ECON 815
Macroeconomic Theory
Winter Term 2012/13

Assignment 4
*Due: In class (DH 213) on **March 25th** 2013*
No late submissions will be accepted
No group submissions will be accepted
No electronic submissions will be accepted

*Remarks: Write clearly and concisely. Devote some time to give the graphs, plots and tables a format easy to understand. Also the way you present your answers matter for the final grade. Even if a question is mainly analytical, **briefly** explain what you are doing, stressing the economic meaning of the various steps. Being able to convey your thoughts effectively is an asset also in real life.*

1 The Empirics of Growth and the Cobb-Douglas assumption

In this exercise you are asked to work with some simple data on growth. We will start by replicating some results in the paper "*A Contribution to the Empirics of Economic Growth*," by Mankiw, Romer, and Weil, *QJE* (1992).

Part 1)

(1a) Neoclassical growth theories predict convergence in income per capita across countries, unlike endogenous growth theories. The neoclassical model implies:

$$\ln y_T - \ln y_0 = \left(1 - e^{-\lambda T}\right) \ln y^* - \left(1 - e^{-\lambda T}\right) \ln y_0$$

where y is GDP per adult, T is the terminal year, 0 the initial year, λ is the rate of convergence, and y^* is the steady-state output. Neoclassical theory predicts that $\lambda > 0$ and thus $\left(1 - e^{-\lambda T}\right) > 0$. Using a software of your choice, load the dataset "Mrw.dta" (this dataset is in STATA format.)

Keep in the sample only the 98 "non-oil" countries (these are the countries for which the variable *sn* equals 1.)

To test for absolute convergence, rewrite the previous equation as a linear regression:

$$g_i = \beta_0 + \beta_1 \ln y_{1960,i} + \varepsilon_i$$

The parameters (both assumed to be constant) are $\beta_0 = \left(1 - e^{-\lambda T}\right) \ln y^*$, and $\beta_1 = -\left(1 - e^{-\lambda T}\right)$, and ε is the error term. Notice that you need to generate the log of the GDP variables and build the dependent variable $g_i = \ln y_{1985,i} - \ln y_{1960,i}$.

(1b) Suppose that ε satisfies the classical assumptions: $E(\varepsilon) = 0$, $Var(\varepsilon) = \sigma^2$, and $E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j$. Get the OLS estimates of β_0, β_1, and σ^2.

(1c) In addition assume that ε is normally distributed. Test the null hypothesis $H_0 : \beta_1 = 0$. Also construct a 95% confidence interval for β_1. What is the value of λ (the rate of convergence) implied by the OLS estimate of β_1?

Part 2)

Neoclassical theory predicts convergence only after controlling for the determinants of the steady state, that is "conditional" convergence. With a Cobb-Douglas production function $Y(t) = K(t)^{\alpha} (A(t) L(t))^{1-\alpha}$, the basic Solow model implies:

$$\ln \left[\frac{Y(t)}{L(t)}\right] = \ln A(0) + xt + \left(\frac{\alpha}{1-\alpha}\right) \ln s - \left(\frac{\alpha}{1-\alpha}\right) \ln (n + x + \delta)$$

when $A(t) = A(0) e^{xt}$, and $L(t) = L(0) e^{nt}$.

(2a) Assume that the sum of the depreciation rate and the rate of technological growth is 5%. Rewrite the previous equation as a linear regression and use the series *workpop* (growth rate of working-age population), and *invest* (investment as a fraction of output) to create the explanatory variables:

$$\ln y_i = \beta_0 + \beta_1 \ln\left(\frac{invest_i}{100}\right) + \beta_2 \ln\left(\frac{workpop_i}{100} + 0.05\right) + \varepsilon_i$$

(2b) Suppose that ε satisfies the classical assumptions and get the OLS estimates of $\beta_0, \beta_1, \beta_2$, and σ^2. Note that, according to the theory, $\beta_2 = -\beta_1$. Test the the null hypothesis $H_0 : \beta_2 = -\beta_1$ versus the alternative $\beta_2 \neq -\beta_1$.

(2c) Re-estimate the model subject to the previous restriction, i.e., write the regression in the restricted form:

$$\ln y_i = \beta_0 + \beta_1 \left[\ln\left(\frac{invest_i}{100}\right) - \ln\left(\frac{workpop_i}{100} + 0.05\right)\right] + \varepsilon_i$$

Obtain the OLS estimates of β_0, β_1 and σ^2.

