

**Trading off Tax Distortion  
and Tax Evasion<sup>+</sup>**

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**Running head:** „Tax Distortion and Tax Evasion“

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**Abstract**

Income tax evasion is modeled as a risky activity and integrated into a standard optimal tax problem in which there is a good whose sales are observable. If the penalty for evasion is proportional to the tax evaded (the Yitzhaki scheme), the optimal tax structure is unaffected by evasion. If the penalty is proportional to unreported income (the Allingham-Sandmo scheme), it is efficient to tax both the observable good and income. The cost of the risk of tax evasion is traded off against the distortion from taxing the observable good. For equal penalties, Allingham/Sandmo is more efficient than Yitzhaki.

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## 1. Introduction

It is apparent that different taxes are evaded to a different extent, and optimal tax design should clearly take such differences into account. In fact, there is relatively little literature on the choice of an optimal tax structure in an economy where evasion exists. This is rather surprising given that the basic analysis of the positive effects of taxation on evasion goes back to Allingham and Sandmo (1972). In this paper, we analyze whether and how the principles of efficient tax design must be revised to take account of the differential ease with which various taxes can be evaded. There may well be a trade-off between the efficiency cost of tax distortion and the efficiency cost of tax evasion. This is shown in a simple context in which a tax applying on a narrow base (a good) is more difficult to evade than one on a broad but less distorting base (income). This way of formulating the problem sheds light on the classical issue of when uniform taxation should be supplemented by differential commodity taxes. In some deeper sense, our analysis makes clear that optimal tax policy cannot and should not be separated from the policy to penalize detected tax evasion.

The existing literature on tax design in the presence of tax evasion has two main thrusts. The first of these exploits the idea that the choice of a tax mix can be motivated by tax evasion considerations. If an otherwise ideal tax base can be evaded, obtaining some revenues from a parallel tax base that overlaps to some extent can mitigate the problem. Thus, Boadway, Marchand and Pestieau (1994) analyze the case of a direct tax used for redistributive purposes, and show how the possibility of evasion of that tax can lead to an argument for a commodity tax system, perhaps with differential rates. Cremer and Gahvari (1993) conduct a similar exercise for different commodity taxes. A drawback to these papers is that they introduce tax evasion in a rather crude way by defining an ad hoc cost-of-evasion function. Thus, a fundamental feature of tax evasion — its riskiness to the taxpayer — is suppressed. It is precisely the cost of risk-taking that conditions evasive behavior in the Allingham-Sandmo approach and that one would expect to be important from a tax design perspective.

The second approach in the literature is to incorporate the possibility of tax evasion into the standard model of optimal redistribution under asymmetric information due to Mirrlees (1971). The emphasis here is on how the inability of the government to monitor income perfectly (unlike in the Mirrlees case where incomes are perfectly observable, but labor supply and wage rates are not) compromises its ability to redistribute. In this case, optimal policy is modeled using the Revelation Principle, following the standard optimal income tax methodology. Both the tax structure and the penalty structure are chosen so that households are induced to reveal their true incomes. Thus, there is no evasion in the optimum, and therefore no costs of risk-taking. The issue of how to design the tax structure to minimize evasion does not arise. See, for example, Cremer and Gahvari (1996), Marhuenda and Ortuno-Ortin (1997), and Chandar and Wilde (1998).<sup>1</sup>

Our paper is most closely related to the first of these approaches. We focus on the implications of tax evasion for the design of an efficient tax system. Unlike the previous literature, we incorporate explicitly the cost of risk-taking resulting from the decision to evade taxes. As a result, we are able to identify a fundamental trade-off in efficient tax design between mitigating tax distortions and mitigating tax evasion.

It is important to be explicit about the standing of the cost of risk-taking in our analysis. Even though the risk comes about by a household's decision to evade taxes illegally, nonetheless we treat it as a source of welfare loss. It is in fact the government that produces risk opportunities by employing random auditing, a strategy that is necessitated by cost considerations. As in Allingham and Sandmo, individuals respond rationally to these risk opportunities by evading taxes, and in so doing incur a cost of risk-taking. We treat this as an efficiency cost associated with the tax system. It might be argued that fighting evasion activities is an objective the government should pursue as such, and that the costs of risk-taking should not be afforded welfare status on the grounds that they are illegal. This is obviously a matter of judgment. We adopt the position in this paper that the objective of an efficient tax system is to obtain revenue in a way that imposes the

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<sup>1</sup> An exception to this is the recent analysis by Boadway and Sato (2000) where the Revelation Principle may fail because of the fact that detection involves errors, either by the taxpayers or by the tax administrators. However, even with the possibility of errors, the Revelation Principle will hold and no one will intentionally evade if the government has full freedom to include rewards for truthful reporting in the penalty structure.

least welfare cost on households. Just as tax avoidance reduces household utility by changing consumption patterns, so the risk to which tax evasion activity gives rise is a source of utility loss. It represents a private cost that is socially wasteful.<sup>2</sup> The analysis will make it clear how optimal tax design can mitigate the cost of risk-taking induced by random auditing, albeit at the expense of introducing distortions on the taxpayer's behavior.

Modeling tax evasion as a risky activity clearly complicates matters. To keep the analysis tractable, we adopt simplifications that earlier studies of tax design under evasion could avoid. Unlike Cremer and Gahvari (1996), Marhuenda and Ortuno-Ortin (1997), Chandar and Wilde (1998) and Boadway and Sato (2000), we assume auditing is exogenous. In these papers, the government is able to vary the intensity of auditing depending on the income reported. A typical result is that it is not necessary to audit the highest income-earners, for whom the marginal tax rate is zero, but random auditing still applies to those reporting lower incomes. In our model, allowing the government to increase the rate of auditing at a cost would add little to the analysis. The basic results on the structure of the optimal tax would still apply as long as there is some evasion in the optimum. Also unlike the above authors, our analysis ignores the equity objective. Our model is one of a representative household so focuses solely on efficient tax design. Their analysis, which studies the mix of direct and indirect taxation in a heterogeneous-agent setting, can be viewed as a qualification to Atkinson and Stiglitz (1976). Ours studies deviations from proportionality because of tax evasion and can be viewed as a qualification of Atkinson and Stiglitz (1972).

Like most of the existing literature, we work with exogenously given penalty structures. This is a critical assumption since, as we show, our results heavily depend on the penalty structure applied in case of detection. We work with two alternative penalty structures. In one, associated with Allingham and Sandmo (1972), the penalty imposed is proportional to the amount of income evaded. In the other, following Yitzhaki (1974), the penalty is proportional to the amount of tax evaded. In either case, we assume that

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<sup>2</sup> This position has also been adopted by Yitzhaki (1987). He argues that tax evasion generates a social cost which adds to the excess burden of tax distortion, although he does not analyze the optimal trade-off between

there is an upper bound on penalty rates, so that penalties cannot be indefinitely high. It is well known that tax evasion can be avoided without cost by adopting the Becker (1968) solution, which involves imposing a maximal sanction and allowing the rate of audit to approach zero.<sup>3</sup> We find that the choice of the penalty structure has a critical effect on optimal tax policy. When the penalty is based on income unreported, efficiency is enhanced by imposing a distortionary excise tax on a good that is difficult to evade. When the penalty is based on tax evaded, that will not be the case. We show that the former yields higher efficiency when penalty levels are identical.

