Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2011/12

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Final Exam
April 17th 2012
180 minutes: 9:00 a.m. to 12:00 p.m.

- Instructions:
- This examination is THREE HOURS in length.
- The total number of marks is 100.
- You get one mark just for seating the exam.
- The exam consists of three questions.
- Each question is worth 33 marks.
- Do **ANY THREE** of the *five* questions.
- For questions that involve a numerical part be sure to show your calculations and intermediate steps.
- Please answer the questions in the answer booklets provided.
- Hand held calculators are allowed.
- Read the questions carefully. The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.

Question 1: A Solow Model with Workers' Types (33 Marks)

Consider a variant of the Solow model. As usual, define: Y = aggregate output, K = aggregate capital, δ = capital depreciation rate. There is no population growth and no technological progress, that is $n = x = 0$. For simplicity, the population size is normalized to 1, hence $L = 1, \forall t$.

Differently from the standard model, in this economy there are two household types ($i = H, L$) that differ in their saving rates. Each household type has mass $= \frac{1}{2}$, and their (exogenous) saving rates are $1 > s_H > s_L > 0$. Both households have access to the same production function, which is $Y_i(t) = F(K_i(t)) = AK_i(t) + B, i = H, L$. A and B are two constant parameters, $A > 0$ and $B > 0$, common to both households, and the i index denotes the household type.

1. Study graphically this economy: does it possess a steady-state?
2. Find a restriction on the parameter δ such that a (non-trivial) steady-state exists, and compute the steady-state values for aggregate capital K^*, output Y^*, investment I^* and consumption C^*.
3. Study the dynamic behavior of K, relating it to different initial values for capital K_0. Can the economy enjoy a positive growth rate forever? Is this model consistent with absolute convergence?
4. Now assume that there is a decrease in the constant term in the production function, which becomes $B = 0$: how does this economy differ from the previous one? Are the dynamics of capital different in this case?

Question 2: Growth with Spillover Effects (33 Marks)

Consider a version of Romer (1986) model. This economy is in continuous time and there is perfect foresight. The representative household maximizes his utility:

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-(\rho-n)t} dt,$$

where ρ is the time discount rate, $c(t)$ is consumption per capita, and n is the population growth rate, with $L(0) = 1$.

Assume that the production function for firm i is $Y_i = A(K_i)^\alpha (L_i)^{1-\alpha} K^\lambda$, where $0 < \alpha < 1$, $0 < \lambda < 1$, and K is the aggregate stock of capital. Assume also a depreciation rate of $0 < \delta < 1$.

1. Characterize the solution of the decentralized economy when $\lambda < 1 - \alpha$ and L is constant ($n = 0$). Are the transitional dynamics similar to those of the Ramsey model? What is the steady-state growth rate of Y, K, and C in this case?
2. If $\lambda < 1 - \alpha$ and L grows at the rate $n > 0$, what is the steady-state growth rate of Y, K, and C?
3. What happens if $\lambda = 1 - \alpha$ and L grows at the rate $n > 0$?

Question 3: Growth through Expanding Variety of Products (33 Marks)

Consider the benchmark version of the growth model with expanding varieties as described in class. Assume no population growth and suppose that agents have CEIS utility:

$$U = \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

The households' budget constraint is:

$$\dot{B} = rB + wL - C$$

where B stands for households assets.

Suppose a different production function for the final good sector (competitive) firms, which now is:

$$Y_i = A L_i^{1-\alpha} \left(\sum_{j=1}^{N} X_{ij}^{\sigma} \right)^{\frac{\alpha}{\sigma}}$$

where $0 < \sigma < 1$. The parameter σ, rather than α, will now determine the elasticity of demand for each type of intermediate.

As usual, the intermediate good sector (monopolistic) producer of the single intermediate good j maximizes:

$$\pi_j = p_j X_j - X_j$$

The cost of one innovation is equal to η.

1. How are intermediate inputs priced, and what is the quantity of each intermediate X_j?
2. What is the free entry condition for the *R&D* firms, and how is the rate of return determined?
3. What are the growth rates of N, X_j and total output Y along a BGP?
4. Is this growth rate pareto optimal?

Question 4: Growth and Quality of Capital (33 Marks)

Consider an extension of the growth model with expanding varieties as described in class. We will not consider the households' problem: we focus only on the production side of the economy.

Suppose a different production function for the final good sector (competitive) firms, which now is:

$$Y = A^Y \left(\sum_{j=1}^{N} X_j^{\gamma} \right)^{\frac{1}{\gamma}}$$

where $\gamma < 1$.

The intermediate good sector producers of the intermediate good j maximize their profits. Intermediate goods are produced with capital and labor:

$$X_j = A_j^X K_j^{\alpha} L_j^{1-\alpha}$$

Crucially, the A_j^X are product specific. In particular, the efficiency level varies across intermediates for identical levels of capital and labor inputs. Labor is homogeneous and mobile. Intermediate good producers are price takers in the factor markets. Therefore, they consider the wage w and the rental rate of capital r as given. Aggregate endowments of capital and labor are given by $K = \sum_{j=1}^{N} K_j$ and $L = \sum_{j=1}^{N} L_j$.

1. Derive the demand function for intermediate good X_j. Using this demand function, write the conditions for the intermediate producer's optimal demand of labor and capital, L_j and K_j.
2. Show that the capital-labor ratio is equal in all intermediate sectors and that $\frac{K_j}{K} = \frac{L_j}{L}$ for all j.
3. Let ϕ_j be the share of sector j capital in the country's total capital stock: $\phi_j = \frac{K_j}{K}$. Find the expression for the labor share in sector j as a function of A_j^X, ϕ_j, α and γ.
4. Using the result in part 3, find the value of ϕ_j as a function of the A_j^X. (Hint: use the guess $\phi_j = \frac{\left(A_j^X\right)^{\varepsilon}}{\sum_{j=1}^{N}\left(A_j^X\right)^{\varepsilon}}$ and solve for ε.)
5. Obtain an expression for per capita output in terms of $k = \frac{K}{L}, A_j^X, A$ and ϕ_j.

Question 5: A Search Model in Discrete Time (33 Marks)

An unemployed worker searches for a job in a labor market with informational frictions. Every period the unemployed worker receives unemployment benefits equal to b.

Differently from the baseline model, an unemployed worker can now draw *two* independently and identically distributed wage offers from the CDF $F(\widehat{w}) = \Pr(w \leq \widehat{w})$. The worker will work forever at the same wage after having accepted an offer.

1. Write the value functions for the unemployed worker (U), and for the employed worker (W).
2. Show graphically that the optimal policy for the unemployed is a reservation wage strategy.
3. Characterize the reservation wage equation for the agent.
4. Prove that the worker's reservation wage is higher than the case with the worker facing the same b but drawing only one offer from the same CDF $F(\widehat{w})$.