ECON 815
Macroeconomic Theory
Winter Term 2012/13

Assignment 3
Due: In Class on ***March 12th*** *2013*
No late submissions will be accepted
No group submissions will be accepted
No electronic submissions will be accepted

Remarks: Write clearly and concisely. Devote some time to give the graphs, plots and tables a format easy to understand. Also the way you present your answers matter for the final grade. Even if a question is mainly analytical, ***briefly*** *explain what you are doing, stressing the economic meaning of the various steps. Being able to convey your thoughts effectively is an asset also in real life.*

1 Government and growth

Consider a competitive economy where all households are identical, supply labor inelastically, and choose the consumption path to maximize their utility:

$$U_0 = \int_0^\infty \log\left(c\left(t\right)\right) \cdot e^{-\rho t} dt$$

subject to the intertemporal budget constraint:

$$\dot{a}\left(t\right) = \left(1 - \xi\left(t\right)\right) r\left(t\right) a\left(t\right) + w\left(t\right) - c\left(t\right) - \tau\left(t\right)$$

and a given level of initial assets a_0. $\tau(t)$ is a lump-sum tax and $\xi(t)$ is the proportional tax rate on asset income. Each household takes the return on assets $r(t)$, the real wage $w(t)$ and the tax rates as given. Factor markets are competitive. The production technology is given by:

$$y\left(t\right) = \left(k\left(t\right)\right)^{\alpha}\left(g^{I}\left(t\right)\right)^{\beta}$$

where $y(t)$ is output per capita, $g^I(t)$ is public services per capita and $k(t)$ is the capital-labor ratio. We assume that $\alpha < 1$, $\beta < 1$, there is neither population growth nor depreciation. The initial capital stock is given by k_0, determined by history. Firms take $g^I(t)$ as given.

The total government per capita purchases are equal to $g^C(t) + g^I(t)$, of which $g^I(t)$ units of output per capita are turned into $g^I(t)$ units of productive services, whereas $g^C(t)$ units of output per capita are simply consumed by the government with no utility for private agents. To finance its purchases, the government levies taxes on asset income as well as a lump-sum tax, with the constraint that its budget must balance in each period. Formally:

$$g^{C}\left(t\right) + g^{I}\left(t\right) = \tau\left(t\right) + \xi\left(t\right) r\left(t\right) a\left(t\right)$$

Wages and rental rates for capital are determined in a competitive fashion. The other market equilibrium conditions are given by:

$$y\left(t\right) = c\left(t\right) + g^{C}\left(t\right) + g^{I}\left(t\right) + \dot{k}\left(t\right)$$

$$k\left(t\right) = a\left(t\right)$$

1. Assume that $g^C(t) = g^C, g^I(t) = g^I$ and $\xi(t) = \xi$ are constant over time. Derive the dynamic equations characterizing the equilibrium in the above economy. Also, explain diagrammatically how the economy moves from the initial state to the steady-state, assuming that the initial capital-labor ratio k_0 is smaller than the steady-state level.

2. Maintaining the set-up of question 1 and starting from the steady-state:

(a) Analyze diagrammatically (with the aid of a phase diagram) the dynamic effects of an unexpected increase of government consumption, g^C, financed by an increase in the lump-sum tax.

(b) Assume that at time t_0 it becomes known to all agents that the government is going to reduce the purchase of goods for consumption (g^C) permanently from some time t' in the future, together with an offsetting reduction in the lump-sum tax. Describe the effects on the paths of the aggregate economic variables before and after t_0. Explain diagrammatically and provide an intuitive explanation as well.

3. We now abandon the set-up of question 1, and assume instead that $g_t^C = \tau(t) = 0$, while $\xi(t) = \xi$ is still constant over time. Derive the dynamic equations characterizing the economy under these assumptions. Study the dynamic path of the economy both when $\beta < (1-\alpha)$ and when $\beta = (1-\alpha)$. Explain why the two cases are different.

2 A Growth Model with Leisure

Consider a Ramsey growth model where the representative consumer has instantaneous preferences over consumption c and hours of work h given by

$$u(c,h) = \log c - \theta \frac{h^{1+\eta}}{1+\eta}$$

and discounts future at rate ρ. Workers have a time endowment of one unit, so $0 \le h \le 1$ and $l + h = 1$, where l denotes leisure. The chosen hours worked contribute to production through a Cobb-Douglas technology

$$y = k^{\alpha} h^{1-\alpha}$$

and capital accumulation follows

$$\dot{k} = i - \delta k$$

(a) State the planner's problem for this economy, write down the Hamiltonian and characterize the optimal allocations by use of optimal control techniques.

(b) Express consumption and the capital stock in terms of units of labor, and call the new normalised variables $\widehat{c}$ and $\widehat{k}$. Are the steady-state levels of consumption $\widehat{c}^*$ and capital $\widehat{k}^*$ different from the standard Ramsey model without endogenous labour supply? Explain.

(c) Obtain an expression for $\frac{\dot{h}}{h}$ as a function of $\frac{\dot{\widehat{k}}}{\widehat{k}}$ and $\frac{\dot{\widehat{c}}}{\widehat{c}}$ and interpret it.

(d) By using the dynamic equations of $\frac{\dot{\widehat{k}}}{\widehat{k}}$ and $\frac{\dot{\widehat{c}}}{\widehat{c}}$ show that $\frac{\dot{h}}{h} = \frac{\alpha}{\alpha+\eta}\left[\frac{\widehat{c}^*}{\widehat{k}^*} - \frac{\widehat{c}}{\widehat{k}}\right]$

(e) It can proved that, when $\widehat{k}_0 < \widehat{k}^*$, $\frac{\widehat{c}}{\widehat{k}}$ declines monotonically along the saddle-path to a steady-state. Taking this result as granted, can you say if the speed of convergence of $\widehat{k}$ to the steady-state is larger or smaller than that of per capita capital in the Standard Ramsey model? Explain your intuition.