Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2010/11

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Midterm Exam
March 4th 2011
80 minutes: 11.30pm-12.50pm

- The exam consists of two questions, and the total number of marks is 100. Break a leg.

Question 1: Dynamics in a Solow-type model (40 Marks)

Consider a variant of the Solow model. As usual, define: y = output per capita, k = capital per capita, s = (exogenous) saving rate, δ = capital depreciation rate. There is no population growth and no technological progress, that is $n = x = 0$.

Differently from the standard model, now the per capita production function is $y(t) = f(k(t)) = k^2(t) + \kappa$. κ is a constant parameter, and $\kappa > 0$.

Assume that $\kappa = \frac{1}{4}\left(\frac{\delta}{s}\right)^2$:

1. Study graphically this economy: does it possess a steady-state?
2. If so, compute the steady-state values for capital per capita k^*, output per capita y^*, investment per capita i^* and consumption per capita c^*.
3. Study the dynamic behavior of k, relating it to different initial values for capital k_0. Can the economy enjoy a positive growth rate forever? Is this model consistent with absolute convergence?
4. Now assume that there is a decrease in the constant term in the per-capita production function, which becomes $\kappa' = \frac{1}{8}\left(\frac{\delta}{s}\right)^2 < \kappa$: how does this economy differ from the previous one? Are the dynamics of capital different in this case? Compute the steady-state values for capital per capita, output per capita, investment per capita, and consumption per capita.

Question 2: A Ramsey Model (60 Marks)

Assume that infinite-horizon households maximize a utility function of the form:

$$U(0) = \int_0^\infty [\beta \log c_1(t) + (1-\beta) \log c_2(t)] \cdot e^{-\rho t} dt$$

where the utility function $u(c_1(t), c_2(t))$ now depends on two different consumption goods, $c_1(t)$ and $c_2(t)$, say durables vs. non-durables. Their prices are denoted with $p_1(t)$ and $p_2(t)$. For simplicity, assume that in any time period t the prices are the same and are normalized to 1, i.e. $p_1(t) = p_2(t) = 1$. Moreover, $0 < \beta < 1$.

There is no population growth, i.e. $n = 0$, and $L(t) = L(0) = L > 0$.

The production side is similar to the one in the Ramsey model. There is zero technological progress ($x = 0$), and capital ($K(t)$) and labor ($L(t)$) are the inputs in a production function. However, the production function is not neoclassical, being $Y(t) = F(K(t), L(t)) = K(t)^\alpha L(t), 0 < \alpha < 1$. Markets for the inputs are perfectly competitive, there is no disutility from working and capital depreciates at rate δ.

1. Write the optimal control problem and find the first-order conditions for the representative household (i.e. you have to solve the "decentralized" problem) with preferences given by this form of $u(.,.)$. Find the expressions for consumption growth and discuss them.

2. Derive the General Equilibrium of the model, by solving the problem of the representative firm and by aggregating the households' intertemporal budget constraint.

3. Compute the steady-state values for capital per capita k^*, and for consumption per capita for each of the two consumption goods, i.e. c_1^* and c_2^*.

4. Draw the phase diagram of this economy with aggregate per capita consumption $c(t)$ on the y-axis and aggregate per capita capital on the x-axis. Does this economy differ from the standard Ramsey model?