Social Insurance and Redistribution with Moral Hazard and Adverse Selection[*]

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Abstract

This paper studies the joint role of social insurance and redistributive taxation when the government has a utilitarian objective. We allow for private insurance to compete with social insurance with moral hazard and adverse selection. We show that government intervention in insurance markets can be welfare-improving. This depends on whether or not redistributive taxation generates a labour distortion and on the correlation between labour productivity and loss probability.

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1 Introduction

Traditionally, there are three types of reasons for public intervention in the field of insurance: transactions costs, market failures and redistribution.[1] In the health care sector and in the pensions area, private insurance exhibits higher transaction costs than social insurance. This is partly related to administrative costs.[2] Market failures, the second reason, arise primarily from asymmetric information, such as that between insurers and insurees (adverse selection and moral hazard) and that between providers and insurers in the health care sector. Our main interest is in the third reason, that is, the role of social insurance as a redistributive device.

In a full-information first-best world, there is little reason for redistribution using social insurance. The so-called distributive and allocative functions of the government can be separated, so one would expect income taxation to achieve all the desired redistribution, and social insurance to operate according to the market rule of actuarial fairness. However, in a second-best world with either imperfect insurance or distortionary taxation, we will show that social insurance is a necessary component of redistribution policy, supplementing the tax-transfer system. In fact, perhaps more surprising, in the presence of imperfect private insurance, not only will the use of social insurance be optimal, but so will the use of distortionary taxation, even if the government is able to redistribute using lump-sum taxes.

It has been established in the literature that if risks are negatively related to income so that the poor face higher risks on average, then we have an obvious redistributive argument for social insurance. As shown by Rochet (1989) and Cremer and Pestieau (1996), social insurance combined with a standard distortionary income tax can redistribute more effectively than the latter alone. The reason is that redistributing through social insurance operates on a different margin.[3] The tax-transfer system redistributes between persons of different productivity, while social insurance also redistributes between persons of different states of nature. Given that a higher proportion of low-productivity persons will be in a bad state of nature, social insurance can

[1] For a survey of the arguments for public intervention in health insurance, see Cutler (2002).

[2] These costs are linked to the small scale of private insurance firms and to their advertisement costs. On this, see Diamond (1992) and Mitchell (1998). The point goes back to Arrow (1963).

[3] See also Petretto (1999).

enhance the amount of redistribution without aggravating the tax distortion.

This result has been developed in a setting where the risk probability is given and any loss can be compensated for in a lump-sum way for all persons regardless of their risk class. In other words, both moral hazard and adverse selection were assumed away. When these are taken into account, it might appear that the case for social insurance is not as strong, and that, unlike in the above analyses, full coverage is no longer necessarily socially desirable. The purpose of this paper is to study the implications of moral hazard and adverse selection in an economy in which redistributive taxation and social insurance can be used jointly along with competitive private insurance.[4] Individuals are di¤erentiated according to two dimensions: their labor productivity and their riskness. More speci…cally, we assume that they can be either 'good risks' (low probability of loss) or 'bad risks' (high probability. We assume that private insurers are well informed about household productivities: they can o¤er separate insurance policies to persons of di¤erent income classes. Since risk class cannot be observed, insurance policies must be designed to separate high- from low-risk persons. In our base case, insurance companies will be better informed than the government, which can observe neither risk class nor productivity. This gives an advantage to private insurers thereby making the case for social insurance as strong as possible.

This paper is a complement to Boadway et al. (2002) which focused solely on moral hazard. However, two types of moral hazard were considered — ex ante moral hazard involving discretionary preventative expenditures that a¤ect the probability of the bad state and ex post moral hazard involving health care expenditures in the event of a bad state of nature. In this paper, we focus on ex post moral hazard, but with adverse selection as well. In addition, we consider the implications of di¤erent information assumptions for the government. It turns that the case for social insurance as a redistributive device is strengthened as the government is better informed about household characteristics.

The paper is organized as follows. Section 2 presents the basic model and assumptions. Sections 3 considers our base case, involving both moral hazard and adverse selection and distortionary income tax. Section 4 considers

[4] Blomqvist and Horn (1984) bears some similarities with our paper even though it is not concerned with moral hazard. These authors also examine the case for public insurance when actuarially fair private insurance is available and individuals di¤er in both labour productivity and illness probability. No labour is supplied when ill, and public insurance consists of a uniform lump-sum bene…t to the ill.

some special cases of our base-case model. Section 5 then relaxes the informational assumptions by allowing the government to make lump-sum taxes and transfers conditional on productivity type. Surprisingly, this turns out to strengthen the case for social insurance. Finally, Section 6 offers some concluding remarks.

2 Model and assumptions

The economy consists of three types of decision-makers — households, insurance firms and the government. Households face an idiosyncratic risk of accident, but are able to take actions that affect the size of the loss in the event of an accident (ex post moral hazard). These actions cannot be directly controlled by the government. As mentioned, households differ both in productivity and in accident risk. Insurance companies can observe productivity but not household risk, and provide insurance competitively and with actuarial fairness.[5] The government's objective is to redistribute income among households, but because it cannot observe productivities, it is restricted to using distortionary policy instruments. Decision-making can be thought of as occurring sequentially. The government chooses its policies first, followed by the insurance firms, and then households. In each case, the outcomes of subsequent stages are fully anticipated, so that equilibria of interest will be sub-game perfect.

To be more specific, we use as an example the case of health insurance, though the analysis would apply more generally to other types of personal risks faced by households. We consider two states of the world $j = 0; 1$, where 0 denotes good health (no illness) and 1 ill health. There are 2n types of individuals indexed by ir $(i = 1; \cdots; n;\ r = L; H)$, each characterized by a wage rate w_i and a risk probability π_r, with $w_{i+1} > w_i$ and $\pi_H > \pi_L$. The proportion of households of type ir is given by f_{ir}, where $\sum_{i;r} f_{ir} = 1$.

