Hence: $$-\frac{\partial}{\partial t}\int_t^{\tau} r(w)\,dw = -\left[\emptyset - 1\cdot r(t) + \int_t^{\tau} \underbrace{\frac{\partial}{\partial t} r(w)}_{\emptyset}\,dw\right] = r(t)$$

$$-\left[\frac{\partial}{\partial t}\int_t^{\tau} r(w)\,dw\right]\cdot \mathrm{EXP}\left\{-\int_t^{\tau} r(w)\,dw\right\} = r(t)\cdot \mathrm{EXP}\left\{-\int_t^{\tau} r(w)\,dw\right\}$$

[*] $$\int_t^{\infty} \frac{\partial}{\partial t}\mathrm{EXP}\left\{-\int_t^{\tau} r(w)\,dw\right\} d\tau = \int_t^{\infty} r(t)\,\mathrm{EXP}\left\{-\int_t^{\tau} r(w)\,dw\right\} d\tau$$

It follows that

$$\dot{V}(t) = \pi\cdot\left[-1 + \int_t^{\infty} r(t)\,\mathrm{EXP}\left\{-\int_t^{\tau} r(w)\,dw\right\} d\tau\right] = \pi\left[-1 + r(t)\int_t^{\infty}\mathrm{EXP}\left\{-\int_t^{\tau} r(w)\,dw\right\} d\tau\right]$$

$$\dot{V}(t) = -\pi + r(t)\underbrace{\int_t^{\infty} \pi\cdot\mathrm{EXP}\left\{-\int_t^{\tau} r(w)\,dw\right\} d\tau}_{=V(t)} = -\pi + r(t)\,V(t)$$

$$r(t)\,V(t) = \pi + \dot{V}(t)$$

$$r(t) = \frac{\pi}{V(t)} + \frac{\dot{V}(t)}{V(t)}$$