A THEORY OF VERTICAL FISCAL IMBALANCE

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ABSTRACT

This paper examines how sequential decision-making by different levels of government can result in vertical fiscal imbalances (VFI). Federal-regional transfers serve to equalize the marginal cost of public funds between regions hit by different shocks. The optimal vertical fiscal gap minimizes the efficiency cost of taxation in the federation as a whole. The analysis shows how the existence of vertical fiscal externalities, leading regional governments to overprovide public goods, can induce the federal government to create a VFI by selecting transfers that differ from the optimal fiscal gap. When the federal government can fully commit to its policies before regional governments select their level of expenditures, the VFI will generally be negative. In the absence of commitment, the equilibrium transfer is unambiguously larger than the optimal fiscal gap, resulting in a positive VFI. In an intertemporal setting, the VFI has implications for the sharing of debt between the federal and regional governments.

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1. Introduction

Transfers from the federal government to sub-national governments—which we shall call regional governments—are commonplace in federations and fulfill various potential roles. They may be purely passive responses to the asymmetric decentralization of expenditure and revenue-raising authority. More important, they may be proactive policy instruments in their own right used to achieve various national policy objectives in a decentralized setting. For one thing, they may be used to equalize the fiscal capacity of the regions to avoid inefficient migration of persons and businesses among regions and to foster horizontal equity in the federation as a whole (Boadway et al 2002). For another, they may be used in conditional forms to counter fiscal externalities imposed by regional governments on other regions, as well as to achieve national standards in social programs and to induce efficiency in the internal economic union of the federation (Dahlby 1996). They may also be used as instruments for insuring regions against idiosyncratic shocks to their fiscal capacities (Lockwood 1999). All of these objectives call for an asymmetry between federal revenues relative to its spending responsibility, typically referred to as a *vertical fiscal gap*.

Although the size of the vertical fiscal gap is endogenously determined by the joint fiscal decisions of the federal and regional governments, the federal government is typically taken to have a leadership role. There has been concern in some countries that this leadership role has been exercised in a way that puts the regional governments at a disadvantage. For example, in Canada, the case with which we are most familiar, there has been much debate about a so-called 'vertical fiscal imbalance' that has emerged in recent years. The argument has been made that the federal government's fiscal response to its structural deficit and debt problems that built up over the 1980s has been a disproportionate reduction in transfers to the provinces, effectively passing on some of its deficit to the latter. The result, as the terminology vertical fiscal imbalance suggests, is alleged to be a situation in which the size of transfers made by the federal government to the provinces falls well short of the amount of federal tax revenues relative to their expenditure responsibilities, that is, what one might think of as the optimal vertical fiscal gap. Moreover, although the federal deficit problem may have been anticipated, the manner in which the federal government responded to it took the provinces by surprise. While there has been some literature documenting the problem of vertical fiscal imbalance, and even whether one might exist,[1] there has been relatively little theoretical literature analyzing either the sources or consequences of vertical imbalance. Indeed, there has been limited progress in formalizing the concept of vertical fiscal imbalance and its relation to the time-honored notion of a vertical fiscal gap, which we take to be the optimal relationship between federal and regional government expenditure and revenue-raising responsibilities, and their reconciliation by federal transfers.

Part of our purpose is to make an initial attempt at developing more formally the concept of a vertical fiscal imbalance. There are some notions of vertical fiscal imbalance in the literature. Hettich and Winer (1986) develop a public choice model to determine the allocation of society's resources among the federal and regional governments and the private sector, and define as a fiscal imbalance the deviation of that allocation from an ideal one taken to be the Lindahl equilibrium allocation. Our approach will be more normative in nature and will attempt to develop a formal notion of vertical imbalance that does not rely on voting or other public choice mechanisms to determine resource allocations. More related to our analysis is the definition of vertical and horizontal fiscal imbalance in revenue-raising by Dahlby and Wilson (1994) as a deviation from a situation in which the marginal cost of public funds is equalized across both regions and levels of government. A similar condition will emerge from our analysis, but the source of the deviations will be explicitly modeled, and we shall be concerned with both optimality in revenue-raising by level and region of government and optimality in public spending.