In the good state, health status is exogenously given as h^0. In the bad state, health status is $h^1 = \bar{h}^1 + m(z)$, where z is health care health care

[5] That is, there is adverse selection within productivity types, but not between. Our assumptions are generally designed to ensure that private insurance firms can provide insurance with constrained efficiency, thereby eliminating insurance market failure as a reason for government intervention. As well, we assume that the proportion of high-risk persons in each productivity class is always large enough such that a separating insurance market equilibrium of the Rothschild and Stiglitz (1976) sort always exists.

expenditure resulting in health improvement, with $m'(z) > 0 > m''(z)$. The expenditure level z that is chosen by the household is undertaken after the state of health is revealed to the household. We assume that $h^1 = \bar{h} + m(z) < h^0$ for all values of z (so $m(\infty) < h^0 - \bar{h}$), so treatment cannot bring health status if ill to a level as high as health status if not ill (i.e. full recovery of health status). Notice that the parameters h^0 and $\bar{h}$, as well as the function $m(z)$, are the same for all types of households. However, the proportions of households of a given productivity class that have π_H and π_L can differ across productivity classes:[6] indeed, that will be part of the motivation for social insurance.

Households have identical state-independent utility functions:

$$u\left(c^j_{ir}, h^j_{ir}, \ell^j_{ir}\right) \tag{1}$$

where c^j_{ir} is consumption and ℓ^j_{ir} is labour supply of a type-ir household in state of health $j = 0, 1$. In illustrating our general results, we shall sometimes assume that utility takes the quasi-linear form: $u\left(c^j_{ir} + h^j_{ir} - g(\ell^j_{ir})\right)$, where $g(\ell^j_{ir})$, the disutility of labour, is increasing and strictly convex. In this case, labour supply depends only on the after-tax wage rate and z on its out-of-pocket price: there are no income or cross-price effects. In particular, labour supply is then state-independent. With a more general utility function, labour could be higher in the bad state if the individual has to compensate for private health care spending or lower if ill health increases the disutility of labour.[7] Naturally, households maximize expected utility, weighted by the probabilities π_r for state 1 (ill health) and $1 - \pi_r$ for state 0 (good health). Households take government policies and private insurance premiums as given. They choose c, ℓ and z after the state is determined.

Insurance firms are perfectly competitive. They offer insurance policies (p_{iH}, P_{iH}) and (p_{iL}, P_{iL}) to households of type i, where p_{ir} is the proportion of health expenditures z_{ir} covered (reimbursed) by private insurance firms and P_{ir} is the total premium paid to insurance firms.[8] The households choose their most preferred policy. Insurance companies anticipate the effect of their

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[6] In other words, $f_{iH}/(f_{iH} + f_{iL})$ — the proportion of high-risk individuals in productivity class i — differs across classes.

[7] A natural extension of this modeling would be to have the labour supply falling to zero in the bad state of nature.

[8] We assume for simplicity that the reimbursement scheme is linear. In principle, since firms can observe z_{ir} they should be able to use non-linear schemes. Allowing that would increase the complexity of our analysis considerably.

insurance policies on health care expenditures z_{ir} (ex post moral hazard). Since we ignore administrative costs, competition entails that premiums are given by:

$$P_{ir} = \pi_r p_{ir} z_{ir} \qquad i = 1; \ldots; n \tag{2}$$

That is, insurance premiums are actuarially fair.

The government has two sorts of policy instruments — tax-transfer policies and social insurance. Tax-transfer policy consists of a linear progressive income tax with marginal tax rate of t and a lump-sum poll subsidy of a per household. Social insurance covers a proportion s of health care expenditures z_{ir}, financed out of general tax revenues. Notice that the same rate of social insurance applies to all households. We denote total insurance coverage by $\sigma_{ir} = p_{ir} + s$. In Section 5 where the government has more complete information, we allow the policies $a; t$ and s to differ by household type.

As mentioned, there are three main stages of decision-making in this economy representing the sequence in which decisions occur:

Stage 1: The government chooses its set of policies $\{t; a; s\}$. It cannot observe individual types or individual demands for goods, leisure or insurance, but can observe incomes. The subsidy on health care expenditures s can be applied indirectly if the government cannot observe z by household. The government knows preferences and the distribution of individuals by type ir. The government anticipates the effect of its policies both on the insurance market and subsequently on households.

Stage 2: The competitive insurance industry sells private insurance to households. Market equilibrium (competition for customers, with zero expected profits) determines p_{ir} and P_{ir}. The insurance industry is unable to observe household risk types, but can observe their productivity. Thus, insurance firms are better informed than the government. In this stage, $\{t; a; s\}$ are taken as given, and household behaviour is correctly anticipated.

Stage 3: Households select $\{c^1_{ir}; \ell^1_{ir}; z_{ir}; c^0_{ir}; \ell^0_{ir}\}$. All variables are state-specific since they are chosen after the state is revealed (z_{ir} is chosen only in the bad state). Households take $\{t; a; s; p_{ir}; P_{ir}\}$ as given from the previous two stages.

Since the equilibrium is assumed to be sub-game perfect, we proceed to solve it by backward induction.