The paper is organized as follows. Section 2 presents a simple model of tax evasion whose features are chosen so that in the absence of evasion, income taxation is efficient. Section 3 analyzes household behavior with respect to both the choice of commodities and tax evasion. Section 4 presents the problem faced by the benevolent government. Sections 5 and 6 analyze in detail the problem when the labor supply decision is suppressed. In Section 5 an explicit formula is derived which allows one to characterize the optimal trade-off between reducing the social cost of tax evasion and the social cost of tax distortion. Section 6 applies this result to the Allingham-Sandmo and Yitzhaki penalty schemes. Section 7 generalizes the results to the case of endogenous labor supply. Section 8 compares the two competing penalty schemes in their interaction with optimal tax policy. Section 9 summarizes and draws some conclusions. Major proofs are relegated to the Appendix.

## 2. A Simple Model with Tax Evasion

Households in this economy are identical, allowing us to focus on issues of efficiency and to abstract from redistributive considerations. The representative household supplies labor  $L$  and consumes two consumption goods —  $C$ , a composite numeraire good, and  $X$ , a specific good that can be taxed. Producer prices of  $C$  and  $X$  are fixed at unity and  $p$

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these costs. See also Cowell (1990).

<sup>3</sup> Cremer and Gahvari (1996), Marhuenda and Ortuno-Ortin (1997) and Chandar and Wilde (1998) all impose arbitrary restrictions on the size of penalties. Boadway and Sato (2000) avoid the need to impose an upper bound by assuming errors of tax administration or reporting.

respectively, and the wage rate is also normalized to unity by choice of labor units. Good  $X$  can be subject to an excise tax at the per unit rate  $a$ , and labor income is taxed at the proportional rate  $t$ . We assume for convenience that the numeraire  $C$  is untaxed. As is well known, a proportional wage tax is equivalent to a proportional tax on  $C$  and  $X$  in this context. There is no possibility of a lump-sum tax by assumption, since in this simple economy it would dominate all other taxes (unless it too could be evaded). By its nature, the tax on  $X$  cannot be evaded. For example, it might be on a good, like petrol, whose transactions can be readily monitored. However, labor income taxation can be evaded.

Let  $q$  be the proportion of labor income that is either not reported or is earned in the underground economy at the going wage rate.<sup>4</sup> Reported income  $(1-q)L$  is subject to income taxation at the rate  $t$ . Tax evasion is detected with some exogenous probability  $\mathbf{d}$ . If detected, the household bears a pre-determined penalty which may be proportional either to the amount of income evaded  $qL$  (the Allingham-Sandmo penalty scheme) or to the amount of tax evaded  $tqL$  (the Yitzhaki penalty scheme). Given the penalty rate  $f$  and the probability of detection  $\mathbf{d}$  (both assumed exogenous), and using a tilde to indicate a stochastic variable, we can write disposable income as  $\tilde{W}(t, q)L$ , where the stochastic net wage rate  $\tilde{W}$  is given by:

$$\tilde{W}(t, q) = 1 - t(1 - q) + \begin{cases} -qf & \text{prob} = \mathbf{d} \\ 0 & \text{prob} = 1 - \mathbf{d} \end{cases} \quad \text{Allingham-Sandmo penalty} \quad (1_{AS})$$

$$\tilde{W}(t, q) = 1 - t(1 - q) + \begin{cases} -qt f & \text{prob} = \mathbf{d} \\ 0 & \text{prob} = 1 - \mathbf{d} \end{cases} \quad \text{Yitzhaki penalty.} \quad (1_Y)$$

As is standard, both specifications assume two mutually exclusive states. In one state, tax evasion is detected, and in the other not. In case of detection, all the income that has been evaded is revealed and a penalty is imposed. Auditing and penalizing is such that specific taxation is not evaded at all, whereas wage income is evaded and detected

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<sup>4</sup> In a more general analysis, we could allow the wage to differ in the market and underground sectors, perhaps compensating for the risk associated with illegal activity. That would complicate the analysis considerably and obscure the point we are trying to make. By the same token, we could allow there to be some evasion of commodity  $X$ , though less than for labor income. We have adopted extreme assumptions to make the analysis as clear as possible.

evasion is penalized as stated above. Since our initial result (Proposition 1) applies for either penalty scheme, we work with the net wage function  $\tilde{W}(t, q)$  in its general form. The specific forms given by (1<sub>AS</sub>) and (1<sub>Y</sub>) are used later as required.

Given disposable income  $\tilde{W}(t, q)L$ , the household's budget constraint can be expressed as

$$\tilde{C} + P\tilde{X} = \tilde{W}(t, q)L \quad (2)$$

where  $P = p + a$  is the consumer price of  $X$ . Note that since the net wage rate is stochastic, so is the consumption of goods  $C$  and  $X$ .

To simplify our analysis and facilitate comparison with the no-evasion case, we assume that utility is additively separable in  $L$  and homothetic in  $C$  and  $X$ . As is well known, under these conditions, taxation of labor income alone is optimal in the absence of evasion: that is,  $t > 0, a = 0$  (Atkinson and Stiglitz 1972, Sandmo 1974). Moreover, we want to assume that the household maximizes expected utility, so we adopt the following cardinal representation of utility:

$$U(\mathbf{F}(C, X)) - D(L) \quad (3)$$

where  $\mathbf{F}(C, X)$  — an index of real consumption — is linear homogenous, the von Neumann-Morgenstern utility function  $U(\cdot)$  is increasing and strictly concave ( $U' > 0 > U''$ ), and the disutility of labor function  $D(L)$  is increasing and strictly convex,  $D', D'' > 0$ . Note that the homotheticity of utility in  $C$  and  $X$  implies that the optimal ratio of consumption  $C/X$  depends solely on the relative consumer price  $P$  and not on income. We exploit that characterization in what follows.

The government obtains revenue from three sources — the income tax  $t$ , the excise tax  $a$ , and the penalty  $f$ . Since auditing policy is exogenous, we can ignore its cost when modeling the government's problem below. We assume that there is no aggregate risk facing the government so that its tax revenues are not stochastic. This reflects the fact that the risks associated with tax evasion by the households are idiosyncratic, given that auditing is purely random. Any variance in the revenues from taxes and penalties can be



assumed to vanish by the law of large numbers. Thus, there is no need to assign a cost to the government from uncertain revenues (Slemrod and Yitzhaki, 2002, fn. 10). At the same time, since tax evasion is an illegal activity, households cannot insure against the risk of being detected.

Decisions and events take place sequentially in this economy, and it is useful to be explicit about them. As is usual in optimal tax analysis, the government chooses its policies first, anticipating household behavior. The government is assumed to be able to commit to its announced tax and enforcement policies. Households then act in two steps. In the first, they choose labor supply  $L$  and the proportion  $q$  of their income to report. Detection then occurs, so taxes and penalties are paid and actual  $W$  is determined. Given  $W$ , disposable income is known, and in the second step households decide how to allocate it between  $C$  and  $X$ . In our analysis, it is useful to treat these two steps sequentially. In fact, since under homotheticity optimal  $C/X$  depends only on  $P$ , it is not crucial to assume that the state of detection is revealed before the household budget is allocated: the household chooses the same  $C/X$  ratio regardless of the amount of disposable income  $\tilde{W}L$  that is revealed. However, it is convenient for pedagogical purposes to suppose that the household chooses consumption at a subsequent stage.

