ECON 815 EGT.

$$V(t) = \underbrace{A^{\frac{1}{1-\alpha}} \cdot \alpha^{\frac{2}{1-\alpha}} L \left(\frac{1-\alpha}{\alpha}\right)}_{\pi} \int_t^{\infty} e^{-\bar{r}(t,v)(v-t)}\, dv \longrightarrow \frac{dV(t)}{dt} = \pi \cdot \frac{\partial}{\partial t} \int_t^{\infty} \operatorname{Exp}\left\{ -\left(\frac{1}{\cancel{v-t}}\right) \cdot \int_t^{v} r(w)\,dw\, \cancel{(v-t)} \right\}$$

Leibnitz:
$$\frac{\partial}{\partial t} \int_{\gamma(t)}^{\phi(t)} g(t,v)\,dv = \phi'(t)\, g(t,\phi(t)) - \gamma'(t)\, g(t,\gamma(t)) + \int_{\gamma(t)}^{\phi(t)} \frac{\partial}{\partial t} g(t,v)\, dv$$

Define: $\gamma(t) = t \rightarrow \gamma'(t) = 1$

"$\phi(t) = \infty$" $\rightarrow \phi'(t)\, g(t,\phi(t)) = 0$

$$g(t,v) = \operatorname{Exp}\left\{ - \int_t^{v} r(w)\,dw \right\}$$

Hence:
$$\frac{\partial}{\partial t} \int_t^{\infty} \operatorname{Exp}\left\{ - \int_t^{v} r(w)\,dw \right\} dv = 0 - \underbrace{1 \cdot \operatorname{Exp}\Big\{ - \underbrace{\int_t^{t} r(w)\,dw}_{=0} \Big\}}_{=1} + \underbrace{\int_t^{\infty} \frac{\partial}{\partial t} \operatorname{Exp}\left\{ - \int_t^{v} r(w)\,dw \right\}}_{\text{See [*]}}$$

Apply Leibnitz ~~again~~ because: $\frac{\partial}{\partial t} \operatorname{Exp}\left\{ - \int_t^{v} r(w)\,dw \right\} = \frac{\partial}{\partial t}\left[- \int_t^{v} r(w)\,dw \right] \cdot \operatorname{Exp}\left\{ - \int_t^{v} r(w)\,dw \right\}$

$$\frac{\partial}{\partial t}\left[- \int_t^{v} r(w)\,dw \right] = \tilde{\phi}'\, \tilde{g}(t,\tilde{\phi}(t)) - \tilde{\gamma}'(t)\, \tilde{g}(t,\tilde{\gamma}(t)) + \int_{\tilde{\gamma}(t)}^{\tilde{\phi}(t)} \frac{\partial}{\partial t} \tilde{g}(t,w)\, dw$$

with $\tilde{\gamma}(t) = t \rightarrow \tilde{\gamma}'(t) = 1$; $\tilde{\phi}(t) = v \rightarrow \tilde{\phi}'(t) = 0$; $\tilde{g}(t,w) = \hat{g}(w) = r(w)$