3 The base case optimum

3.1 Stage 3: Household choice

Households of type ir make their choices given the public policy parameters t, a and s chosen by the government in Stage 1 and the private market premium P_{ir} and coverage p_{ir} determined by the insurance market equilibrium in Stage 2. Actually, they are only concerned by total coverage defined by $\sigma_{ir} \equiv s + p_{ir}$. The budget constraints in the two states of health are given by:

$$c^1_{ir} = (1 - t)w_i \ell^1_{ir} + a - (1 - \sigma_{ir})z_{ir} - P_{ir} \tag{3}$$

and

$$c^0_{ir} = (1 - t)w_i \ell^0_{ir} + a - P_{ir}. \tag{4}$$

The type-ir households' problem is thus

$$\max_{\{\ell^j_{ir}, z_{ir}\}} \pi_r u((1 - t)w_i \ell^1_{ir} + a - P_{ir} - (1 - \sigma_{ir})z_{ir}; h^1 + m(z_{ir}); \ell^1_{ir})$$
$$+ (1 - \pi_r)u((1 - t)w_i \ell^0_{ir} + a - P_{ir}; h^0; \ell^0_{ir}).$$

The first-order conditions are as follows, where we use the convention that when dealing with functions, subscripts are used for partial derivatives and superscripts for the type of individuals and the state of nature:

$$z_{ir}: \quad (1 - \sigma_{ir})u^{1,ir}_c = m'(z_{ir})u^{1,ir}_h$$
$$\ell^j_{ir}: \quad (1 - t)w_i u^{j,ir}_c = -u^{j,ir}_\ell \qquad j = 0, 1.$$

These yield the state-contingent labour supply functions $\ell^0_{ir}(t, a - P_{ir})$ and $\ell^1_{ir}(t, a - P_{ir}, \sigma_{ir})$, the demand function for health care spending $z_{ir}(t, a - P_{ir}, \sigma_{ir})$, and the indirect expected utility $v_{ir}(t, a - P_{ir}, \sigma_{ir})$. Applying the envelope theorem gives:

$$v^{ir}_t = -w_i E_j[\ell^j_{ir} u^{j,ir}_c]; \quad v^{ir}_a = E_j[u^{j,ir}_c]; \quad v^{ir}_\sigma = \pi_r z_{ir} u^{1,ir}_c.$$

The operator E_j is taken over the two states of health, 0 and 1.

3.2 Stage 2: Private insurance market equilibrium with adverse selection

We assume that private insurers operate in a competitive environment. They observe ability w_i but not risk π_r. This gives an informational advantage to the private sector relative to the government, which does not observe either characteristic. This is a relatively standard adverse selection problem. For each level of productivity i, we look for the separating Nash equilibrum of the Rothschild and Stiglitz (1976) type consisting of two specific contracts:

$$(p_{iH}; P_{iH}) \quad \text{and} \quad (p_{iL}; P_{iL}):$$

The separating equilibrium involves depicting household preference and the zero-profit condition of insurance firms in $(p; P)$–space. Consider each in turn

For a separating equilibrium to apply, the relevant single-crossing property involving the slope of household indifference curves in $(p_{ir}; P_{ir})$-space must be satisfied. Differentiating $v_{ir}(t; a - P_{ir}; \sigma_{ir})$ along an indifference curve we obtain:

$$\left.\frac{dP_{ir}}{dp_{ir}}\right|_{v} = -\frac{v_p^{ir}}{v_P^{ir}} = \frac{\pi_r z_{ir} u_c^{1;ir}}{E_j\left[u_c^{j;ir}\right]} = \frac{z_{ir} u_c^{1;ir}}{u_{cr}^{1;ir} + u_c^{0;ir}(1 - \pi_r)/\pi_r} > 0:$$

As can be seen, the slope of the indifference curve is increasing in π_r, so the single-crossing property is satisfied. We also assume that indifference curves are concave, which will be the case if moral hazard is not too intensive.[9] In a separating competitive equilibrium, expected profits for each contract offered is driven to zero. This implies that the contracts $(p_{ir}; P_{ir})$ will be actuarially fair for both risk types, or

$$P_{ir} = \pi_r p_{ir} z_{ir}(t; a - P_{ir}; s + p_{ir}); \qquad r = H; L$$

which yields

$$P_{ir} = P_{ir}(t; a; s; p_{ir})$$

[9] In the absence of moral hazard, z is fixed so indifference curves have the slope $\left[1 + \left(u_c^{0;ir}/u_{cr}^{1;ir}\right)(1 - \pi_r)/\pi_r\right]^{-1}$, which is expected to decrease as insurance coverage p increases and marginal utilities of consumption in the two states become mor equal. The rate at which the slope of the indifference curve decreases can be expected to fall once z is endogenous since z decreases in p.

with

$$P_p^{ir} = \frac{\pi_r z_{ir} + \pi_r p_{ir} z_\sigma^{ir}}{1 + \pi_r p_{ir} z_a^{ir}}; \quad P_a^{ir} = \frac{\pi_r p_{ir} z_a^{ir}}{1 + \pi_r p_{ir} z_a^{ir}}; \quad \text{and} \quad P_s^{ir} = \frac{\pi_r p_{ir} z_s^{ir}}{1 + \pi_r p_{ir} z_a^{ir}}: \qquad (5)$$

Note that $d^2 P_{ir}/dp_{ir}^2$ is generally of ambiguous sign because of terms involving $z_{\sigma\sigma}^{ir}$ and $z_{a\sigma}^{ir}$. In what follows, we assume that $d^2 P_{ir}/dp_{ir}^2 > 0$, which means that the actuarial fairness curve linking P_{ir} to p_{ir} is convex.[10] Furthermore, this curve is steeper for high-risk than for low-risk individuals. This, combined with the single-crossing property, ensures that the separating equilibrium is unique.

