Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2011/12

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Midterm Exam
March 9th 2012
80 minutes: 10.00am-11.20am

- The exam consists of two questions, and the total number of marks is 100. Break a leg.

Question 1: Dynamics in a Solow-type model (50 Marks)

Consider the effect of unemployment in the basic Solow Growth model. The main difference from the standard model is how the labor input enters the aggregate production function, which is as follows:

$$Y(t) = (K(t))^{\alpha} \left[(1-\overline{u})\, L(t) \right]^{1-\alpha}$$

where $Y(t)$ is total output, and $K(t)$ is the aggregate capital stock, $L(t)$ is the size of the labor force, and $\overline{u}$ is the natural rate of unemployment. The labor force grows at rate n, capital depreciates at rate δ, and the households save a constant fraction s of their income. $k(t) = K(t)/L(t)$ is the capital-labor ratio. Focus on the case where the natural rate of unemployment is constant over time, and where all households perfectly share the unemployment risk among them.

1. Write the condition for a steady-state and find the expression for the steady-state capital-labor ratio k^*. How much is the steady-state value of output per worker y^*? What is the effect of the natural rate of unemployment on k^*?
2. Study graphically this economy.
3. Solve explicitly the differential equation that describes the law of motion of $k(t)$, given an initial condition $k_0 > 0$.
4. Now assume that the unemployment rate is no longer constant, and that the labor force *does not* grow at a constant rate n. The unemployment rate is cyclical and it is such that $u(t) = \cos(t) + 1$. Find a transformation of capital $\widehat{k}$ together with a dynamic behavior of the labor force such that you can still represent the economy in the standard graph with $\widehat{k}$ on the x-axis.

Question 2: A Ramsey Model (50 Marks)

Assume that infinite-horizon households maximize a utility function of the form:

$$U(0) = \int_0^{\infty} \log C(t) \cdot e^{-\rho t} dt$$

where $\log C(t)$ is the istantaneous utility function and $e^{-\rho t}$ is the discount factor.

There is no population growth, i.e. $n = 0$, and $L(t) = L(0) = 1$.

The production side is similar to the one in the Ramsey model. There is zero technological progress ($x = 0$), and capital ($K(t)$) is the only input in an aggregate production function, $Y(t) = F(K(t)) = K(t)^{\alpha}, 0 < \alpha < 1$. There is no disutility from working and capital depreciates at rate $\delta(K) = \frac{\overline{\delta}}{2}K, \overline{\delta} > 0$. This equation says that the capital level in the economy affects how much capital is destroyed in the production stage, which has to be replaced by new investment. By assumption, the more developed the economy, the higher its depreciation rate.

1. Write the optimal control problem and find the first-order conditions for the social planner (i.e. you have to solve the "centralized" problem) with his preferences corresponding to the households' ones. Find the expression for consumption growth and discuss it.
2. Draw the phase diagram of this economy with aggregate consumption $C(t)$ on the y-axis and aggregate capital on the x-axis. How does this economy differ from the standard Ramsey model?
3. Compute the steady-state values for capital K^*, and for consumption C^*. Is K^* higher or lower than the value implied by the golden rule?
4. Linearize the system around the steady-state and discuss its dynamic properties.