The sequence of decisions taken by the government and the representative household can then be summarized as follows:

Stage 1 Government policies: For a given penalty scheme, the government chooses  $\{t, a\}$  to maximize its expected revenues subject to a given level of expected utility for households,<sup>5</sup> anticipating how  $\{t, a\}$  affects household behavior.

Stage 2 Household labor supply and evasion: The household takes  $\{t, a\}$  as given and chooses  $\{L, q\}$  to maximize expected utility  $E[U(\cdot)] - D(L)$ , anticipating how disposable income will be allocated to  $C$  and  $X$  in Stage 3. This yields household labor supply and evasion functions  $\{L(t, a), q(t, a)\}$ .

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<sup>5</sup> The solution to this problem is equivalent to its dual of maximizing expected utility of the representative household subject to a government revenue constraint. For expositional purposes, it is more convenient to proceed as in the text.

Stage 3 Household budget allocation: The extent of detection has been revealed so net income  $WL$  is now given. The household chooses  $\{C, X\}$  to maximize real consumption  $\mathbf{F}(C, X)$  subject to its budget  $C + PX = WL$ .

The problem is solved by backward induction. The next section treats the two steps in the household problem. In section 4, we turn to government policy.

### 3. Household Behavior

We begin first with Stage 3 and then go back to Stage 2.

#### Stage 3: Budget Allocation

The detection state and therefore  $W$  have been revealed. Given  $WL$  from Stage 2, the representative household's budget allocation problem, using budget constraint (2), is:

$$\max_X \mathbf{F}(WL - PX, X) . \quad (\text{S3})$$

The first-order condition is:

$$\mathbf{F}_X = P\mathbf{F}_C . \quad (4)$$

It is well known that for a homothetic function like  $\mathbf{F}(C, X)$ ,  $\mathbf{F}_X / \mathbf{F}_C = H(c)$ ; that is, the marginal rate of substitution depends only on the consumption ratio  $c \equiv C / X$  regardless of the level of disposable income. Hence the solution of (4) may be written as:

$$c = H^{-1}(P) \equiv h(P) .$$

Given that  $\mathbf{F}(C, X)$  is linear homogeneous, it is straightforward to show that:<sup>6</sup>

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<sup>6</sup> By Euler's Theorem,  $X\mathbf{F}(c,1) = C\mathbf{F}_C(c,1) + X\mathbf{F}_X(c,1)$ . Condition (4) may be written  $\mathbf{F}_X(c,1) = P\mathbf{F}_C(c,1)$ , or using the previous equation,  $\mathbf{F}(c,1) = (P + c)\mathbf{F}_C(c,1)$ . Differentiating with respect to  $P$  and  $c$ , we obtain (5).

$$\frac{dc}{dP} = h'(P) = - \frac{\Phi_c^2}{\Phi \Phi_{CC}} \Big|_{c=h(P)} > 0. \quad (5)$$

Moreover, linear homogeneity of  $\mathbf{F}$  also implies that:  $F_C(C, X) = F_C(c, 1) = F_C(h(P), 1) \equiv \mathbf{j}(P)$ . Differentiating this by  $P$ , we obtain  $\mathbf{j}'(P) = F_{CC}h' = -F_C^2 / \mathbf{F}$ . Therefore, for later reference,

$$\frac{\mathbf{j}(P)}{\mathbf{j}'(P)} = - \frac{\mathbf{F}}{F_C} = - \frac{F_X}{F_C} - c = - (P + h(P)). \quad (6)$$

Using  $\mathbf{F}_C = \mathbf{j}(P)$ , condition (4) and the household's budget constraint, the household's index of real consumption, given the optimal choice of  $\{C, X\}$  at this stage, can be written:

$$\mathbf{F}(C, X) = C\mathbf{F}_C + X\mathbf{F}_X = \mathbf{F}_C(C + PX) = \mathbf{j}(P)WL.$$

Thus,  $\mathbf{j}(P)WL$  is the maximum value function for the index of real consumption resulting from Stage 3.

## Stage 2: Labor Supply and Evasion

At this stage, the detection state has not yet been revealed, so disposable income  $\tilde{W}L$  is stochastic. Using the anticipated outcome of Stage 3, the problem of the household is:

$$\max_{L, q} \{E[U(\mathbf{j}(P)\tilde{W}(t, q)L)] - D(L)\}. \quad (S2)$$

Assuming an interior solution (see below), the first-order conditions with respect to  $q$  and  $L$  may be written:

$$E[U'(\mathbf{j}\tilde{W}L)\tilde{W}_q] = 0 \quad (7)$$

$$\mathbf{j}E[U'(\mathbf{j}\tilde{W}L)\tilde{W}] = D'(L) \quad (8)$$

where  $\tilde{W}_q$  denotes  $\partial\tilde{W}/\partial q$ . Our assumptions ensure that the second-order conditions are fulfilled. The solution to (7) and (8) yields the labor supply and evasion functions,  $L(t, a)$  and  $q(t, a)$ . The properties of these functions will depend on the form of the penalty scheme, as we shall see in later sections.

Two comparative static properties of the household's problem are worth noting. Suppose that the utility function exhibits decreasing absolute risk aversion (DARA) so  $-U''/U'$  is decreasing in its argument, and increasing relative risk aversion (IRRA) so  $-\mathbf{j}(P)WLU''/U'$  is increasing. Then, the following properties apply:

$$E[U''(\cdot)\tilde{W}_q] \geq 0 \quad (9a)$$

$$E[U''(\cdot)\tilde{W}\tilde{W}_q] \leq 0. \quad (9b)$$

Equation (9a) follows from DARA, while (9b) follows from IRRA.<sup>7</sup>

In the above discussion and in what follows, we assume an interior solution for  $q \in (0, 1)$ . A positive value for  $q$  will be chosen if, and only if, the marginal return from evasion, evaluated at  $q=0$ , is positive:  $E[\tilde{W}_q]|_{q=0} > 0$ . That is, tax evasion is better than a fair bet.

If  $E[\tilde{W}_q]|_{q=0} \leq 0$ , it does not pay to bear the risk of tax evasion, so  $q$  is set optimally at zero.<sup>8</sup> For  $q = 0$ ,  $\tilde{W} = W = 1 - t$ , and the individual faces no risk. In this case, the problem is the standard optimal commodity-tax one with two goods and leisure. As mentioned above, for a utility function of the form (3), the optimal tax on goods is proportional, so  $a=0$ .

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<sup>7</sup> The proof of (9a) relies on the observation that:  $E[U''\tilde{W}_q] = -E[(-U''/U')U'\tilde{W}_q]$ . By (7) this expression equals zero if  $-U''/U'$  is constant. What remains to observe is that  $E[(-U''/U')U'\tilde{W}_q]$  is non-positive if  $-U''/U'$ , being a decreasing function, puts less weight on large values of  $U'\tilde{W}_q$ . Similarly Arrow (1970) shows that (9b) follows from increasing relative risk aversion (IRRA).

<sup>8</sup> This assumes that over-reporting is not rewarded, which is the case in practice.