Figure 1 uses these indifference curves and the actuarial fairness constraints to depicts a separating competitive equilibrium for given values of government policies. The equilibrium has standard features. An equilibrium contract makes zero expected profits, while any other contract that consumers prefer would make losses if it were offered. Since indifference curves are steeper for the high-risk individuals, the contract $(p_{iH}; P_{iH})$ can be characterized as the actuarially fair one that maximizes the expected utility for individuals of type–$(i; H)$, while the contract $(p_{iL}; P_{iL})$ is actuarially fair for individuals of type $(i; L)$ and satisfies the self-selection constraint that it must not be attractive for type $(i; H)$ (high-risk) individuals:

$$v_{iH}(t; a - P_{iH}(t; a; s; p_{iH}); s + p_{iH}) \geq v_{iH}(t; a - P_{iL}(t; a; s; p_{iL}); s + p_{iL})$$

where government policies $t; a$ and s are chosen in the previous stage. Given the relationship $P_{ir}(t; a; s; p_{ir})$ obtained from the condition of actuarial fairness, we can then characterize the separating equilibrium by the levels of p_{iH} and p_{iL} determined by the insurance industry. Consider each in turn. To simplify the notation, we suppress the productivity subscript i for the time being since the same analysis applies for all i.

3.2.1 Choice of p_H

As Figure 1 indicates, the insurance policy offered to the high-risk households maximizes their expected utility along the actuarial fairness curve

[10] In the absence of moral hazard, the slope of the actuarial fairness line is constant. Once moral hazard is introduced, its slop is expected to increase since higher coverage p induces more health care spending, which induces the premium P to increase.

$P_H(t, a, s, p_H)$. The representative insurance firm's problem can then be written as:

$$\max_{p_H} \; v^H(t, a - P_H(t, a, s, p_H), s + p_H).$$

This yields the first-order condition:

$$v^H_\sigma - v^H_a P^H_p = 0, \tag{6}$$

where $v^H_\sigma = \pi_H z_H u^{1H}_c$ and $v^H_a = E_j[u^{j,H}_c] = \pi_H u^{1H}_c + (1 - \pi_H) u^{0H}_c$ by the envelope conditions derived above. Combining (5) and (6), we obtain

$$u^{1H}_c = \frac{E_j[u^{j,H}_c]}{\pi_H z_H} \left(\frac{\pi_H z_H + \pi_H p_H z^H_\sigma}{1 + \pi_H p_H z^H_a} \right). \tag{7}$$

If $z^H_\sigma = z^H_a = 0$ (no moral hazard), then $u^{1H}_c = E_j[u^{j,H}_c]$ or $u^{1H}_c = u^{0H}_c$. That is, without moral hazard there is full insurance,[11] as expected. Otherwise, $u^{1H}_c > u^{0H}_c$ and in general there is less than full insurance.

It is interesting to consider the circumstances under which private insurance would be introduced given the presence of social insurance. To see that, differentiate the objective function for the representative firm's problem at $p_H = 0$ and use (5) to obtain:

$$\left.\frac{dv_H}{dp_H}\right|_{p_H=0} = \pi_H z_H (u^{1H}_c - E_j[u^{j,H}_c]) = \pi_H z_H (1 - \pi_H)(u^{1H}_c - u^{0H}_c).$$

This is positive as long as the marginal utility of income is higher in the bad than in the good state of health. With a separable utility function and less than full social insurance ($s < 1$), this will be the case as is well known.[12] It may seem odd that private insurance will be offered whatever the value of $s < 1$ (in particular when it is close to unity). This can be understood by recognizing that private coverage p_H imposes a negative externality on the cost of social insurance, which is not internalized by private insurers at the insurance market equilibrium: increasing coverage p_H makes health care spending z_H and so the costs of social insurance sz_H rise.

[11] Full insurance is reached when $p_H + s$ is chosen such that the marginal utility of consumption is identical in the two states of health. With an additive utility function, this requires that $p_H + s = 1$.

[12] One might expect that these results would generally also apply without separability, but it is not impossible that $u^{1H}_c < u^{0H}_c$ because of substitutability among c, ℓ and h.

The solution of the above problem for each productivity class yields private coverage $p_H(t,a,s)$ and the value function denoted $V_H(t,a,s)$, that is, the expected utility of the high-risk households. Applying the envelope theorem, we obtain:

$$V_\beta^H = v_\beta^H - v_a^H P_\beta^H; \quad \beta = t, a, s. \tag{8}$$

This outcome will be anticipated by the government in the first stage.

3.2.2 Choice of p_L

The equilibrium value for p_L is determined by the point on the actuarial fairness curve for the low-risk households where the self-selection contraint is just binding, or:

$$V_H(a,t,s) = v_H(t, a - P_L(t,a,s,p_L), s + p_L). \tag{9}$$

The solution to this condition yields the private insurance coverage $p_L = p_L(t,a,s)$ offered to the low-risk households and their value function $V_L(t,a,s) \equiv v_L(t, a - P_L(s, p_L(\cdot)), s + p_L(\cdot))$. By differentiating the latter we obtain:

$$V_\beta^L = v_\beta^L - v_a^L P_\beta^L + (v_p^L - v_a^L P_p^L) p_\beta^L; \quad \beta = t, a, s. \tag{10}$$

The term in brackets $(v_p^L - v_a^L P_p^L)$, which is positive, reflects the information externality arising from adverse selection: high-risk individuals impose a negative externality on low-risk individuals whose risks cannot be covered as much as in world with observability of types (p_L is too low).

In stage 1, to which we now turn, the government will use the value functions $V_{iH}(t,a,s)$ and $V_{iL}(a,t,s)$ just derived for all productivity types $(i = 1, \cdots, n)$ to find the optimal values of tax and social insurance parameters.