As these papers recognize, any notion of vertical imbalance must use as a benchmark

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[1] In the Canadian context, a synthesis and evaluation of the recent debate on the existence of a vertical fiscal imbalance may be found in Lazar, St-Hilaire and Tremblay (2004). The most forceful argument for the existence of an imbalance may be found in Commission on Fiscal Imbalance (2001). Bird and Tarasov (2004) computed different indicators of VFI for eight OECD federations—Australia, Austria, Belgium, Canada, Germany, Spain, Switzerland and the USA—based on static notions of budget balance for each order of government. The notion of a vertical fiscal imbalance in the sense in which we define it may be found in Hunter (1974).

a situation in which vertical fiscal relations are in balance. The benchmark we use differs somewhat from the recent literature on vertical fiscal externalities and optimal vertical transfers (e.g., Boadway and Keen 1996, Dahlby 1996, Boadway et al 1998, Sato 2000). In this literature, the allocation of spending responsibilities is taken as pre-determined, and the issue is how should revenue-raising and federal-regional transfers be designed so as to achieve a second-best optimum in a decentralized setting, given that taxes are distortionary. In simple models, the second-best optimum can be achieved by an appropriate choice of revenue-raising assignment and transfers, and the issue of imbalance does not arise. In our model, the notion of imbalance is related to the inability to achieve a second-best optimum in a decentralized federation, and the distinction between the vertical fiscal gap and vertical fiscal imbalance (VFI) reflects that inability. Specifically, the vertical fiscal gap is taken to be the optimal level of transfers when the second best is achieved by a hypothetical central planner, or equivalently a unitary national government that can take coordinated decisions for both levels of government. A vertical fiscal imbalance (VFI) is then defined as any deviation—positive or negative—from the optimal vertical fiscal gap. These deviations will occur in a decentralized setting because of the fact that regional governments emit fiscal externalities on one another through well-known vertical fiscal externalities (Keen 1998) and are unable to coordinate their decisions. Moreover, the federal government will be unable to completely offset these fiscal externalities because of constraints we impose on its instruments. The existence of a VFI will be an optimal response of the federal government to this coordination failure between regional governments, and will be efficiency-enhancing. But second-best efficiency will not be achieved.

Our model has several features that are introduced in order to highlight the possibility of a VFI. The key one, meant to reflect the source of VFI problems allegedly imposed by federal governments on regional governments, is the fact that the fiscal capacity of the regions—taken to be two in number for simplicity—and the nation as a whole depend on shocks to economic fundamentals that do not occur until after governments have committed to at least some fiscal decisions. The shocks are of given magnitude and can be positive or negative. They hit the regions independently with the result that from a national point of view, the shocks can be symmetric in nature, either positive or negative, or asymmetric in the sense that one region faces a positive shock and the other a negative one. Regional governments have to make expenditure decisions before the shocks are revealed, and cannot change them afterwards. Taxes and transfers can, however, be changed after shocks are revealed, and given the predetermined level of spending that has to be financed, some combination of regional and federal taxes must be changed ex post to ensure provincial budgets are balanced. The possibility and nature of a VFI then depends on the decision-making constraints we impose on the federal government, which determine the relative size of federal and regional taxes and the amount of transfers that are implemented ex post, and the level of public spending that is chosen by the regional governments ex ante. (For simplicity, public goods provision is all at the regional level since national public goods add little of interest to our analysis.) Two key constraints are imposed on federal fiscal policies. The first is that federal transfers to any region must be non-negative, a constraint that reflects the reality of decentralized federations. The second is that the federal tax system is uniform across the nation, while regional taxes can be region-specific. Indeed, one of the reasons for decentralization, emphasized for example by Oates (1972), is to allow fiscal policies to be differentiated among regions.

