Queen's University
Faculty of Arts and Science
Department of Economics
ECON 815
Macroeconomic Theory
Winter Term 2008/09

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Final Exam
April 13th 2009
180 minutes: 9:00 a.m. to 12:00 noon

- Instructions:
- This examination is THREE HOURS in length.
- The total number of marks is 100.
- You get one mark just for seating the exam.
- The exam consists of five questions and two parts, A and B.
- Each question is worth 33 marks.
- Do **THREE** of the *five* questions, but:
- Do at least **ONE** of the *two* Part A (labor markets) questions.
- Do at least **ONE** of the *three* Part B (growth theory) questions.
- For questions that involve a numerical part be sure to show your calculations and intermediate steps.
- Please answer the questions in the answer booklets provided.
- Hand held calculators are allowed.
- Read the questions carefully. The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.
- Break a leg.

PART A: Labor Markets.

Answer at least **ONE** of the *two* questions. Each question is worth 33 marks.

Question 1 - A: Search Equilibrium (in discrete time) (33 Marks)

An unemployed worker searches for a job in a labor market with informational frictions. The wage offer distribution in the economy is described by the CDF $F(\widehat{w}) = \Pr(w \leq \widehat{w})$.

Every period the unemployed worker receives unemployed benefits equal to b. When searching, with probability α he draws an offer from the wage distribution, but with probability $(1-\alpha)$ he receives no offer, so he remains unemployed.

When employed, he is paid the wage w he accepted, but with some probability λ he can be laid off and he becomes unemployed again.

1. Write the value functions for the unemployed and the employed worker.
2. Show graphically that the optimal policy for the unemployed is a reservation wage strategy.
3. Characterize the reservation wage equation for the agent.
4. Prove that an increase in α will increase the reservation wage. Explain the intuition for this result.

Question 2 - A: Matching Equilibrium (33 Marks)

Consider a labor market where vacancies v and unemployed workers u are matched together into new jobs m through an aggregate Cobb-Douglas, constant returns to scale, matching function:

$$m = Av^{\alpha}u^{1-\alpha}$$

Therefore, unemployed workers meet vacant jobs at rate $p(\theta) = \frac{m}{u} = A\theta^{\alpha}$, where $\theta = \frac{v}{u}$ is the labor market tightness. Unemployed workers receive a welfare payment z, and when they find a job, they receive a wage w. Firms when vacant pay a constant recruiting cost c every period, and meet idle workers at rate $q(\theta) = \frac{m}{v} = A\left(\frac{1}{\theta}\right)^{1-\alpha}$. Free entry of firms drives the value of a vacancy to zero in equilibrium. A worker-firm pair who engage in production generate output p. Jobs break down exogenously at rate λ. Every period a firm matched with a worker makes profits equal to $\pi = p - w$. There is no capital and the interest rate r is exogenously given. Time is continuous. The wage is determined by cooperative Nash bargaining between the worker and the firm, where the bargaining power of the worker is assumed to be β.

1. Write down the value functions for the unemployed U and employed workers W, and for vacant V and operating firms J.
2. The wage which solves the Nash bargaining as a function of market tightness is:

   $$w = (1-\beta)z + \beta(p + \theta c)$$

   Interpret this expression.
3. Obtain the two equations (from the free entry condition and the steady-state unemployment condition) that describe the steady-state equilibrium of the model in the unemployment-vacancies (u, v) space, i.e. the Beveridge space.
4. Call these two curves the "free entry condition" (the one positively sloped) and the Beveridge curve (the one negatively sloped). In a number of OECD Countries, in the past 30 years, the Beveridge curve has shifted outward (i.e. to the right) in the (u, v) space. Identify at least two changes in the parameters of the model $\{A, \alpha, \beta, \lambda, c, p, r, z\}$ that can rationalize this outcome and explain your logic.

PART B: Growth Theory.

Answer at least **ONE** of the *three* questions. Each question is worth 33 marks.

Question 3 - B: Government and Growth (33 Marks)

Consider the following model of growth with a public good. The economy is populated by a constant population $L = 1$ of identical agents endowed with the following logarithmic preferences:

$$U(0) = \int_0^{\infty} \log(c(t)) \cdot e^{-\rho t} dt$$

subject to the budget constraint:

$$\dot{a}(t) = r(t) a(t) + w(t) - c(t)$$

and by a continuum of competitive firms producing a unique final good. The government taxes production and uses the proceedings to provide free public services G that can be used by all firms simultaneously, with no congestion effect. The production function for an individual firm i is:

$$Y_i = A L_i^{1-\alpha} K_i^{\alpha} G^{1-\alpha}$$

where Y_i, L_i, K_i denote firm-level variables (time indexes are omitted), and $0 < \alpha < 1$. The government runs a balanced budget, $G = \tau Y$, where τ is the tax rate which is assumed to be levied on the value of production of each firm. Capital is assumed not to depreciate, i.e. $\delta = 0$.

