ECON 815
Macroeconomic Theory
Winter Term 2012/13

Assignment 2
Due: In Class (DUN 213) on ***February 26th*** *2012*
No late submissions will be accepted
No group submissions will be accepted
No electronic submissions will be accepted

Remarks: Write clearly and concisely. Devote some time to give the graphs, plots and tables a format easy to understand. Also the way you present your answers matter for the final grade. Even if a question is mainly analytical, ***briefly*** *explain what you are doing, stressing the economic meaning of the various steps. Being able to convey your thoughts effectively is an asset also in real life.*

1 A Solow model

Consider the Solow model with a Cobb-Douglas technology $Y = K^{\alpha}(TL)^{1-\alpha}$. Define: $s =$ (exogenous) saving rate, $\delta =$ depreciation of capital, $n =$ population growth rate, $x =$ growth rate of labor augmenting technology, $TL = e^{(x+n)t} =$ effective labor supply. Furthermore, let $k = K/L$, $y = Y/L$, and $\widehat{k} = K/(TL)$, $\widehat{y} = Y/(TL)$.

The gross resource constraint for the (closed) economy is

$$K^{\alpha}(TL)^{1-\alpha} = C + I = C + \dot{K} + \delta K$$

where C is consumption, $0 < \alpha < 1$, and $\dot{K} \equiv \frac{dK}{dt}$. We can rewrite the resource constraint as a saving relation:

$$S = sY = Y - C = \dot{K} + \delta K = I.$$

1. Rewrite the resource constraint in terms of $\widehat{k}$, i.e. write $\dot{\widehat{k}}$ as a function of $\widehat{k}$ and exogenous variables.

2. Define $\gamma_{\widehat{k}} = \frac{\dot{\widehat{k}}}{\widehat{k}}$ as the growth rate of $\widehat{k}$. By definition, in a steady-state all the variables grow at a constant rate (i.e. we are considering a Balanced Growth Path). Show that in steady-state $\gamma_{\widehat{k}} = 0$. Can you relax a model assumption so to have positive long run growth? What is γ_K in the steady-state?

3. Find the steady-state value for k (expressed as a function of exogenous parameters).

4. From now on, assume $x = 0$. Express the economy's growth rate of GDP per capita $\frac{\dot{y}}{y}$ (γ_y) at a given point in time as a function of the parameters s, n, δ, α, and the current output level $y(t)$.

5. Solve explicitly the differential equation that describes the law of motion of k, found in (1), given an initial condition $k_0 > 0$. (Hint: this is a Bernoulli equation which can be solved with the substitution $z = k^{1-\alpha}$). Interpret your results.

6. Derive the speed of convergence of capital to the steady-state, defined as $\frac{\dot{k}}{k^* - k}$. (Hint: take a linear approximation of the law of motion for k around the steady-state value.) How do s, n, δ, α affect the speed of convergence? Explain.

2 Another Solow type model

Consider a Solow model with exogenous saving rate s, where the growth rate of the population $n(k)$ depends on the development of the economy and it is not a constant. In particular:

$$n = \begin{cases} 0.08 & \text{if } k < 5,000 \\ \frac{400}{k} & \text{if } 5,000 \leq k < 40,000 \\ 0.01 & \text{if } k \geq 40,000 \end{cases}$$

where k is capital per worker. The production function is $y = k^{0.75}$, and agents save in each period 20% of their income. Moreover $\delta = 0$, that is the rate of depreciation of physical capital is zero, and $x = 0$, that is there is no technological change.

1. Find the law of motion of capital per capita k, and characterize graphically the equilibrium dynamics.

2. Calculate the growth rate of income per capita at time zero for an economy with an initial GDP per capita equal to $y_L(0) = 53.18$. What is the GDP per capita of this economy in the long run? Calculate the growth rate of income per capita at time zero for an economy with an initial GDP per capita equal to $y_H(0) = 3,089.65$. What is the GDP per capita of this economy in the long run?

3. Briefly discuss the predictions of this variant of the Solow model in terms of the "convergence debate".

3 A Ramsey Model with Exponential Utility

Assume that infinite-horizon households maximize a utility function of the form:

$$U(0) = \int_0^\infty u(c(t)) \cdot e^{-(\rho - n)t} dt$$

where $u(c(t))$ is now given by the exponential form:

$$u(c(t)) = -\frac{1}{\eta} e^{-\eta c(t)}$$

and $\eta > 0$.

The behavior of firms is the same as in the Ramsey model, with zero technological progress ($x = 0$). Capital ($K(t)$) and labor ($L(t)$) are the inputs in the Cobb-Douglas production function $Y(t) = K(t)^\alpha L(t)^{1-\alpha}$.

There is no disutility from working and $L(t) = e^{nt}$ and capital depreciates at rate δ.

1. Relate η to the concavity of the utility function and to the desire to smooth consumption over time. Compute the intertemporal elasticity of substitution. How does it relate to the level of per capita consumption, $c(t)$?

2. Write the optimal control problem and find the first-order conditions for a representative household (i.e. you have to solve the decentralized problem) with preferences given by this form of $u(c(t))$.

3. Combine the first-order conditions for the representative household with those of firms and with the other equilibrium conditions to describe the behavior of $c(t)$ and $k(t)$ over time. [Assume that $k(0)$ is below its steady-state value.]

4. Draw the phase diagram of this economy and linearize the system of DE around the steady-state. Is there saddle-path stability?

5. Obtain the (linearized) solution for $k(t)$. How does the speed of convergence depend on the parameter η?