity of substitution between factors for the production function (i.e. the relative change in the input ratio (K/L) due to a relative change in their marginal productivity (f_K/f_L)). Interpret the different cases: $\sigma \to 1$, $\sigma \to 0$, $\sigma \to -\infty$.

(b) Rewrite the resource constraint in terms of k, i.e., write $\dot{k}$ as a function of k and exogenous parameters.

(c) Assume $0 < \sigma < 1$. Find $\gamma_k = \frac{\dot{k}}{k}$ as a function of k. What is the steady-state (asymptotic) growth rate of k and K if we assume $sA\eta^{\frac{1}{\sigma}} > n + \delta$?

(d) Find the economy's growth rate of GDP per capita as a function of parameters and the capital level.

(e) Analyze the transitional dynamics with the help of a graph. Is there conditional convergence in this economy?

(f) Suppose now that $\sigma < 0$. Can the economy sustain positive steady-state growth? What goes wrong? Provide an intuition for your result.

2 Factor rewards and the golden rule

For a neoclassical production function, show that each factor of production earns its marginal product. Show that if owners of capital save all their income and workers consume all their income, then the economy reaches the golden rule of capital accumulation. Explain the result.

3 Dynamics in a Solow-type model

Consider a Solow econmy that is on its Balanced Growth Path (BGP). Assume for simplicity that there is no technological progress. Now suppose that the rate of population growth falls.

(a) What happens to the BGP values of capital per worker, output per worker, and consumption per worker? Sketch the paths of these variables as the economy moves to its new BGP.

(b) Describe the effect of the fall in population growth on the path of total output (in levels).

4 A Simple Ramsey Model

The analysis of government policies we saw in class assumes that government purchases do not affect utility from private consumption. The opposite extreme is that government purchases and private consumption are perfect substitutes. Specifically, suppose that the representative household's utility function is modified to be:

$$U = B \int_0^{\infty} \frac{[c(t) + G(t)]^{1-\theta}}{1-\theta} e^{-(\rho - n - (1-\theta)g)t} dt,$$

where ρ is the time discount rate, $c(t)$ is consumption per capita, n is the population growth rate, and g is the growth rate of labor augmenting technology.

If the economy is initially on its BGP, what are the effects of a temporary increase in government purchases on the paths of capital, consumption and the interest rate?