The benchmark against which the VFI is defined is the unitary nation in which a central government makes decisions on behalf of the regions and the nation as a whole, but is otherwise unable to achieve a first-best because of the fact that taxes are distortionary. This unitary nation outcome can be decentralized in a federation in which the federal and regional governments behave cooperatively, but it requires a set of transfers. In fact, the optimal level of transfers will be indeterminate in this case: the division of the total tax burden between federal and regional governments is irrelevant since changes in the division can be offset by changes in transfers. We therefore adopt the convention of defining the optimal vertical fiscal gap as the minimal non-negative transfers needed to decentralize the unitary state outcome in a cooperative federation. It turns out to be the case that transfers are only needed when asymmetric outcomes occur, and then only to the region suffering the negative shock. This benchmark outcome could be achieved in a decentralized non-cooperative setting if there are no restrictions on federal policy instruments, and if the federal government can commit to the ex post taxes and transfers before the regions make their fiscal choices. However, this turns out to require that the federal government impose negative transfers in some states of nature. Once we rule that out, the benchmark optimum cannot be decentralized, and a VFI will emerge. We study the nature of that VFI under different assumptions about the ability of the federal government to commit. We then extend the analysis to the case of two periods so as to allow for the possibility of deficit financing in response to economic shocks.

A number of general results emerge from the analysis. As argued by Dahlby and Wilson (1994), in our setting where interregional equity is not a concern, we find that in the social optimum the marginal cost of public funds should be equalized across the two regions for any given state of nature (but not across states of nature). The optimal ex ante choice of regional public spending should be to equate marginal benefits from regional public goods with the expected marginal cost of public funds. In a decentralized non-cooperative federation, it will no longer be optimal to equalize the marginal cost of public funds over regions under asymmetric shocks: regions hit with negative shocks should end up with a higher marginal cost of public funds than the region that obtains a positive shock. If the federal government can commit to fiscal policies, this will be the outcome. Marginal costs will not be equalized and the VFI will be negative: transfers will be lower than in the social optimum. However, if the federal government cannot commit, marginal costs of public funds will be equalized, regions will overspend, and the VFI will be positive. In the extreme case of no commitment, there will be a soft budget constraint. Similar results will carry over to the two-period case.

We proceed by first outlining the basic one-period model and deriving the cooperative, or second-best, outcome. Next, the possibility of a VFI is considered when the federal government can fully commit to its fiscal policies. Then, we turn to the case in which the federal government cannot commit to any policies ex ante. Finally, we extend the analysis to the two-period case.

2. The Basic One-Period Model

The federation we consider is a very simple one. It consists of a federal government and two ex ante identical regional governments. Since both regions behave identically, it is convenient to consider one of them as the representative region for purposes of analysis. Variables for the second, or 'other', region will be denoted by bars when necessary. Each region is populated by the same number of identical and immobile households, which we normalize to one per region for simplicity. The level of production in the representative region consists of two parts: a deterministic component y chosen by the resident household, and an exogenous stochastic component z. Total production $y+z$ accrues to the household, and serves as the tax base that is used by the federal and regional governments alike. In the other region, the analogous production components are $\overline{y}$ and $\overline{z}$.