3.3 Stage 1: Government policy

The government chooses the linear tax parameters t, a and the level of social insurance s to maximize the sum of expected utilities subject to its budget constraint, anticipating the outcomes of the subsequent stages. The

Lagrangean expression is:

$$\mathcal{L} = \sum_{ir} f_{ir} V_{ir}(t;a;s) + \lambda \sum_{ir} f_{ir} \{ t w_i [\pi_r \ell^1_{ir}(t; a - P_{ir}(\cdot); s + p_{ir}(\cdot))$$
$$+ (1 - \pi_r) \ell^0_{ir}(t; a - P_{ir}(\cdot))] - a - s \pi_r z_{ir}(t; a - P_{ir}(\cdot); s + p_{ir}(\cdot)) \}.$$

where $p_{ir}(\cdot) = p_{ir}(t;a;s)$ and $P_{ir}(\cdot) = P_{ir}(t;a;s;p_{ir}(\cdot))$ are determined in stage 2, and λ is the multiplier associated with the revenue constraint. The first-order conditions with respect to t, a and s are:

$$\sum f_{ir} V^{ir}_t + \lambda \sum f_{ir} \left\{ w_i E_j \left[\ell^j_{ir} \right] + t w_i E_j \left[\frac{d\ell^j_{ir}}{dt} \right] - s \pi_r \frac{dz_{ir}}{dt} \right\} = 0 \quad (11)$$

$$\sum f_{ir} V^{ir}_a + \lambda \sum f_{ir} \left\{ -1 + t w_i E_j \left[\frac{d\ell^j_{ir}}{da} \right] - s \pi_r \frac{dz_{ir}}{da} \right\} = 0 \quad (12)$$

$$\sum f_{ir} V^{ir}_s + \lambda \sum f_{ir} \left\{ -\pi_r z_{ir} + t w_i E_j \left[\frac{d\ell^j_{ir}}{ds} \right] - s \pi_r \frac{dz_{ir}}{ds} \right\} = 0. \quad (13)$$

Combining (11) and (12) we would obtain a standard formula for the linear tax rate, amended to include a term involving the indirect effect of the tax system on the cost of social insurance. We can reasonably expect this to yield an interior solutions for t and a. But this is not the concern of this paper. We are only interested by the value of s. To obtain it, we combine (12) and (13). After some manipulations, we obtain:

$$\begin{aligned} s = {} & \Delta^{-1} \mathrm{cov}_{ir}(b_{ir}; \pi_r z_{ir}) \\ & + \Delta^{-1} t \sum_{ir} f_{ir} w_i \left(E_j \left[\frac{d\ell^j_{ir}}{ds} \right] - \pi_r z_{ir} E_j \left[\frac{d\ell^j_{ir}}{da} \right] \right) \\ & + \Delta^{-1} \frac{1}{\lambda} \sum_i f_{iL} (v^{iL}_p - v^{iL}_a P^{iL}_p)(1 + p^{iL}_s - E_{ir}[\pi_r z_{ir}] p^{iL}_a) \end{aligned} \quad (14)$$

where

$$\Delta = \sum_{ir} f_{ir} \pi_r \left(\frac{dz_{ir}}{ds} - \pi_r z_{ir} \frac{dz_{ir}}{da} \right) = \sum_{ir} f_{ir} \pi_r \frac{d\tilde{z}_{ir}}{ds} > 0.$$

In these expressions, cov_{ir} is the covariance taken over all types ir; $d\tilde{z}_{ir}/ds$ is a compensated total change in the demand for health care spending with respect to s (taking account of indirect effects through $p_{ir}(t; a; s)$), and b_{ir} is the marginal net expected social valuation of income of type-$(i; r)$ individuals (divided by λ) and ir. That is:

$$b_{ir} \equiv \frac{v_a^{ir}}{\lambda}(1 - P_a^{ir}) - s\pi_r \frac{dz_{ir}}{da} + tw_i E_j \left[\frac{d\ell_{ir}^j}{da} \right] \qquad (15)$$

The interpretation of b_{ir}, which is familiar from optimal tax theory, is that if $b_A > b_B$ for two individuals A and B, redistributing income from B to A would be socially desirable. From (12) and $V_a^{ir} = v_a^{ir}(1 - P_a^{ir})$ by (10), we obtain $E_{ir}[b_{ir}] = 1$, where E_{ir} refers to the expected value taken over individual types ir.

The expression (14) is analogous to the standard expression for the optimal linear income tax rate, except that it includes additional terms reflecting the various margins of distortion in our economy. The denominator Δ is an efficiency effect arising from the ex post moral hazard induced by social insurance. As noted it can be shown to be positive in sign, as expected. The numerator includes i) an equity concern (the covariance term), ii) an efficiency concern arising from the indirect effect of social insurance on the distorted labour market, and iii) an efficiency concern arising from the distortion imposed in low-risk types due to the adverse selection. We consider these various effects in turn in the next section.

4 Interpretation of optimal social insurance policy

The solution to the government's optimality conditions (11)–(13) yields the optimal choice of both the linear tax structure $(t; a)$ and the level of social insurance s. Since our interest is in the latter, that will be our focus. The characterization of the optimal tax structure is similar to that obtained in the standard optimal tax literature. Each of the three terms in (14) will be considered in turn.

4.1 The equity term

The equity term involves the covariance between the marginal net expected social valuation of income b_{ir} and expected health care spending $\pi_r z_{ir}$. Theoretical considerations are not of much help in signing this covariance. Even if we assume a positive covariance between b_{ir} and π_r we still have to verify whether taking $\pi_r z_{ir}$ instead of π_r changes the sign. If we assume that π_r and w_i are 'sufficiently' negatively correlated, and that z_{ir} does not increase much with w_i, then the covariance term is positive. Ultimately, assessing the sign of this covariance term and its magnitude requires investigating empirically the relation between individual income and health care spending.[13]

The equity term would be the only one in (14) if there were no adverse selection — so the last term disappears — and if utility is of the quasi-linear form mentioned earlier — so the second term involving cross effects on labour supply disappears. In these circumstances, the expression for optimal social insurance becomes simply:

$$s = \Delta^{-1} \mathrm{cov}_{ir} \left(b_{ir}; \pi_r z_{ir} \right)$$

This is analogous to the standard expression for the optimal linear tax rate. The numerator is an equity effect, while the denominator is an efficiency effect arising from the induced effect of s on health care expenditures z for all households, that is, the moral hazard effect. As noted, $s > 0$ if risk and productivity are sufficiently negatively correlated and health care expenditures do not increase much with productivity.