(2d) Compute the value of α implied by the OLS estimate of β_1.

Part 3)

Mankiw, Romer, and Weil's neoclassical framework extended Solow's model, to consider the role of human capital $H(t)$. With a Cobb-Douglas production function $Y(t) = K(t)^{\alpha} H(t)^{\gamma} (A(t) L(t))^{1-\alpha-\gamma}$, their model implies:

$$\ln\left[\frac{Y(t)}{L(t)}\right] = \ln A(0) + xt + \left(\frac{\alpha}{1-\alpha}\right)\ln s - \left(\frac{\alpha}{1-\alpha}\right)\ln(n + x + \delta) + \left(\frac{\gamma}{1-\alpha}\right)\ln h$$

(3a) As before, rewrite the previous equation as a linear regression and use the series *workpop* (growth rate of working-age population), *invest* (investment as a fraction of output), and *school* (a measure of human capital) to create the explanatory variables:

$$\ln y_i = \beta_0 + \beta_1 \ln\left(\frac{invest_i}{100}\right) + \beta_2 \ln\left(\frac{workpop_i}{100} + 0.05\right) + \beta_3 \ln school_i + \varepsilon_i$$

(3b) Suppose that ε satisfies the classical assumptions and get the OLS estimates of $\beta_0, \beta_1, \beta_2, \beta_3$ and σ^2.

(3c) Note again that $\beta_2 = -\beta_1$. Re-estimate the model subject to this restriction, i.e., write the regression in the restricted form:

$$\ln y_i = \beta_0 + \beta_1 \left[\ln\left(\frac{invest_i}{100}\right) - \ln\left(\frac{workpop_i}{100} + 0.05\right)\right] + \beta_3 \ln school_i + \varepsilon_i$$

Obtain the OLS estimates of $\beta_0, \beta_1, \beta_3$ and σ^2.

(3d) Compute the value of α and γ implied by your OLS estimates.

Part 4)

We are interested to see whether the Cobb-Douglas assumption is supported by our cross-sectional data. The production function, in the basic Solow model, will now be a CES:

$$Y(t) = \left[\alpha K(t)^{\rho} + (1-\alpha)(A(t) L(t))^{\rho}\right]^{\frac{1}{\rho}}$$

where the elasticity of substitution is $\sigma = \frac{1}{1-\rho}$. Kmenta, IER (1967), proposed to consider a second-order Taylor series expansion around $\rho = 0$ (the C-D case, because $\sigma = 1$). Following this procedure leads to:

$$\ln\left[\frac{Y(t)}{L(t)}\right] = \ln A(0) + xt + \left(\frac{\alpha}{1-\alpha}\right)\ln\left(\frac{s}{n+x+\delta}\right) + \frac{1}{2}\frac{\alpha}{(1-\alpha)^2}\rho\left[\ln\left(\frac{s}{n+x+\delta}\right)\right]^2$$

(4a) Rewrite the previous equation as a linear regression and use the series *workpop* (growth rate of working-age population), and *invest* (investment as a fraction of output) to create the explanatory variables. What is the value of σ implied by the OLS estimates? Is it statistically different from 1?

2 Growth through expanding variety of products

Consider the benchmark version of the growth model with expanding varieties as described in class. Assume no population growth and suppose that agents have CEIS utility:

$$U = \int_0^\infty \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

The households' budget constraint is:

$$\dot{B} = rB + wL - C$$

where B stands for households assets.

The profits for the final good sector (competitive) firms are (X_j stands for the quantity of intermediate good j):

$$\pi = AL^{1-\alpha} \sum_{j=1}^{N} X_j^{\alpha} - wL - \sum_{j=1}^{N} p_j X_j$$

while the intermediate good sector (monopolistic) producer of the single intermediate good j maximizes:

$$\pi_j = p_j X_j - X_j$$

Rather than being equal to η, the cost of one innovation is now $\eta - T$, where T is a subsidy to research. The subsidy is financed with a lump-sum tax on the consumers (expressed in terms of the final good in the budget constraint).

(a) How are intermediate inputs priced, and what is the quantity of each intermediate X_j?

(b) What is the free entry condition for the *R&D* firms, and how is the rate of return determined?

(c) What are the growth rates of N, X and total output Y along a BGP?

(d) Is this subsidy-taxation policy enough to achieve the first best?