Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2008/09

Instructor: Marco Cozzi

Midterm Exam
March 6th 2009
80 minutes: 1.00pm-2.20pm

- The exam consists of two questions, and the total number of marks is 100. Break a leg.

Question 1: The AK model with an Exogenous Saving Rate (50 Marks)

Consider the Romer (1986) production function for firm j :

$$
\begin{aligned}
y_j(t) &= k_j^{\alpha}(t)\,A^{\eta}(t) \text{ with } 0 < \alpha < 1; \\
A(t) &= A_0 \frac{\sum_{j=1}^{N} k_j(t)}{N}
\end{aligned}
$$

where y and k are output and capital per worker, and N is the total number of firms.

Suppose that s is the constant saving rate, n is the constant population growth rate, and δ the rate of depreciation of physical capital.

1. Find the differential equation for k when all firms are identical.
2. Represent graphically the growth rate of the model for the cases where the production function exhibits: (i) diminishing returns to scale $\alpha + \eta < 1$, (ii) constant returns to scale $\alpha + \eta = 1$, (iii) increasing returns to scale $\alpha + \eta > 1$.
3. Examine the effect on the long-run growth rate of a change in the saving rate for each of the three cases.
4. Consider the effect of a shock. Suppose an earthquake destroys half of the capital stock of the economy. Examine what happens in each of the three cases to: the growth rate immediately after the shock, the long-run growth rate, and the level of income that would have been reached if there had been no shock. Do shocks have temporary or permanent effects?

Question 2: A Ramsey Model with Exponential Utility (50 Marks)

Assume that infinite-horizon households maximize a utility function of the form:

$$U(0) = \int_0^{\infty} u(c(t)) \cdot e^{-(\rho-n)t} dt$$

where $u(c(t))$ is now given by the exponential form:

$$u(c(t)) = -\frac{1}{\eta} e^{-\eta c(t)}$$

where $\eta > 0$.

The behavior of firms is the same as in the Ramsey model, with zero technological progress ($x = 0$). Capital ($K(t)$) and labor ($L(t)$) are the inputs in the Cobb-Douglas production function $Y(t) = K(t)^{\alpha} L(t)^{1-\alpha}$. There is no disutility from working and $L(t) = e^{nt}$ and capital depreciates at rate δ.

1. Relate η to the concavity of the utility function and to the desire to smooth consumption over time. Compute the intertemporal elasticity of substitution. How does it relate to the level of per capita consumption, $c(t)$?
2. Write the optimal control problem and find the first-order conditions for a representative household (i.e. you have to solve the decentralized problem) with preferences given by this form of $u(c(t))$.
3. Combine the first-order conditions for the representative household with those of firms and with the other equilibrium conditions to describe the behavior of $c(t)$ and $k(t)$ over time. [Assume that $k(0)$ is below its steady-state value.]
4. Draw the phase diagram of this economy and linearize the system of DE around the steady-state. Is there saddle-path stability?
5. Obtain the (linearized) solution for $k(t)$. How does the speed of convergence depend on the parameter η?