A further special case is obtained when there is neither moral hazard not adverse selection. In this simple case, r is observed by insurers. Let us assume that z is fixed at $\hat{z}$ such that $\overline{h} + m(\hat{z}) = h^0$. This is the case originally studied by Rochet (1989). In such a setting, it is straightforward to show that the insurance market equilibrium implies $p_{ir} = 1 - s$. In other words, with actuarially fair private insurance there is always full insurance: social insurance fully crowds out private insurance, and private insurance exerts no externality on the cost of social insurance in the absence of moral hazard. Then, using the first-order condition with respect to a, we have $d\$/ds = \mathrm{cov}_{ir}(b_{ir}; \pi_r \hat{z})$; with the obvious conclusion that if $\mathrm{cov}_{ir}(b_{ir}; \pi_r) > 0$, it is optimal to push the coverage of social insurance up to 100 percent. The intuition is that if people with high probability of loss happen to be those

[13] See, for example, Manning et al (1987).

with high marginal net social value of income (and so low income generally), then providing uniform social insurance paid in a lump-sum way is desirable and indeed should alleviate part of the income tax distortion. As we show below, when the tax is non-distortionary the case for social insurance is even stronger.

4.2 The labour market efficiency term

The second term involves the indirect effect of social insurance on the labour market, which itself is distorted. It can be written $\Delta^{-1} t \left[\sum_{ir} f_{ir} w_i E_j \left(d\ell_{ir}/ds \right)\right]$, where the derivative is again an income-compensated total effect, whose sign is ambiguous in general. This can be interpreted as a second-best effect. Given a positive tax on labour income, the social value of an increase in labour supply — the pre-tax wage rate reflecting the productivity of an additional unit of labour supplied — exceeds the net cost to households of supplying the labour — the after-tax wage rate. In these circumstances, anything that increases labour supply provides a net increase in social welfare. An alternative interpretation is that if ℓ increases with a compensated increase in s, an increase in s will indirectly increase income tax revenues. Since the social value of an additional unit of tax revenues is greater than one, this would enhance the case for social insurance.

The expression for the optimal social insurance rate will involve only the first two terms in the special case where there is moral hazard but no adverse selection. In this case, equation (14) simplifies to:

$$s = \Delta^{-1}\left\{ \underbrace{cov_{ir}(b_{ir}, \pi_r z_{ir})}_{\text{equity}} + \underbrace{t \sum_{ir} f_{ir} w_i E_j \left[\frac{d\ell_{ir}}{ds} \right]}_{\text{second-best}} \right\}.$$

This is the case considered in more detail in Boadway et al (2002). The formula for s differs from the general case only by the absence of the adverse selection effect. As before, the sign is generally ambiguous, with the second best term adding to the ambiguity already found for the equity term.

4.3 The adverse selection efficiency term

The third term arises because of adverse selection. As indicated when discussing (10), the presence of high-risk individuals (and so of their self-selection constraint (9)) prevents low-risk ones from reaching their most preferred level of private insurance coverage (p_{iL}). Increasing this coverage would increase their expected utility by $v_p^{iL} - v_a^{iL} P_p^{iL} > 0$. An increase in s results in this gain if it relaxes the self-selection constraint (9), which occurs if $1 + p_s^{iL} - E_{ir}[\pi_r z_{ir}] p_a^{iL} > 0$. This condition involves the compensated effect of a change in s on total insurance coverage. It will be satisfied if and only if

$$\frac{v_a^{iH}}{1 + \pi_H p_{iH} z_a^{iH}}[\pi_H z_{iH} - E(\pi z)] > \frac{\hat{v}_a^{iH}}{1 + \pi_L p_{iL} z_a^{iL}}[\pi_L z_{iL} - E(\pi z)]$$

where $\hat{v}_a^{iH}$ is the marginal utility of income of L mimicking H. A sufficient condition for this inequality to hold is

$$\pi_L z_{iL} < E[\pi z] < \pi_H z_{iH}.$$

If this is satisfied for every $i = 1, \cdots, n$, the information externality term is positive and pushes s upwards. If $\pi_L z_{iL} < E[\pi z] < \pi_H z_{iH}$ did not hold for some i, $\pi_H z_{iH} > \pi_L z_{iL}$ and $v_a^{iH} \simeq \hat{v}_a^{iH}$ (since $v^{iH} = \hat{v}^{iH}$) would make the above inequality very likely to hold.

The expression for social insurance, equation (14), involves only the equity and adverse selection terms if preferences are quasi-linear. As mentioned, in this case, labour supply is independent of s and is the same in both health states. Moreover, z_{ir} depends only on $\sigma_{ir} = s + p_{ir}$: more precisely, $dz_{ir}/d\sigma_{ir} = -1/m''(z_{ir}) > 0$. Again, the sign of s is generally ambiguous. Even in this case where z_{ir} depends only on insurance coverage, there is no guarantee that the covariance term is positive even if there is a negative correlation between π_r and w_i, although there might be a general presumption to that effect. As well, the adverse selection term remains of ambiguous sign.

The upshot of our discussion so far is that in general we would expect $s \neq 0$, but it could take either sign depending on the signs of the various equity and efficiency effects. Although there might be a presumption that $s > 0$ under reasonable circumstances, it is certainly possible that it might be optimal to impose a tax on health care expenditures rather than partially covering them. Once we relax the informational constraints that we have imposed on the government, the case for social insurance is strengthened, as the next section shows. The reason is that social insurance effectively distributes between both productivity types and risk types. Once the information of the government is enhanced so that the tax system is better able to redistribute between productivity classes, social insurance is left to redistribute between risk classes, and in this task its sign turns out to be unambiguous.