Note that for specifications (1<sub>AS</sub>) and (1<sub>Y</sub>) the marginal return from evasion is constant in  $q$ . Furthermore,  $E[\tilde{W}_q] > 0$  holds if and only if:

$$t > \mathbf{j}^* \text{ for Allingham-Sandmo penalty schemes,} \quad (10_{AS})$$

$$1 > \mathbf{j}^* \text{ for Yitzhaki penalty schemes, assuming } t > 0. \quad (10_Y)$$

#### 4. The Government Problem

In Stage 1, the government foresees household behavior as characterized by problem (S2) and the associated first-order conditions (7) and (8), where  $\mathbf{j}(P)$  is known from Stage 3. Government revenues depend on household choices  $\{L(t, a), q(t, a)\}$  in Stage 2, as well as commodity purchases  $\tilde{X}(t, a)$  in Stage 3. Using the household budget constraint, consumption of the taxable good  $\tilde{X}$  can be expressed in terms of disposable income:<sup>9</sup>

$$\tilde{X} = \frac{\tilde{W}(t, q)L}{P + h(P)} \text{ with } P = p + a, L = L(t, a) \text{ and } q = q(t, a).$$

The government maximizes its expected revenues per household subject to some given level of household expected utility and the optimal household choices  $\{L(t, a), q(t, a)\}$ . The revenues come from the three sources: the income tax  $t$ , the excise tax  $a$ , and the penalty  $f$ . The sum of expected income tax and penalty revenue is necessarily equal to the gross wage income minus the expected net income accruing to the household,  $E[1 - \tilde{W}(t, q)]L$ . Expected revenue from the excise tax is  $aE[\tilde{X}] = aE[\tilde{W}(t, q)]L/(P + h(P))$ . Hence the government's problem can be written as:

$$\max_{t, a} \left[ 1 - \frac{p + h(p + a)}{p + a + h(p + a)} E[\tilde{W}(t, q)] \right] L \quad (S1)$$

subject to

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<sup>9</sup> To see this, note that  $\tilde{W}L = \tilde{C} + P\tilde{X} = (c + P)\tilde{X} = (h(P) + P)\tilde{X}$ .

$$V(t, a) \equiv E[U(\mathbf{j}(p+a)\tilde{W}(t, q)L)] - D(L) \geq v \quad (11)$$

where  $v$  is the given level of expected utility, and  $q$  and  $L$  are functions of  $t$  and  $a$ .

The solution to this problem yields the optimal — that is, efficient — tax policy  $\{t, a\}$ . We start by characterizing the optimal tax policy when labor supply is fixed but the net wage function  $\tilde{W}(t, q)$  is arbitrary. We derive an explicit formula that can be used to determine optimal policy for the Allingham/Sandmo and Yitzhaki penalty schemes. Subsequently, we consider the more general case of endogenous labor supply.

## 5. Optimal Taxation When Labor Supply is Constant

Although the solution to the government's problem is in general complicated, it is possible to obtain clear-cut, intuitive results by assuming that the labor supply is constant.<sup>10</sup> In this case, the characterization of optimal tax policy is facilitated by distinguishing two key welfare effects of imposing an excise tax — its social benefit and its social cost. The social benefit consists of its effect on the cost of risk-taking borne by the household when evading the wage tax. The social cost reflects the standard tax distortion imposed as a result of deviating from the uniform wage tax, which is non-distortionary when labor supply is fixed. To the extent that an excise tax ameliorates the cost of risk-taking, a tax distortion will be tolerated. We consider these two effects in turn.

### *i) Benefit of the excise tax: reduced cost of risk*

Tax evasion gives rise to private risk in the form of variability in the net wage  $\tilde{W}$ . Its cost is the maximum premium the household would be willing to pay to eliminate the risk. For given  $L$ , this risk premium  $\mathbf{P} = \mathbf{P}(t, a)$  is implicitly defined by setting

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<sup>10</sup> Even if households are free to choose  $L$ , they will choose a constant value if the utility of real consumption is logarithmic,  $U(\mathbf{F}) = \log \mathbf{F}$ . In this case, the first-order condition (8) can be written as  $D'(L) = 1/L$ , whose solution is independent of prices and policy parameters.

$$U(\mathbf{j}(P)LE[\tilde{W}(t,q)] - P) = E[U(\mathbf{j}(P)\tilde{W}(t,q)L)].$$

For a utility-compensated increase in  $a$ , the right-hand side remains constant. By implicit differentiation of the left-hand side, we obtain

$$\left. \frac{dP}{da} \right|_{V=\text{const}} = \frac{d}{da} (\mathbf{j} LE[\tilde{W}]) \Big|_{V=\text{const}} = \mathbf{j}' LE[\tilde{W}] + \mathbf{j} L \frac{d}{da} (E[\tilde{W}(t,q)]) \Big|_{V=\text{const}}. \quad (12)$$

In general the sign of the right-hand side of (12) is ambiguous: the first term is negative, but the second depends on the form of the penalty structure. If (12) is negative, there will be a marginal benefit in the form of a reduced cost of private risk from a compensated increase in the excise tax rate  $a$ .

### *ii) Cost of the excise tax: increase in tax distortion*

The excise tax distorts consumption choice in Stage 3 of the household's problem. Recall that the maximum value function from Stage 2 was  $\mathbf{j} WL$ , where at this stage disposable income was already determined. Let  $C(P,u)$  and  $X(P,u)$  be compensated demand functions obtained from the dual to the Stage 3 problem. They are obtained as solutions of

$$\mathbf{F}(C, X) = u \quad \text{and} \quad \frac{\mathbf{F}_X(C, X)}{\mathbf{F}_C(C, X)} = P.$$

Where  $u$  is some given value of real consumption. Since  $\mathbf{F}$  is linear homogenous, the compensated demand for  $X$  satisfies  $X\mathbf{F}_C(c,1) = \mathbf{F}(C, X) = u$ , or

$$X(P, u) = \frac{u}{\mathbf{F}(h(P), 1)} \quad (13)$$

where  $h(P)=c$  has been defined earlier. The substitution effect is clearly negative,

$$\frac{\partial X(P, u)}{\partial P} = -\frac{uh'\mathbf{F}_C}{\mathbf{F}^2} = -\frac{Xh'\mathbf{F}_C}{\mathbf{F}} < 0. \quad (14)$$

Equation (14) is derived from the Stage 3 problem when detection, and therefore disposable income, have been determined. From the point of view of the effect of

government policy, it is the ex ante effect that is relevant since policies are undertaken before the outcome of detection is revealed. This means that the government should internalize the effect that a utility-compensated increase of  $a$  has on expected sub-utility  $\bar{u} = \mathbf{j} LE[\tilde{W}]$ . The marginal efficiency effect of the excise tax is the change in the size of the distortion, or

$$\begin{aligned}
 a \frac{d}{da} X(p + a \mathbf{j} LE[\tilde{W}(t, q)]) \Big|_{V=\text{const}} \\
 &= a \frac{\partial X(P, \bar{u})}{\partial P} + a \frac{\partial X(P, \bar{u})}{\partial \bar{u}} \frac{d}{da} (\mathbf{j} LE[\tilde{W}(t, q)]) \Big|_{V=\text{const}} \\
 &= a \frac{\partial X(P, \bar{u})}{\partial P} + \frac{a}{\Phi} \frac{d\Pi}{da} \Big|_{V=\text{const}} .
 \end{aligned} \tag{15}$$

The latter equality makes use of (12) and (13). Since the compensated price effect in the first term on the right-hand side is negative, (15) will be negative for  $a > 0$  if (12) is negative. In other words, if a compensated increase in  $a$  reduces the cost of risk-taking, it will also increase the efficiency cost of the excise tax.

