Queen's University<br>Faculty of Arts and Science<br>Department of Economics<br>ECON 815<br>Macroeconomic Theory<br>Winter Term 2009/10<br>Instructor: Marco Cozzi

Final Exam
April 16th 2010
180 minutes: 9:00 a.m. to 12:00 noon

- Instructions:
- This examination is THREE HOURS in length.
- The total number of marks is 100 .
- You get one mark just for seating the exam.
- The exam consists of four questions and two parts, A and B.
- Each question is worth 33 marks.
- Do THREE of the four questions, but:
- Do the Part A (labor markets) question, which is compulsory.
- Do TWO of the three Part B (growth theory) questions.
- For questions that involve a numerical part be sure to show your calculations and intermediate steps.
- Please answer the questions in the answer booklets provided.
- Hand held calculators are allowed.
- Read the questions carefully. The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.
- Break a leg.


## PART A: Labor Markets.

This question is compulsory. It is worth 33 marks.

## Question 1 - A: Search Equilibrium (in discrete time) (33 Marks)

An unemployed worker searches for a job in a labor market with informational frictions. Every period the unemployed worker receives unemployed benefits equal to $b$.

Differently from the baseline model, now there are two types of jobs in the economy: full-time jobs and part-time ones.

The wage offer distribution of full time jobs in the economy is described by the $\operatorname{CDF} F(\widehat{w})=$ $\operatorname{Pr}(w \leq \widehat{w})$. There is a minimum wage $\underline{w}$, i.e. the lower bound of $F($.$) is \underline{w}$ (which satifies $\underline{w}>b$ ), and every part-time job pays exactly the minimum wage.

A worker who accepted a part-time job can still be searching for a full-time job, and he draws wage offers from $F($.$) , just like any unemployed worker.$

A worker that accepted a full-time job cannot search any longer for other jobs.
There is no destruction of jobs, apart from the voluntary quits.
When searching as an unemployed, with probability $\gamma$ the agent draws an offer from part-time jobs, but with probability $(1-\gamma)$ he receives no such offer.

Similarly, with probability $\alpha$ the agent draws an offer from full-time jobs, but with probability ( $1-\alpha$ ) he receives no such offer. The same applies to the job searcher who is currently employed part-time. Finally, the two sets of offers are independent events.

1. Write the value functions for the unemployed worker $(U)$, for the worker employed in a part-time job $(P)$, and for the worker employed in a full-time job $(W)$.
2. Show graphically that the optimal policy for the unemployed is a reservation wage strategy. How many reservation wages a rational agent is going to set?
3. Characterize the reservation wage equation(s) for the agent.

## PART B: Growth Theory.

Answer TWO of the three questions. Each question is worth 33 marks.

Question 2 - B: Government, Time-varying Consumption Tax Rates, and Growth (33 Marks)

Consider the baseline Ramsey model with no technological change, no population growth, and a neoclassical production function.

Part A)
Start with a situation in which the government does not tax capital income or purchase goods and services: $\tau_{a}=\tau_{f}=G=0$. Moreover, the consumption tax rate, $\tau_{c}$, is constant. Suppose that the government switches to a rising path of $\tau_{c}$ (i.e. $\dot{\tau}_{c}>0$ ) while maintaining $\tau_{a}=\tau_{f}=G=0$.

1. How does this change affect the households' first-order condition for consumption growth?
2. Focus on the special case where the tax growth rate is constant, that is $\frac{\dot{\tau}_{c}}{\tau_{c}}=\gamma_{\tau}$. Does the economy have a steady state? Does it have a Balance Growth Path? Is the shift to a time-varying consumption tax rate a good idea?

## Part B)

Suppose that the government unexpectedly increases government spending, raising it from $G=0$ to $\bar{G}>0$.
3. Starting from the steady-state, analyze the effects of this increase on the paths of consumption and capital accumulation.
4. Suppose, instead, that the increase in government spending is announced at time $t_{0}$ to take place at a later time $t_{1}$. Characterize the dynamic effects on consumption and capital accumulation from $t_{0}$.

## Question 3 - B: A Ramsey type model (33 Marks)

A hypothetical social planner solves the following Ramsey problem in continuous time under perfect foresight. The social planner maximizes the representative household's utility:

$$
U=\int_{0}^{\infty} \frac{c(t)^{2}}{2} e^{-\rho t} d t
$$

where $\rho$ is the time discount rate, $c(t)$ is consumption per capita, there is no population growth, and $L(t)=1$. Production per capita is $f(k(t))=A k(t)^{\alpha}$, where $0<\alpha<1$, and the resource constraint for the economy is

$$
\dot{k}(t)=f(k(t))-\delta k(t)-c(t)
$$

a starting $k_{0}>0$ is given and $0<\delta<1$ is the depreciation rate of physical capital.

1. Characterize the solution to the problem with two differential equations, one for $k(t)$ and one for $c(t)$.
2. Solve for the steady-state values for the capital stock and consumption. Describe the dynamics around the steady-state in a phase diagram.
3. What happens to the economy if the depreciation rate decreases permanently (i.e. if $\delta$ decreases)? Assume that the economy is initially in a steady-state. Plot the results in a phase diagram.
4. Assume $\delta=0$. Linearize the system found in part (1) around its steady-state and solve for $k(t)$. Examine the stability properties of the linearized system. Argue that the reduced system together with the transversality condition and $k_{0}$ pins down $c(0)$.

## Question 4 - B: Varieties and Labor (33 Marks)

Consider the version of the model of Romer (1990) without physical capital described in class. Agents have logarithmic utility:

$$
U=\int_{0}^{\infty} \ln C(t) e^{-\rho t} d t
$$

The households' budget constraint is:

$$
\dot{A}=r A+w L-C
$$

Assume that labor is not used in final good production, but is used as the unique input in the intermediate goods production. Assume the final good production function is given by:

$$
Y=\int_{0}^{N} x_{j}^{\alpha} d j
$$

Also suppose that $\frac{1}{N}$ units of labor are required to produce one unit of any intermediate good. The number of varieties evolves according to:

$$
\frac{\dot{N}}{N}=\delta\left(L-L_{x}\right)
$$

where $L$ stands for the size of the constant labor force, while $L_{x}$ is the amount of workers employed in the intermediate good sector.

1. What is the equilibrium level of intermediate good production?
2. What is the equilibrium price of a unit of any intermediate good?
3. Looking at a BGP equilibrium in which wages and technology grow at the same rate, compute the maximum profit for intermediate good producers.
4. Write down the research arbitrage condition.
5. Compute the equilibrium growth rate of the economy.