5 Alternative informational assumptions

So far we have assumed that the government could observe neither the risk class of households nor their productivity levels. This a¤orded an advantage to the insurance industry, which could identify the productivity of households. In this section we explore some of the implications of di¤erent informational assumptions on the part of the government. Two cases are of interest. In the …rst, the government has full information over both productivity w_i and risk class ¼$_r$. In the second, it can identify households by productivity but not risk class. These additional information assumptions allow the government to condition its policy instruments on the observable characteristics of households. In particular, lump-sum redistribution across productivity classes is possible: there is no need to levy a distortionary tax t. Moreover, surprisingly the case for positive social insurance is enhanced.

Before proceeding, two limitations to our analysis are worth mentioning, both of which arise from considerations of tractability. First, we do not consider the case in which the insurance industry has less information than in the previous sections. If insurance companies could observe neither the productivity nor the risk class of households, the equilibrium in the insurance industry would involve two-dimensional screening: insurance policies would have to separate households by both risk and productivity. This would involve obvious complications that would detract from the insights that are gained from our previous analysis. And, as mentioned, allowing the insurance industry to observe productivity enhances the e¤ectiveness of private insurance and makes the case for social insurance more demanding, which is useful for our purposes. Second, in the previous sections we do not allow the government to fully exploit the information available to it. It is well known that if the government can observe income, it will be welfare-improving to adopt an optimal non-linear income tax system rather than a linear one. Presumably, by improving its redistributive instruments, the case for social insurance as a supplementary redistributive device would be changed: as we shall show below, it might even be enhanced given that redistribution between producticity types is being better performed by the income tax system. That is, the optimal income tax can redistribute between productivity classes but not between risk classes or states of nature. Social insurance serves to do the latter. This section serves to shed some light oin the case in which the government exploits its informational advantages to the fullest. By being able to observe productivity, it can use lump-sum redistribution, which is an extreme form of non-linear redistributive taxation.

5.1 The full information case

To begin with, consider the case in which the government can observe both productivity and risk class of all households. The results then depend on whether the insurance companies also have full information, or whether they can only observe productivity. Both cases will be discussed in what follows. The properties of household maximization at Stage 3 carry over to this case. As well, the characterization of insurance market equilibrium in Stage 2 for the adverse selection case remains as above. If the insurance companies have full information, the contract offered to the low-risk types will be of the same form as that offered to those with high-risk: the self-selection constraint is not imposed. Therefore, we can go directly to the government's problem in Stage 1. For simplicity and clarity, we concentrate on the case with quasi-linear preferences.

Since the government can observe both types of persons, its policy instruments can be indexed by ir. Otherwise the same Lagrangean expression applies as in Section 3. The first-order conditions can now be written for each type ir, using the assumption of quasi-linear preferences to eliminate cross-price effects on labour supply:

$$V_t^{ir} + \lambda \left\{ w_i \ell_{ir} - t_{ir} w_i^2 \ell'_{ir} - s_{ir}\pi_r \frac{dz_{ir}}{dt_{ir}} \right\} = 0 \quad (16)$$

$$V_a^{ir} - \lambda \left\{ 1 + s_{ir}\pi_r \frac{dz_{ir}}{da_{ir}} \right\} = 0 \quad (17)$$

$$V_s^{ir} - \lambda \left\{ \pi_r z_{ir} + s_{ir}\pi_r \frac{dz_{ir}}{ds_{ir}} \right\} = 0. \quad (18)$$

where labour supply is given by $\ell_{ir} = \ell\left((1 - t_{ir}) w_i\right)$, the same in both health states, and health spending is $z_{ir} = z\left(s_{ir} + p\left(t_{ir}, a_{ir}, s_{ir}\right)\right)$.

Consider first the case where the insurance companies have full information, so there is no adverse selection. The first-order conditions on t_{ir} and a_{ir}, (16) and (17), can be rewritten:

$$\begin{aligned} w_i \ell_{ir} E_j\left[u_c^{j;ir}\right] - \lambda\left\{w_i \ell_{ir} - \left[t_{ir} w_i^2 \ell'_{ir}\right] - s_{ir}\pi_r z_\sigma^{ir} p_t^{ir}\right\} &= 0 \\ E_j\left[u_c^{j;ir}\right] - \lambda\left\{1 + s_{ir}\pi_r z_\sigma^{ir} p_a^{ir}\right\} &= 0 \end{aligned}$$

Both t_{ir} and a_{ir} affect p_{ir} through an income effect alone. Therefore, by differentiating the consumer's budget constraint with respect to t_{ir} and a_{ir} and using the household's first-order condition on labour supply for the separable case $g'(\ell_{ir}) = (1 - t_{ir}) w_{ir}$, we infer that

$$p_t^{ir} = \left(-w_i \ell_{ir} + (1 - t_{ir}) w_i^2 \ell'_{ir} - w_i g'(\ell_{ir}) \ell'_{ir}\right) p_a^{ir} = -w_i \ell_{ir} p_a^{ir}.$$

Using this and combining the previous two equations, we obtain as expected $t_{ir} = 0$. Moreover, from the first order condition on a_{ir}, if $s_{ir} = 0$, then $E_j[u_c^{j;ir}] = \lambda$, so expected utility is equalized across all households (although not across states because of the moral hazard). Finally, it is straightforward to show that $s_{ir} = 0$ in the optimum. From the first-order conditions on s_{ir} and a_{ir}, and using condition (6) from Stage 2, we obtain $(1 + p_s^{ir} - \pi_r z_{ir} p_a^{ir}) s_{ir} = 0$. It can be shown that $1 + p_s^{ir} - \pi_r z_{ir} p_a^{ir} > 0$, implying that $s_{ir} = 0$ in the optimum: there should be no social insurance in the full information case.[14] Private insurance alone is optimal. The intuition for this result is as follows. If the government were to set $s_{ir} \neq 0$, there will be an externality imposed by private insurance on the cost of social insurance, resulting in a non-optimal amount of overall insurance coverage. This can be avoided if the government simply refrains from using social insurance, leaving it to the market to provide. Since there is no adverse selection, the government can provide insurance no better than the private sector.