Optimality in the choice of the excise tax  $a$  involves trading off the benefit of a reduced cost of risk-taking against the cost of an increase in the tax distortion on consumption. In fact, as is shown in the Appendix, at an optimum, the trade-off can be made explicit as follows:

**Proposition 1:** A necessary condition for the optimal choice of  $\{t, a\}$  satisfies

$$a \frac{dX}{da} \Big|_{V=\text{const}} = \frac{1}{\mathbf{j}} \frac{d\Pi}{da} \Big|_{V=\text{const}} . \tag{16}$$

The division by  $\mathbf{j}$  on the right-hand side is needed to transform units of sub-utility in units of income. Given our above demonstration that the left-hand side of (16) will be



negative if a compensated increase in the excise tax reduces the cost of risk-taking, we immediately obtain the following corollary:

**Corollary 1:** It is optimal to set  $a > 0$  if  $\left. \frac{dP}{da} \right|_{V=\text{const}} < 0$ .

It ought to be stressed that Proposition 1 and its corollary apply for any strictly concave utility function  $U$  and for any net wage function  $\tilde{W}(t, q)$ . However, the assumption of exogenous labor supply is critical. If  $L$  is variable, income effects complicate the analysis, since labor supply reacts to variations in its return,  $\mathbf{j} \tilde{W}$ .

As Corollary 1 indicates, the optimality of  $a > 0$  follows from (16) only if a utility-compensated increase in  $a$  reduces the cost of private risk borne by the tax-evading households. In the latter case, negativity of  $dX / da|_V$  follows from (15) so that  $a$  must be positive. Hence we have to study the sign of the right-hand side of (16) for particular cases in more detail.

## 6. Allingham-Sandmo and Yitzhaki Penalty Schemes

In this section, we apply the results of the previous section to Allingham-Sandmo and Yitzhaki penalty schemes. In doing so we retain the assumption of constant labor supply.

### *The Allingham-Sandmo penalty scheme*

Assuming the Allingham-Sandmo penalty scheme as specified by (1<sub>AS</sub>) we obtain  $\tilde{W}_t \equiv \partial \tilde{W} / \partial t = -(1-q)$  and  $\tilde{W} = (1-t) + q\tilde{W}_q$ . Furthermore,  $\tilde{W}_q$  is constant in  $q$  and  $\tilde{W}_{qt} = 1$ . Using these identities as well as (7), implicit differentiation of (11) gives

$$\left. \frac{dt}{da} \right|_{V=const} = - \frac{\partial V / \partial a}{\partial V / \partial t} = - \frac{\mathbf{j}' E[U'(\cdot)\tilde{W}]}{\mathbf{j} E[U'(\cdot)\tilde{W}_t]} = \frac{\mathbf{j}'(1-t)}{\mathbf{j}(1-q)} < 0. \quad (17_{AS})$$

Then, with the help of some straightforward manipulations we obtain

$$\left. \frac{dq}{da} \right|_{V=const} = \frac{\partial q}{\partial a} + \frac{\partial q}{\partial t} \left. \frac{dt}{da} \right|_{V=const} = - \frac{\mathbf{j}'}{\mathbf{j}} q - \frac{\mathbf{j}'(1-t)}{\mathbf{j}^2 L(1-q)} \frac{E[U']}{E[U''(\cdot)\tilde{W}_q^2]}. \quad (18_{AS})$$

Note that the two terms on the right-hand side of (18<sub>AS</sub>) are of opposing signs. The first one is positive, since  $\mathbf{j}' < 0$ , whereas the second one is negative due to the negativity of the denominator  $E[U''(\cdot)\tilde{W}_q^2]$ . The implication is that utility-compensated changes in policies  $\{t, a\}$  have ambiguous effects on tax evasion. It is not clear whether the excise tax helps to fight tax evasion as measured by  $q$ .

However the effects of tax changes on the social cost of tax evasion as measured by  $\mathbf{P}$  are unambiguous. This is shown by making use of (12), (1<sub>AS</sub>), (17<sub>AS</sub>) and (18<sub>AS</sub>):

$$\begin{aligned} \left. \frac{d\mathbf{P}}{da} \right|_{V=const} &= \frac{d}{da} (\mathbf{j} L E[\tilde{W}(t, q)]) \Big|_{V=const} = \frac{d}{da} (\mathbf{j} L [1-t + q(t - \mathbf{d}f)]) \Big|_{V=const} \\ &= \mathbf{j}' L [1-t + q(t - \mathbf{d}f)] - \mathbf{j} L (1-q) \left. \frac{dt}{da} \right|_{V=const} + \mathbf{j} L (t - \mathbf{d}f) \left. \frac{dq}{da} \right|_{V=const} \\ &= - (t - \mathbf{d}f) \frac{\mathbf{j}'(1-t)}{\mathbf{j}(1-q)} \frac{E[U']}{E[U''(\cdot)\tilde{W}_q^2]} < 0. \end{aligned} \quad (19_{AS})$$

Therefore, invoking Corollary 1, we obtain:

**Proposition 2** For fixed labor supply and the Allingham-Sandmo penalty scheme, a utility-compensated increase in the excise tax rate reduces the cost of private risk borne by the household. Hence it is optimal to set  $a > 0$ .

As mentioned, it is not at all clear whether the excise tax helps to reduce  $q$ . Therefore, the rationale for the proposition is not simply to fight tax evasion. Instead, the rationale for setting  $a > 0$  lies in trading off of the social costs associated with tax evasion on the one hand, and tax distortions on the other.

A utility-compensated increase in  $a$  reduces the risk premium  $P$  the taxpayer is willing to pay in order to shed the risk associated with subutility  $\mathbf{j}L\tilde{W}$ . If effects of higher order can be ignored, a reduction in  $P$  must result from reduced variance. This is in fact the case. The variance is  $\text{var}[\mathbf{j}L\tilde{W}] = (\mathbf{j}qL)^2 \text{var}[\tilde{W}_q] = (\mathbf{j}qL)^2 \mathbf{d}(1-\mathbf{d})f^2$ , which can be shown to decrease by making use of (18<sub>AS</sub>):

$$\left. \frac{d(\mathbf{j}q)}{da} \right|_{V=\text{const}} = \mathbf{j}'q + \left. \frac{dq}{da} \right|_{V=\text{const}} = -\frac{\mathbf{j}'(1-t)}{\mathbf{j}L(1-q)} \frac{E[U']}{E[U''(\cdot)\tilde{W}_q^2]} < 0. \quad (20_{\text{AS}})$$

Finally, as noted earlier, some might argue that the risk associated with tax evasion should not be treated as a welfare cost, given the illegality of tax evasion. Ignoring the cost of risk is equivalent to setting the right-hand side of (16) to zero. In this case with fixed labor supply, it follows immediately that  $a = 0$  regardless of how risk averse the household is, and therefore how responsive is the household's evasion to the income tax rate. In the more general case with variable labor supply, it turns out that  $a$  will deviate from zero in the optimum even when the government gives no welfare weight to the risk of evasion.

