Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2009/10

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Midterm Exam
March 3rd 2010
80 minutes: 11.30pm-12.50pm

- The exam consists of two questions, and the total number of marks is 100. Break a leg.

Question 1: Dynamics in Solow-type models (40 Marks)

Part 1)

Consider a variant of the Solow model. As usual, define: y = output per capita, k = capital per capita, s = (exogenous) saving rate, δ = capital depreciation rate. There is no population growth and no technological progress.

Differently from the standard model, now the production function $y(t) = f(k(t))$ has the following characteristics:

$$
\begin{aligned}
f(k) &\geq 0, \text{ and } f(k) = 0 \text{ iff } k = 0 \\
f(\overline{k}) &= \delta \overline{k} \\
f'(k) &\geq 0, \; k \in [0, +\infty) \\
\lim_{k \to 0} f'(k) &= +\infty \\
f''(k) &< 0, \text{ if } k \in \left[0, \overline{k}\right) \\
f''(\overline{k}) &= 0 \\
f''(k) &> 0, \text{ if } k \in \left(\overline{k}, +\infty\right)
\end{aligned}
$$

1. Study graphically this economy: does it possess a steady-state?
2. Study the dynamic behavior of k, relating it to different initial values for capital k_0. Can the economy enjoy a positive growth rate forever? Is this model consistent with absolute convergence?

Part 2)

Consider another variant of the Solow model describing an economy that is starting off with a very low value of k_0 (strictly positive, but close to zero). Differently from before, now the production function is neoclassical. However, capital depreciation is different from the standard model. For relatively low values of capital, say $k \in [0, \overline{k})$, depreciation is geometric: capital depreciates at rate δ. As soon as the economy reaches the threshold value $\overline{k} > k_0$, the process for capital depreciation changes. It is still geometric, but with probability p the economy is going to enjoy a low depreciation rate $\underline{\delta} < \delta$ forever. Otherwise, with probability $1 - p$, the economy is going to have a high depreciation rate $\overline{\delta} > \delta$ forever.

3. Analyze this economy graphically, discuss its steady-states (or lack of), and their associated levels of output and consumption.

Question 2: A Ramsey Model (60 Marks)

Assume that infinite-horizon households maximize a utility function of the form:

$$U(0) = \int_0^\infty u(c(t)) \cdot e^{-\rho t} dt$$

where $u(c(t))$ is now given by the "Stone-Geary" form:

$$u(c(t)) = \frac{(c(t) - \overline{c})^{1-\theta} - 1}{1 - \theta}$$

where $\overline{c} \geq 0$ represents the subsistence level of per capita consumption. There is no population growth, i.e. $n = 0$.

The production side is the same as in the Ramsey model, with zero technological progress ($x = 0$). Capital ($K(t)$) and labor ($L(t)$) are the inputs in a neoclassical production function $Y(t) = F(K(t), L(t))$.

There is no disutility from working and capital depreciates at rate δ.

1. What is the intertemporal elasticity of substitution for this utility function? If $\overline{c} > 0$, how does it relate to the level of per capita consumption, $c(t)$?
2. Write the optimal control problem and find the first-order conditions for a benevolent social planner (i.e. you have to solve the "centralized" problem) with preferences given by this form of $u(c(t))$. Find the expression for consumption growth and discuss it.
3. Compute the steady-state values for consumption and capital, i.e. c^* and k^*. How do they differ from the standard Ramsey model with CEIS preferences?
4. Draw the phase diagram of this economy and linearize the system of DE around the steady-state. Briefly discuss its stability.