Next consider the case in which insurance companies cannot observe risk types so there would be adverse selection in private insurance markets along with moral hazard. In this case, the government has better information that the insurance companies so can achieve the same outcome as above on

[14] In fact, this can be inferred directly from the analogue to expression (14) for s derived earlier. In this case, since there is no adverse selection (because of perfect information) and no induced effect (because of quasi-linearity), only the equity effect is left. And, since the government can separate all ir types of households, the covariance term also disappears.

its own. This would entail it mimicking the behaviour of a fully informed representative insurance firm (subject to moral hazard), setting $t_{ir} = 0$ and using lump-sum transfers a_{ir} to redistribute among all household types. Private insurance companies should be prevented from operating since if they did, they would impose an insurance externality that would distort resource allocation.

5.2 Household productivity observable

The final case to consider is that in which both the government and the insurance industry can observe productivity types, but not risk types. Once again, the Lagrangean expression for the government's problem is the same as before, with the exception that policies can be conditioned on productivity alone — $t_i; a_i; s_i$. The first-order conditions can be written for each productivity type i:

$$\sum_r f_{ir}V_t^{ir} + \lambda \sum_r f_{ir}\left\{ w_i\ell_i - t_i w_i^2 \ell'_i - s_i\pi_r \frac{dz_{ir}}{dt_i}\right\} = 0 \tag{19}$$

$$\sum_r f_{ir}V_a^{ir} - \lambda \sum_r f_{ir}\left\{ 1 + s_i\pi_r \frac{dz_{ir}}{da_i}\right\} = 0 \tag{20}$$

$$\sum_r f_{ir}V_s^{ir} - \lambda \sum_r f_{ir}\left\{ \pi_r z_{ir} + s_i\pi_r \frac{dz_{ir}}{ds_i}\right\} = 0. \tag{21}$$

Analogously to above, the first-order conditions on t_i and a_i can be rewritten:

$$\sum_r f_{ir} w_i \ell_{ir} E\left[u_c^{j;ir}\right] - \lambda \sum_r f_{ir}\left\{ w_i\ell_{ir} - t_i w_i^2 \ell'_{ir} - s_i\pi_r z_\sigma^{ir} p_t^{ir}\right\} = 0$$

$$\sum_r f_{ir} E\left[u_c^{j;ir}\right] - \lambda \sum_r f_{ir}\left\{ 1 + s_i\pi_r z_\sigma^{ir} p_a^{ir}\right\} = 0$$

Using again the relationship $p_t^{ir} = -w_i\ell_{ir}p_a^{ir}$ as before and combining these two conditions, we again derive $t_i = 0$. Again, not surprisingly, there is no need for a distorting tax when the government can redistribute among productivity types using lump-sum transfers. Moreover, if $s_i = 0$, the condition on a_i would imply full redistribution across productivity types: $\sum_r f_{ir}E_j[u_c^{j;ir}] = \lambda$ for all i.

However, in this case s_i will generally not be zero. The easiest way to see that is to adapt expression (14) for s derived earlier to the case where the government can make its policies contingent on productivity:

$$
\begin{aligned}
s_i \;=\;& \Delta_i^{-1}\mathrm{cov}_r\,(b_{ir}, \pi_r z_{ir}) \\
&+ \Delta_i^{-1}\frac{1}{\lambda}(v_p^{iL} - v_a^{iL} P_p^{iL})(1 + p_s^{iL} - E_{ir}[\pi_r z_{ir}] p_a^{iL})
\end{aligned}
$$

where $\Delta_i = \sum_r f_{ir}\pi_r de_{ir}/ds > 0$. This expression is now unambiguously positive. The covariance across states of health for a given productivity person is positive. The second term involving the adverse selection effect is also positive, since $1 + p_s^{iL} - E_{ir}[\pi_r z_{ir}] p_a^{iL} > 0$. The intuition is that since social insurance is contingent on productivity type, it redistributes only across states of health. It is welfare-improving to redistribute from low-risk to high-risk households, which positive social insurance will do. Of course, if the government cannot for whatever reason differentiate s by productivity class, this unambiguous result will no longer necessarily apply.

Returning to the case studied by Rochet (1989) in which neither adverse selection nor moral hazard are present, the results are striking. We now have that the covariance term is positive regardless of the sign of the correlation between w_i and π_r, so the outcome $s = 1$ always holds.

6 Conclusion

The starting point of this paper was the finding of Rochet (1989) that with distortionary income taxation, social insurance is desirable as a redistributive device. The gist of his argument was the distortionary feature of income taxation. Our purpose was to see how robust this finding was when introducing moral hazard and adverse selection.

We considered *ex post* moral hazard and showed that the case for public intervention in insurance markets remains. However, while in Rochet's analysis, optimal social insurance is complete and crowds out private insurance, in the presence of moral hazard, it is no longer the case. The introduction of adverse selection has the effect of fostering social insurance. We also show that even with lump-sum taxation there could be a case for social insurance so as to redistribute from good to bad risks which income taxation does not do at least directly. Indeed in this case the covariance term is always negative and thus the case for social insurance is stronger with lump-sum taxation than with distortionary income taxation.

A number of extensions to the current analysis could be contemplated. First, it might be interesting to see whether or not an optimal non-linear tax would dampen the case for social insurance. Evidence from related literatures suggests that even when non-linear taxes are set optimally, the case for second-best policy instruments typically remains intact. Second, the viewpoint adopted here was purely normative. It would be interesting to adopt a political economy approach with social insurance being determined by voting.[15]

[15] In that respect, see Hindriks and De Donder (2000).

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