### ***The Yitzhaki penalty scheme***

Assuming the Yitzhaki penalty scheme as specified by (1<sub>Y</sub>), we obtain  $1 + t\tilde{W}_t = \tilde{W} = (1 - t) + q\tilde{W}_q$ . Furthermore,  $\tilde{W}_q$  is constant in  $q$  and  $t\tilde{W}_{qt} = \tilde{W}_q$ . Using these identities as well as (7), implicit differentiation of (11) now gives

$$\left. \frac{dt}{da} \right|_{V=const} = - \frac{\partial V / \partial a}{\partial V / \partial t} = - \frac{\mathbf{j}' E[U'(\cdot)\tilde{W}]}{\mathbf{j} E[U'(\cdot)\tilde{W}_t]} = (1 - t) \frac{\mathbf{j}'}{\mathbf{j}} < 0. \quad (17_Y)$$

As before, with the help of some straightforward manipulations, we obtain

$$\left. \frac{dq}{da} \right|_{V=const} = \frac{\partial q}{\partial a} + \frac{\partial q}{\partial t} \left. \frac{dt}{da} \right|_{V=const} = - \frac{\mathbf{j}'}{\mathbf{j}} \frac{q}{t} > 0. \quad (18_Y)$$

Note that, contrary to the Allingham-Sandmo regime, a utility-compensated increase in the excise tax  $a$  *unambiguously increases* tax evasion for *arbitrary risk preferences*. That is, the level of evasion  $q$  increases for all  $U$  in response to a reform that substitutes a tax that cannot be evaded for one that can!

The explanation for this seemingly counter-intuitive result can be explained as follows. Under the Yitzhaki penalty scheme, as is well-documented in the literature (Slemrod and Yitzhaki, 2002, p. 1429; Cowell, 1990, Ch. 4), an isolated decrease in  $t$  tends to increase evasion. In fact,  $\partial q / \partial t < 0$  is obtained if absolute risk aversion is decreasing (DARA). If the decrease in  $t$  is complemented by an increase in  $a$ , the increasing effect on  $q$  is reinforced. Because of  $\mathbf{j}' < 0$ , an increase in the specific tax  $a$  works like insurance against the risk of uncertain wage income which tends to encourage risk-taking (Domar and Musgrave, 1944). In fact,  $\partial q / \partial a > 0$  is obtained if absolute risk aversion is constant. Equation (18<sub>Y</sub>) states that the overall increasing effect on  $q$  turns out to be unambiguous and independent of specific risk preferences if an increase in  $a$  and a decrease in  $t$  are combined in such a way that expected utility remains constant. But given that the tax reform increases evasion, we should not be surprised to learn that it has no beneficial effect on the social cost of tax evasion as measured by  $\mathbf{P}$ . In fact, we can show that the

marginal effect actually vanishes. This follows from (12) by making use of (1<sub>Y</sub>), (17<sub>Y</sub>) and (18<sub>Y</sub>):

$$\begin{aligned}
\left. \frac{dP}{da} \right|_{V=const} &= \left. \frac{d}{da} (\mathbf{j} LE[\tilde{W}(t, q)]) \right|_{V=const} = \left. \frac{d}{da} (\mathbf{j} L[1-t + qt(1-\mathbf{d}f)]) \right|_{V=const} \\
&= \mathbf{j}' L[1-t + qt(1-\mathbf{d}f)] - \mathbf{j} L[(1-q)(1-\mathbf{d}f)] \left. \frac{dt}{da} \right|_{V=const} \\
&\quad + \mathbf{j} L t (1-\mathbf{d}f) \left. \frac{dq}{da} \right|_{V=const} = 0.
\end{aligned} \tag{19_Y}$$

By Proposition 1, we immediately obtain:

**Proposition 3:** For fixed labor supply and the Yitzhaki penalty scheme, a utility-compensated increase in the excise tax rate leaves the cost of private risk borne by the household unchanged. Hence it is optimal to set  $a = 0$ .

Since a utility-compensated increase of the excise tax leaves the social cost of tax evasion unchanged, we may expect the variance of subutility,  $\text{var} [\mathbf{j} L \tilde{W}(t, q)] = (\mathbf{j} q L)^2 \text{var} [\tilde{W}_q] = (\mathbf{j} q L)^2 \mathbf{d} (1-\mathbf{d}) t^2 f^2 = (\mathbf{j} q t)^2 \mathbf{d} (1-\mathbf{d}) L^2 f^2$ , to be unchanged. This is straightforward to confirm:

$$\left. \frac{d}{da} (\mathbf{j} q t) \right|_{V=const} = \mathbf{j}' q t + \mathbf{j} q \left. \frac{dt}{da} \right|_{V=const} + \mathbf{j} t \left. \frac{dq}{da} \right|_{V=const} = 0.$$

The last equality follows after substituting (17<sub>Y</sub>) and (18<sub>Y</sub>). The variance of subutility remains constant because three marginal effects of increasing  $a$  are exactly offsetting. The first two, which are reflected by the first two terms in the equation, can be interpreted as insurance effects of the reform. The first one is a direct effect of increasing  $a$  while the second one works indirectly via reducing  $t$ . Both effects are negative and reduce the variance of subutility. It is obviously a specific feature of the

Yitzhaki penalty scheme that the joint insurance effects are exactly canceled by the positive behavioral reaction as reflected in the third term of the equation.

## 7. Endogenous Labor Supply

When labor supply is endogenous, it seems to be no longer possible to characterize the optimal tax structure in the form of a trade-off between social costs of tax evasion and the social cost of tax distortion. We were not able to derive formula (16) when labor supply is endogenous. Hence it is not clear how to characterize the optimal trade off between the social costs of tax evasion and distortion. However those parts of Propositions 2 and 3 that relate to the optimal choice of the excise tax do generalize in a straightforward way to the case of endogenous labor supply. Proofs are relegated to the Appendix.

**Proposition 4:** For endogenous labor supply and the Allingham-Sandmo penalty scheme, it is efficient to set  $a > 0$  if preferences satisfy DARA and IRRA.

**Proposition 5:** For endogenous labor supply and the Yitzhaki penalty scheme, it is efficient to set  $a = 0$ .

Propositions 4 and 5 demonstrate in remarkable generality that the choice of penalty schemes may interact with optimal tax policy. Separability of tax and penalty policies applies only if the Yitzhaki scheme is adopted. Separability does not hold, however, in the case of the Allingham-Sandmo scheme. The Allingham-Sandmo penalty scheme makes it necessary to reconsider the efficient choice of tax instruments. The wage tax is more broadly based, but it suffers from tax evasion by assumption. The specific excise tax, on the other hand, cannot be evaded but it imposes an efficiency cost relative to the optimal uniform tax. Increasing  $a$  incrementally above zero is welfare-improving because it imposes no first-order efficiency cost but helps to reduce the cost of private risk

borne by the tax evader. Eventually, the cost of the additional distortion imposed by the specific tax has to be traded off against the benefit of reduced risk.

