Q1

1) $\bar{W}(w) = w + \beta\left[(1-\lambda)\max\{\bar{W}(w), U\} + \lambda U\right] = w + \beta(1-\lambda)\bar{W}(w) + \beta\lambda U$ [1]

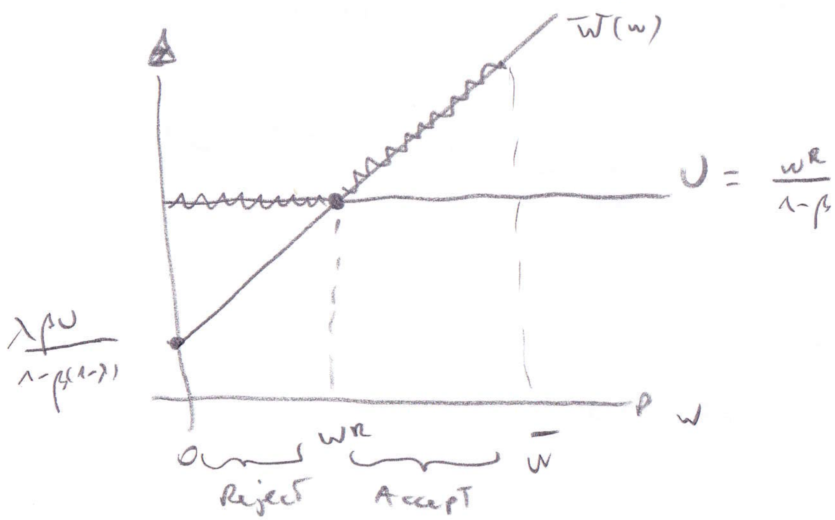
$$U = b + \beta\lambda E_w \max\{\bar{W}(w), U\} + \beta(1-\lambda)U = b + \beta\lambda\int_0^{\bar{w}} \max\{\bar{W}(w), U\}\,dF(w) + \beta(1-\lambda)U$$

From [1] we get $\bar{W}(w) = \dfrac{w + \beta\lambda U}{1-\beta(1-\lambda)}$

At the reservation wage $\bar{W}(w^R) = U$ and

$$\frac{w^R + \beta\lambda U}{1-\beta(1-\lambda)} = U \;\rightarrow\; w^R = (1-\beta)U \;\rightarrow\; U = \frac{w^R}{1-\beta}$$

2)



$\equiv \max\{\bar{W}(w), U\}$

3) There are different versions... One is

$$\frac{w^R}{1-\beta} = U = b + \alpha\beta \int_0^{\bar{w}} \max\left\{ \bar{W}(w), \frac{w^R}{1-\beta} \right\} dF(w) + (1-\alpha)\beta \frac{w^R}{1-\beta}$$

$$\downarrow$$

$$\frac{w}{1-\beta(1-\lambda)} + \frac{\beta\lambda w^R}{(1-\beta)[1-\beta(1-\lambda)]}$$

The most parsimonious one breaks the integrals in $[0, w^R]$ and $(w^R, \bar{w}]$ to be able to solve for the $\max\{\cdot,\cdot\}$ operator, and integrates by parts. This gets to

$$w^R = b + \alpha\beta \int_{w^R}^{\bar{w}} \frac{d\bar{W}(w)}{dw} \cdot [1-F(w)]\, dw = b + \frac{\alpha\beta}{1-\beta(1-\lambda)} \int_{w^R}^{\bar{w}} [1-F(w)]\, dw$$

4) Apply Leibnitz:

$$dw^R = \frac{\partial}{\partial\alpha}\left[\frac{\alpha\beta}{1-\beta(1-\lambda)} \int_{w^R}^{\bar{w}} [1-F(w)]\, dw \right] d\alpha + (-1) \cdot \frac{\alpha\beta}{1-\beta(1-\lambda)} [1-F(w^R)]\, dw^R$$

$$dw^R = \left[\frac{\beta}{1-\beta(1-\lambda)} \int_{w^R}^{\bar{w}} [1-F(w)]\, dw \right] \cdot d\alpha - \frac{\alpha\beta}{1-\beta(1-\lambda)} [1-F(w^R)]\, dw^R$$

$$\frac{dw^R}{d\alpha} = \frac{\frac{\beta}{1-\beta(1-\lambda)} \int_{w^R}^{\bar{w}} [1-F(w)]\, dw}{1 + \frac{\alpha\beta}{1-\beta(1-\lambda)} \cdot [1-F(w^R)]} = \frac{>0}{>0} \rightarrow >0$$

Q2

(1)

$$\bar{W}(w) = w + \beta \max\{\bar{W}(w), U\} = w + \beta \bar{W}(w) \rightarrow \bar{W}(w) = \frac{w}{1-\beta}$$

$$U = b + \beta E_{w_1 w_2} \max\{\bar{W}(w_1), \bar{W}(w_2), U\} = \beta E_{w_1, w_2} \max\{\bar{W}(\max\{w_1, w_2\}), U\}$$

Note that the event $\max\{w_1, w_2\} < w$ is the event $(w_1 < w) \cap (w_2 < w)$.

Therefore, $\text{Prob}\{\max(w_1, w_2) < w\} = \text{Prob}\{w_1 < w\} \cdot \text{Prob}\{w_2 < w\} = F(w) \cdot F(w) = F^2(w)$

The worker will limit his choice to the larger of the two offers each period, and the Bellman equation is

$$U = b + \beta \int_0^{\bar{w}} \max\{\bar{W}(w), U\}\, dF^2(w)$$

(2)

$$w_R = b + \left(\frac{\beta}{1-\beta}\right) \cdot \int_{w_R}^{\bar{w}} (w - w_R)\, dF^2(w)$$

Integrating by parts we get

$$w_R = b + \left(\frac{\beta}{1-\beta}\right) \int_{w_R}^{\bar{w}} [1 - F^2(w)]\, dw \rightarrow h(w_R) = (1-\beta) w_R - b - \left(\frac{\beta}{1-\beta}\right) \int_{w_R}^{\bar{w}} [1 - F^2(w)]\, dw$$

$$\frac{dh(w_R)}{dw_R} = (1-\beta) + \left(\frac{\beta}{1-\beta}\right) \cdot [1 - F^2(w_R)] > 0$$

with only one offer we have the standard case with $h_1(\hat{w}_R) = (1-\beta)\,\hat{w}_R - b - \left(\frac{\beta}{1-\beta}\right)\int_{\hat{w}_R}^{\bar{w}} [1 - F(w)]\,dw$

Again $h_1'(\cdot) > 0$ and

$$h(w^e) \leq h_1(w^e) \quad \text{because} \quad F^2(w) \leq F(w), \quad \text{and}$$

$$h_1(\hat{w}_R) = 0 = h(w_R) \leq h_1(w_R) \quad \text{and} \quad \hat{w}_R \leq w_R \quad \text{because } h_2(\cdot)$$

is an increasing function.

The worker could always choose to ignore the second offer, even though this could be sub-optimal. This restricted case would look like the standard problem.

The value of being employed in the unrestricted case is at least as high as the value of being employed in the restricted one.

The reservation wage is such that we equate the value of accepting the job to the value of rejecting the offer: $\frac{w}{1-\beta} = b + \beta\,\underbrace{E \max\{W(w), U\}}_{[*]}$.

A higher expected value of search ([*]) leads to a higher reservation wage.