1. Assume that τ is exogenous. Characterize the profit maximizing firms' behavior, and determine the equilibrium interest rate r and wage rate w as functions of τ and other exogenous parameters.
2. Write the utility maximization problem for consumers, solve it, and characterize the equilibrium growth rate as functions of τ and other exogenous parameters. Is the growth rate increasing or decreasing in τ?
3. Consider now a benevolent social planner who chooses the paths of consumption, capital and public good provision to maximize the representative agent's present discounted value utility. The planner takes as constraints the initial level of K_0 and the aggregate resource constraint. Characterize the allocation chosen by this planner (i.e. the Pareto Optimum) and the corresponding growth rate.
4. Can a government whose only policy instrument is taxation on production (as above) achieve a Pareto Optimum? Explain why.

Question 4 - B: Solow type models (33 Marks)

Part 1)

Consider a Solow model with exogenous saving rate s, where the growth rate of the population $n(k)$ depends on the development of the economy and it is not a constant. In particular:

$$n = \begin{cases} 0.08 & \text{if } k < 5,000 \\ \frac{400}{k} & \text{if } 5,000 \leq k < 40,000 \\ 0.01 & \text{if } k \geq 40,000 \end{cases}$$

where k is capital per worker. The production function is $y = k^{0.75}$, and agents save in each period 20% of their income. Moreover $\delta = 0$, that is the rate of depreciation of physical capital is zero.

1. Find the law of motion of capital per capita k, and characterize graphically the equilibrium dynamics.
2. Calculate the growth rate of income per capita at time zero for an economy with an initial GDP per capita equal to $y_L(0) = 53.18$. What is the GDP per capita of this economy in the long run? Calculate the growth rate of income per capita at time zero for an economy with an initial GDP per capita equal to $y_H(0) = 3,089.65$. What is the GDP per capita of this economy in the long run?
3. Briefly discuss the predictions of this variant of the Solow model in terms of the "convergence debate".

Part 2)

Consider a different Solow model with both physical capital K and human capital H. Output can be used on a one-to-one basis for consumption or investment in either type of capital. People save a constant fraction s, of their gross income. The production function is the usual Cobb–Douglas, however the inputs now are physical capital K, human capital H, and "raw" labor L:

$$Y = AK^{\alpha}H^{\eta}L^{1-\alpha-\eta}$$

1. Find the expression for output in per unit of effective labor.
2. Under the assumption that all input markets are competitive, find the expressions for their prices: R_K (rental rate of physical capital), R_K (rental rate of human capital), and w (raw labor services).
3. Assume that the two capital goods depreciate at the same constant rate δ, and that there is no population growth or technological progress. Find the law of motion for this economy.

Question 5 - B: An extension to Romer (1990) (33 Marks)

Consider a version of the Romer (1990) model, without physical capital described in class. Agents have logarithmic utility:

$$U = \int_0^\infty e^{-\rho t} \ln C(t)\, dt$$

The households' budget constraint is:

$$\dot{A} = rA + wL - C$$

Differently from the baseline model, assume that there are two different types of intermediate goods: x_1 and x_2. The number of varieties N_i for each intermediate evolves according to:

$$\frac{\dot{N}_i}{N_1 + N_2} = \delta_i L_{x_i}, i = 1, 2$$

where L_{x_i} is the amount of researchers employed in the discovery of new varieties in the i intermediate good sector, L stands for the size of the constant labor force, while L_y is the amount of workers employed in the final good sector.

The profits for the final good sector (competitive) firms are ($x_i(j)$ stands for the quantity of variety j of the intermediate good i):

$$\pi = L_y^{1-\alpha} \left[\sum_{i=1}^{2} \int_0^{N_i} [x_i(j)]^\alpha \, dj \right] - wL_y - \sum_{i=1}^{2} \int_0^{N_i} p_i(j)\, x_i(j)\, dj$$

while the intermediate good sector (monopolistic) producers of the single intermediate good j maximizes:

$$\pi_i(j) = p_i(j)\, x_i(j) - x_i(j), i = 1, 2$$

1. Characterize the laissez-faire (decentralized) equilibrium of this economy. Consider the case where $\delta_1 = \delta_2 = \delta$. More precisely, obtain the expressions for $p_i(j), x_i(j), \pi_i(j), L_{x_i}$, together with the growth rate.
2. Argue what would happen in the case where $\delta_1 \neq \delta_2$. Would the problem of the households change? Do NOT solve the model in this case; just use the free entry condition and your economic intuition.
3. Write the Social Planner Problem for this economy, and take the first order conditions. Give an economic interpretation to these equations. However, do NOT solve for the allocations.
4. Why the competitive equilibrium is not Pareto Optimal? What would be the effects of a policy trying to foster the innovation process by subsidizing the wage costs for innovating firms at rate $s_{R\&D}$?