## 8. Allingham-Sandmo versus Yitzhaki

The analysis of the preceding two sections shows that the nature of the penalty structure has implications for the structure of optimal taxes, in particular the usefulness of differential tax rates as a device for combating evasion. The fact that the Allingham-Sandmo scheme generally calls for differential taxation while the Yitzhaki scheme calls for no deviation from proportionality naturally leads one to ask whether one scheme can be preferred over the other on efficiency grounds. In general, that depends on how the comparison is made, that is, on what one takes to be comparable penalty levels. A natural comparison to make is between schemes that have the same power of deterrence. We shall argue in this section that for given powers of deterrence — that is, penalties in the event of detection — efficiency will be higher under the Allingham-Sandmo scheme than under the Yitzhaki scheme.

Let  $F$  be the size of the penalty in the event of detection, and denote the relevant values of penalties in the two schemes by  $f_{AS}$  and  $f_Y$ . The schemes to be compared will have the same size of penalties. Thus, by  $(1_{AS})$  and  $(1_Y)$ , these will satisfy  $F = f_{AS} = tf_Y$ . Obviously the choice of  $f_Y$  depends on a benchmark tax rate  $t$ , and it turns out to be useful to select as a value of  $t$  that which would be chosen under the Yitzhaki penalty scheme where  $t > 0$ ,  $a = 0$ . The net wage under either scheme will be given by:

$$\tilde{W}(t, q, F) = 1 - t(1 - q) + \begin{cases} -qF & \text{prob} = d \\ 0 & \text{prob} = 1 - d \end{cases}$$

Suppose first that only a wage tax is in place ( $a = 0$ ). Moreover, suppose that  $t > dF$  to ensure an interior solution in  $q$ . The solution to the first-order conditions (7) and (8) of the household's maximization problem can be expressed as  $q(t, F)$  and  $L(t, F)$ , which lead to an expected wage  $\tilde{W}(t, q(t, F), F)$ , and therefore an expected utility  $E[U(\mathbf{j}(p)\tilde{W}(t, q(t, F), F)L(t, F))] - D(L(t, F))$ . Moreover, expected government

revenue  $E[(1 - \tilde{W}(t, q(t, F), F)L(t, F))]L(t, F)$  is determined. Therefore, any values of  $f_{AS}$  and  $f_Y$  that yield the same penalty value  $F$  will result in the same allocation when  $a = 0$ , so:

**Lemma 1:** Given  $t > 0$ ,  $a = 0$ , an Allingham-Sandmo penalty scheme and a Yitzhaki penalty scheme that have the same penalty value ( $f_{AS} = tf_Y$ ) will yield the same outcome.

Now, Propositions 4 and 5 (or 2 and 3 in the fixed labor supply case) imply that beginning at this common equilibrium, welfare can be improved by increasing  $a$  under the Allingham-Sandmo scheme, but not under the Yitzhaki scheme. Moreover, the increase in  $a$  in the former case will leave the equal-penalty condition  $f_{AS} = tf_Y$  unchanged. Thus we have:

**Proposition 6:** For a given penalty value  $F$ , optimal taxation under the Allingham-Sandmo scheme is welfare-superior to optimal taxation under the Yitzhaki scheme.

The above comparison does suffer from an asymmetry. The choice of tax rates is endogenous whereas the choice of the penalty  $F$  is exogenous. However, endogenizing the latter has similar implications under either scheme. If the government were free to choose the penalty value  $F$ , it would be efficient to choose it such that  $F = t/\mathbf{d}$ . Under either penalty scheme, there would be no evasion so there would be no need to impose a distorting excise tax. The two schemes would be observationally equivalent.

## 9. Conclusions

Evading taxes is a risky activity that arises because of government tax enforcement policies: tax evaders run the risk of being detected and punished. The risk is a private



one, but it gives rise to a social cost. The specific feature of the Yitzhaki penalty scheme is that the cost of private risk borne by the tax evader can be ignored in optimal tax design. This is not the case if the Allingham-Sandmo penalty scheme is adopted. In the latter case, tax policy has to consider the extent to which different taxes can be evaded. This enlarges the scope for efficiency enhancing tax design. However, there is a price to be paid. Taxes that cannot be evaded are likely to be narrow ones that impose significant distortions. Given the Allingham-Sandmo penalty scheme, the government is therefore faced with trading off the costs of risk borne by taxpayers who rationally decide to engage in the tax evasion lottery against the costs of distortion arising from choosing the tax mix so as to reduce the opportunities for evasion. The analysis of this paper shows how these two costs must be balanced in the optimum. The optimal trade-off is illustrated using a specific model in which a broad-based income tax is efficient but prone to evasion, while a narrow-based distortionary excise tax that cannot be evaded is available. In this context, the existence of risk aversion alone is sufficient to warrant introducing the excise tax. Moreover, the exact scope of tax evasion is irrelevant: the result holds however small the proportion of income that is evaded. The intuition is that introducing the excise tax initially produces a second-order welfare cost, while at the same time inducing a first-order reduction in the private cost of tax evasion.

In formulating our argument, our presumption has been that narrow tax bases may be more difficult to evade than broad ones, and we have interpreted our findings as casting doubt on the standard arguments for uniform commodity or income taxation. It is not sufficient just to assume appropriate separability and homotheticity conditions for utility functions. When designing optimal taxation one cannot ignore tax evasion even if these conditions on utility apply unless the penalty for detected evasion is of the Yitzhaki-type. It is apparent that alternative tax bases will differ not only in their distortionary effect but also in the ease with which they may be evaded. If the Allingham-Sandmo penalty scheme applies, fully efficient taxation must take account of the incremental effects of each type of tax on both the cost of distortions and the costs of risk-taking. When all taxes can be evaded to some extent, the analytical task is challenging. It would become even more so if households were heterogeneous.

Our task was made much simpler, and our results much sharper, by the various simplifying assumptions we have made. A crucial one might seem to be that utility is additive separable in labor and homothetic in consumption. This not only ensured that the benchmark case with no evasion was uniform taxation. It also simplified the analysis. In fact, the extension to the case where utility is only weakly separable in labor would presumably be straightforward. Propositions 1-3 will clearly continue to apply, since labor is assumed fixed. One may conjecture that Proposition 5 will also apply, since weak separability still ensures that the benchmark case entails uniform taxation. The proof would certainly be more complicated. However, Propositions 4 and 5 will clearly not continue to hold in their stated form when weak separability and homotheticity are violated. In this case,  $a$  will generally be non-zero as part of the Ramsey optimal tax system. The results of the present paper can only be expected to hold relative to those that can be derived when tax evasion is excluded. This may well imply that an excise tax is (not) efficient because of Ramsey-type considerations even in circumstances where the present analysis would rule this out.

