Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2012/13

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Midterm Exam
March 12th 2013
80 minutes: 1.00pm-2.20pm.

- The exam consists of two questions, and the total number of marks is 100. Break a leg.

Question 1: Dynamics in a Solow-type model (50 Marks)

Consider an extension to the basic Solow Growth model. Differently from the standard model, the aggregate production function is as follows:

$$Y(t) = AK(t)$$

where $Y(t)$ is total output, $K(t)$ is the aggregate capital stock, and A is the constant total factor productivity. The households save a constant fraction s of their income, and $k(t) = K(t)/L(t)$ is the capital-labor ratio. Capital does not depreciate (i.e. $\delta = 0$), and the labor force does not grow at a constant rate n. In order to capture the decrease in fertility rates observed over the development stages, the population growth rate depends on the capital-labor ratio $n(k) = \frac{\overline{n}}{k^2}$, where $\overline{n}$ is a positive number.

1. Write the condition for a steady-state and find the expression for the steady-state capital-labor ratio k^*. How much is the steady-state value of output per worker y^*? What is the effect of $\overline{n}$ on k^*?
2. Study graphically this economy.
3. Solve explicitly the differential equation that describes the law of motion of $k(t)$, given an initial condition $k_0 > 0$.
4. What is the dynamic behavior if $k_0 > k^*$? What is the prediction of the model if $k_0 < k^*$ instead?

Question 2: Optimal Capacity Utilization in a Ramsey Model (50 Marks)

Assume that infinite-horizon households maximize a utility function of the form:

$$U(0) = \int_0^\infty \log C(t) \cdot e^{-\rho t} dt$$

where $\log C(t)$ is the istantaneous utility function and $e^{-\rho t}$ is the discount factor. There is no disutility from working, and there is no population growth, i.e. $n = 0$, and $L(t) = L(0) = 1$.

Also the production side is similar to the one in the Ramsey model. There is no technological progress ($x = 0$), and capital ($K(t)$) and labor ($L(t)$) are the inputs in a neoclassical production function. This function, however, is extended to consider the effect of the capacity utilization rate $u(t)$. Intuitively, production units (i.e. plants) have to remain idle for a fraction of each period, because they need maintenance. More formally, $Y(t) = u(t) AK(t)^\alpha L(t)^{1-\alpha}, 0 < \alpha < 1$ and $0 \leq u(t) \leq 1$. A stands for the total factor productivity, and it is constant over time.

Moreover, the higher the capacity utilization, the faster the physical capital's decay, with the depreciation rate now being $\delta(u(t)) = u(t)^{\widetilde{\delta}}, \widetilde{\delta} > 1$. This equation says that the intensity of capital use affects how much capital is destroyed in the production stage, which has to be replaced by new investment. By assumption, the higher the capacity utilization rate, the higher the capital depreciation rate. Focus on the case where $A > \frac{1}{\widetilde{\delta}} > \alpha$.

1. Write the optimal control problem and find the first-order conditions for the households (i.e. you have to solve the "decentralized" problem). Find the expression for consumption growth and discuss it.

2. Write the optimization problem of the representative firm and find the first-order conditions. (Hint: as usual, the firm demands labor and capital services, but it also decides on the capacity utilization rate.)

3. Study the General Equilibrium of the economy and draw the phase diagram with aggregate consumption $C(t)$ on the y-axis and aggregate capital $K(t)$ on the x-axis. Does this economy differ from the standard Ramsey model?

4. Compute the steady-state values for both capital K^* and consumption C^*. Linearize the system around the steady-state and discuss its dynamic properties.