Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2010/11

Instructor: Marco Cozzi

Final Exam
April 25th 2011
180 minutes: 9:00 a.m. to 12:00 p.m.

- Instructions:
- This examination is THREE HOURS in length.
- The total number of marks is 100.
- You get one mark just for seating the exam.
- The exam consists of three questions.
- Each question is worth 33 marks.
- Do **THREE** of the *four* questions.
- For questions that involve a numerical part be sure to show your calculations and intermediate steps.
- Please answer the questions in the answer booklets provided.
- Hand held calculators are allowed.
- Read the questions carefully. The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.

Question 1: A Ramsey model with different workers (33 Marks)

This economy is in continuous time and there is perfect foresight. There are two types of households $i = \{s, u\}$ that maximize their utility:

$$U_i = \int_0^\infty \frac{c_i(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, i = \{s, u\}$$

where ρ is the time discount rate, $c_i(t)$ is consumption per capita for each type, there is no population growth, and $L_s(0) = 1 - \mu, L_u(0) = \mu$. The two types of households supply two different kinds of labor, say skilled (s) Vs. unskilled (u) labor. Their wages w_s and w_u are determined in segmented competitive markets.

The aggregate production function is $Y(t) = K^\alpha(t)\left(BL_u^\eta(t) + (1-B)L_s^\eta(t)\right)^{\frac{1-\alpha}{\eta}}$, where $0 < B < 1$ is a constant, $0 < \alpha < 1$ and $0 < \eta < 1$. A starting $k_0 > 0$ is given and $0 < \delta < 1$ is the depreciation rate of physical capital.

1. Characterize the decentralized solution to the problem with a system of differential equations. Can you say which household type is going to have the faster consumption growth?
2. Combine the previous equations with equilibrium factor prices, and derive the equilibrium path. Impose a restriction on B in order for $w_s > w_u$.
3. Is the competitive equilibrium Pareto optimal?
4. Assume that $-\infty < \eta < 0$ instead. What has happened to wage inequality? Would the social planner choose a different allocation in this case?

Question 2: Growth and Scale Effects (33 Marks)

Consider a version of Romer (1986) model. This economy is in continuous time and there is perfect foresight. The representative household maximizes his utility:

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-(\rho-n)t} dt,$$

where ρ is the time discount rate, $c(t)$ is consumption per capita, and n is the population growth rate, with $L(0) = 1$.

The firm's productivity parameter A_i depends on the economy's average capital per worker, $\frac{K}{L}$. Hence, the production function is now:

$$Y_i = AK_i^\alpha \left[\frac{K}{L} L_i\right]^{1-\alpha}$$

where $0 < \alpha < 1$ is the capital share. The aggregate resource constraint for the economy is:

$$Y(t) = C(t) + I(t)$$

a starting $k_0 > 0$ is given and $0 < \delta < 1$ is the depreciation rate of physical capital.

1. Characterize the solution to the problem with two differential equations, one for capital and one for consumption.
2. Combine these equations with equilibrium factor prices, and derive the decentralized equilibrium path.
3. Characterize the solution to the social planner problem.
4. Does this economy have scale effects?

Question 3: Varieties and Taxes (33 Marks)

Consider Romer (1990) model without physical capital, as described in class. Agents have logarithmic utility:

$$U = \int_0^\infty \ln C(t)\, e^{-\rho t} dt$$

The households' budget constraint is:

$$\dot{A} = rA + wL - C$$

The number of varieties evolves according to (L stands for the size of the constant labor force, while λ is the share of workers employed in the final good sector):

$$\frac{\dot{N}}{N} = \frac{(1-\lambda) L}{\eta}$$

The profits for the final good sector (competitive) firms are (x_j stands for the quantity of intermediate good j):

$$\pi = (\lambda L)^{1-\alpha} \int_0^N x_j^\alpha dj - w\lambda L - \int_0^N p_j x_j dj$$

while the corporate profits of the intermediate good sector (monopolistic) producers are taxed at proportional rate $\tau > 0$. Tax revenues are thrown away, that is they are not used to finance any public expenditure.

1. Characterize the laissez-faire (decentralized) equilibrium. Find, in particular, the growth rate of the economy.
2. Characterize the Pareto optimal allocations.
3. Consider two economies with identical technologies, identical initial conditions but with different corporate tax rates, $\tau' > \tau > 0$. Determine the relative income of these two economies (as a function of time).
4. Imagine that the government uses the tax revenues to subsidize proportionally the use of labor in final production. This means that final firms will have to pay $1 - s$ for each unit of labor used to produce final goods. What are the effects on the growth rate? Is this policy going to restore the first-best Pareto optimum?

Question 4: Varieties and Cournot Competition (33 Marks)

Start from the benchmark version of the growth model with expanding varieties as described in class. Assume no population growth and suppose that agents have CEIS utility:

$$U = \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

The households' budget constraint is:

$$\dot{A} = rA + wL - C$$

where A stands for households assets.

The profits for the final good sector (competitive) firms are (X_j stands for the quantity of intermediate good j):

$$\pi = L^{1-\alpha} \sum_{j=1}^{N} \left[\sigma X_j - \frac{\phi}{2} X_j^2 \right] - wL - \sum_{j=1}^{N} p_j X_j$$

where $0 < \alpha < 1, \sigma > 1$ and $0 < \phi < 2$ are constants.

The intermediate good sector differs from the benchmark case. If innovations are being developed, there are always two new firms becoming active, which form a duopoly in the intermediate sector. These two oligopolists compete a-la Cournot and once they enter the intermediate sector they are going to keep their market power forever. The duopolists, denoted with j_1 and j_2, provide the final good sector with the same variety j (that they have invented simultaneously.) Their marginal cost of production is still equal to 1.

Finally, rather than being equal to η, the cost of one innovation is now $\frac{\eta}{I}$, where $I = 2$ represents the number of oligopolists operating in each intermediate good market.

You can interpret this situation as one in which there are several identical entrepreneurs that apply for a free licence allowing them to innovate, with the competition bureau selecting randomly only two applicants. Once selected, the two innovators share the innovation costs equally between them, they start producing, but they cannot collude in the production stage.

1. How are intermediate inputs priced, and what is the quantity of each intermediate X_j? (Consider only symmetric Nash equilibria.)
2. What is the free entry condition for the *R&D* firms and how is the rate of return determined?
3. What are the growth rates of N, X and total output Y along a BGP?
4. Is this economy efficient? If not, can you suggest a set of policies that could achieve the first best?