## 10. Appendix

To prove Proposition 1, we evaluate the government target function (S1) for a utility-compensated change in the specific tax rate  $a$ . The first-order optimality condition is:

$$\begin{aligned} 0 &= \frac{d}{da} \left( -\frac{p+h}{p+a+h} E[\tilde{W}(t, q(t, a))] \right) \Big|_{V=const} \\ &= \frac{p+h-ah'}{(p+h)^2} E[\tilde{W}] - \frac{p+h}{p+h} \left[ \frac{1}{\mathbf{j}L} \frac{d\mathbf{P}}{da} \Big|_V - \frac{\mathbf{j}'}{\mathbf{j}} E[\tilde{W}] \right]. \end{aligned}$$

where the last equality makes use of (12). Rearranging and using (6) we obtain

$$-\frac{ah'}{p+h} \mathbf{j}^2 LE[\tilde{W}] = (p+h) \mathbf{j} \frac{d\mathbf{P}}{da} \Big|_V = (\mathbf{F} - a\mathbf{j}) \frac{d\mathbf{P}}{da} \Big|_V.$$

Rearranging again we obtain

$$\frac{1}{\mathbf{j}} \frac{d\mathbf{P}}{da} \Big|_V = \frac{a}{\mathbf{F}} \frac{d\mathbf{P}}{da} \Big|_V - \mathbf{j} LE[\tilde{W}] \frac{ah'}{(P+h)\mathbf{F}} .$$

By (6),  $\frac{h'}{(P+h)\mathbf{F}} = \frac{\mathbf{F}_C}{\mathbf{F}^2} h' = -\frac{d}{dP} \frac{1}{\mathbf{F}(h(P))}$ . Hence by (13) and (15),

$$\frac{1}{\mathbf{j}} \frac{d\mathbf{P}}{da} \Big|_V = a \left\{ \frac{1}{\mathbf{F}} \frac{d\mathbf{P}}{da} \Big|_V + \frac{\partial}{\partial P} X(P, \mathbf{j} LE[\tilde{W}]) \right\} = a \frac{d}{da} X(P, \mathbf{j} LE[\tilde{W}]) \Big|_{V=const}$$

which proves Proposition 1.

Proposition 4 is proved by maximizing the government's problem (S1) in  $t, a, q, L$  subject to the constraints (7), (8), and (11). The associated Lagrange multipliers are  $-\mathbf{l}$ ,  $\mathbf{u}$ , and  $-\mathbf{m}$ , respectively. The first-order conditions with respect to  $t$  and  $a$  give:

$$\frac{p+h}{p+a+h} \frac{1}{\mathbf{j}} = \mathbf{l} \left[ \frac{E[U']}{(1-q)L\mathbf{j}} - E[U'' \cdot \tilde{W}_q] \right] + \mathbf{u} \left[ \mathbf{j} (1-t)E[U''] + \frac{E[U']}{(1-q)L} \right] - \mathbf{m}E[U'] ,$$

and

$$\begin{aligned} \frac{p+h-ah'}{(p+a+h)^2} \frac{E\tilde{W}}{(1-t)\mathbf{j}}, &= \mathbf{l} \left[ E[U'' \cdot \tilde{W}_q] + \frac{q}{1-t} E[U'' \cdot \tilde{W}_q^2] \right] \\ &\quad - \mathbf{u} \left[ \mathbf{j} E[U'' \cdot \tilde{W}] + \frac{E[U']}{L} \right] + \mathbf{m}E[U'] . \end{aligned}$$

Eliminating  $\mathbf{m}E[U']$  and using (6), we obtain

$$\begin{aligned} \frac{ah'}{p+a+h} - \frac{q}{1-t} E[\tilde{W}_q] \frac{p+h-ah'}{p+a+h} & \tag{21} \\ &= \mathbf{l} \left[ \frac{q\mathbf{j}}{1-t} E[U'' \cdot \tilde{W}_q^2] + \frac{E[U']}{(1-q)L} \right] + \mathbf{u} \mathbf{j} q \left[ \frac{E[U']}{(1-q)L} - \mathbf{j} E[U'' \cdot \tilde{W}_q] \right] . \end{aligned}$$

The first-order condition with respect to  $q$  yields

$$\frac{p+h}{p+a+h} E[\tilde{W}_q] = -\mathbf{l}\mathbf{j} E[U'' \cdot \tilde{W}_q^2] + \mathbf{u}\mathbf{j}^2 (1-t) E[U'' \cdot \tilde{W}_q] . \quad (22)$$

By making use of (22) we are able to write (21) in the form of

$$\frac{ah'}{p+a+h} \left[ 1 + \frac{q}{1-t} E[\tilde{W}_q] \right] = \frac{\mathbf{l} + \mathbf{u}\mathbf{j}q}{(1-q)L} E[U'] . \quad (23)$$

Solving (22) for  $\mathbf{l}$  and inserting into (23) yields

$$\begin{aligned} \frac{ah'}{p+a+h} \frac{E[\tilde{W}]}{1-t} = & -\frac{p+h}{p+a+h} \frac{E[\tilde{W}_q]}{\mathbf{j}(1-q)L} \frac{E[U']}{E[U'' \cdot \tilde{W}_q^2]} \\ & + \mathbf{u}\mathbf{j} \frac{E[U']}{(1-q)L} \frac{E[U'' \cdot \tilde{W}_q \tilde{W}]}{E[U'' \cdot \tilde{W}_q^2]} . \end{aligned} \quad (24)$$

Based on (24), we obtain a proof of Proposition 2 which is independent of the one given in the main text. Proposition 2 is for exogenous labor supply which is captured by (24) after setting  $\mathbf{u} \equiv 0$ . Proposition 2 follows immediately by noting

$$E[\tilde{W}], E[\tilde{W}_q] > 0, \quad t, q < 1, \quad \mathbf{j} > 0, \quad h' > 0, \quad p+a+h > 0, \quad E[U'] > 0, \quad E[U'' \cdot \tilde{W}_q^2] < 0.$$

In the case of endogenous labor supply, we must determine the sign of  $\mathbf{u}$  in (24). For this purpose, take the partial derivative of the Lagrangean target function with respect to  $L$ . Then substitute for  $\lambda$  by making use of (22). The resulting equation is

$$\begin{aligned} \left[ 1 - \frac{p+h}{p+a+h} E\tilde{W} \right] + \frac{p+h}{p+a+h} E\tilde{W}_q \frac{E[U'' \cdot \tilde{W}_q \tilde{W}]}{E[U'' \cdot \tilde{W}_q^2]} \\ = \mathbf{u}D'' + \mathbf{u}\mathbf{j}^2 \frac{(E[U'' \cdot \tilde{W} \tilde{W}_q])^2 - E[U'' \cdot \tilde{W}_q^2] \cdot E[U'' \cdot \tilde{W}^2]}{E[U'' \cdot \tilde{W}_q^2]} \end{aligned} \quad (25)$$

The first bracketed term on the left-hand side if multiplied by  $L$  is the government's expected net revenue. It is non-negative by assumption. In order to sign the second term on the left-hand side we make use of (9b). It follows that both these terms are non-

negative. Finally, the factors involving  $\mathbf{u}$  on the right-hand side in (25) are positive. The second factor is positive since the numerator and the denominator are (weakly) negative. Weak negativity of the numerator follows from the fact that the squared covariance of two stochastic variables,  $\tilde{W}$ ,  $\tilde{W}_q$ , is never greater than the product of their variances.<sup>11</sup> We conclude that  $\mathbf{u}$  is non-negative just as the factor of  $\mathbf{u}$  in (24). This gives us Proposition 4.

Proposition 5 is proved along the same lines. The details are not necessary since the derivations basically follow the ones given for the Allingham-Sandmo case. The essential difference is that in the Yitzhaki case no partial derivative has to be taken with respect to  $L$  in order to derive the result that  $a=0$  is optimal. After substituting (22) into (21), one obtains  $\frac{ah'}{p+a+h}tE[\tilde{W}] = 0$ , from which the assertion readily follows.

## References

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<sup>11</sup> Interpret  $U$  as a weighting scheme.

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