The stochastic component z takes a very simple form. With probability π, $z = \varepsilon > 0$, while with probability $1-\pi$, $z = -\varepsilon < 0$. The same stochastic structure applies in the other region: $\overline{z} = \{\varepsilon, -\varepsilon\}$. Thus, a region's tax base can be subject to a positive or a negative shock of equal size, whose expected value can be positive or negative depending on the value of π. The shocks are independently distributed across regions, so four possible states of nature can occur for the federation as a whole: $(z, \overline{z}) = \{(\varepsilon, \varepsilon), (-\varepsilon, -\varepsilon), (\varepsilon, -\varepsilon), (-\varepsilon, \varepsilon)\}$. These states of nature will be denoted by the superscript $k = \{hh, \ell\ell, h\ell, \ell h\}$, with associated probabilities $p^k = \{\pi\pi, (1-\pi)(1-\pi), \pi(1-\pi), (1-\pi)\pi\}$, where $\sum_k p^k = 1$. The first two of these will be referred to as symmetric shocks, and the last two asymmetric shocks. This distinction will be important in what follows.[2] Most of the variables in the model will generally vary with the state of nature k, and in what follows this will be indicated either using the superscript k, or using as a superscript the particular state (e.g., $hh, h\ell$, etc.) as appropriate. When the latter is used, the first component refers to the state in the representative region and the second to the state in the other region: for example, $S^{\ell h}$ refers to the federal transfer to the representative region when it receives a bad shock and the other region receives a good one, while $\overline{S}^{\ell h}$ is the transfer to the other region in the

[2] While one might suppose that regions can insure against these shocks, insurance will serve no purpose in our model given our assumption below that households are risk neutral.

same case.

Production in the representative region in each state of nature k, $y^k + z^k$, can be used for household consumption c^k and for the provision of regional public goods g, which as we shall see below is the same for all states of nature. (For simplicity we assume that there is no federal public good.) The regional public good is financed by a regional tax and by a transfer from the federal government. In particular, the representative region levies a proportional tax at the rate t^k on domestic production in state k, and receives a transfer S^k from the federal government. We shall restrict the federal transfer to be non-negative, which will turn out to be important in what follows. Analogous variables $\overline{g}, \overline{t}^k$ and $\overline{S}^k$ apply in the other region. The federal-regional transfers are financed by a federal proportional tax at the rate T^k imposed uniformly on production in both regions. Thus, the federal tax base is identical to regional tax bases, which gives rise to well-known vertical fiscal externalities.[3]

An important property of these fiscal instruments concerns the extent to which they can be adjusted. We assume that once the levels of regional public goods g and $\overline{g}$ are chosen, they cannot be changed. On the other hand, taxes $t^k, \overline{t}^k$ and T^k and transfers S^k and $\overline{S}^k$ can be adjusted instantaneously, including after the state of nature is revealed. This extreme characterization is for simplicity in our analysis. We could have allowed changes in regional public goods with some adjustment costs or imposed adjustment costs on taxes and transfers. The main requirement for our analysis is that public goods be less adjustible than taxes and transfers.

Households in the representative region have a quasilinear additive utility function:

$$u(c^k, g) = c^k - h(y^k) + b(g) \qquad \forall\ k \qquad (1)$$

where $h'(y^k), h''(y^k) > 0$, $b'(g) > 0 > b''(g)$. Two important properties of this utility

[3] The concept of a vertical fiscal externality was first recognized by Johnson (1988), and its implications for the vertical fiscal gap studied by Boadway and Keen (1996) and Dahlby (1996). See also Keen (1998) and Dahlby and Wilson (2003). The interaction between vertical fiscal externalities and horizontal fiscal externalities (tax competition) and the consequences for federal-regional transfers have been analyzed by Smart (1998), Keen and Kotsogiannis (2002), Köthenbürger (2004) and Bucovetsky and Smart (2004).

function should be noted. The first, already mentioned, is that there is no risk aversion so there is no insurance motive in this model.[4] Second, the disutility of supplying output depends only on the deterministic component y^k over which households have some discretion. It is not affected by the exogenous shock z^k. The household budget constraint is, however, affected by the shock, and is given by:

$$c^k = (1 - t^k - T^k)(y^k + z^k) \qquad \forall\ k \qquad (2)$$

Identical analogs of (1) and (2) apply to the other region.

Both the federal government and the regional governments are benevolent. The latter maximize the utility of the representative resident of their region, or the expected utility in the event that regional policies are chosen ex ante. The federal government maximizes the sum of utilities of the residents of the two regions, or the expected sum of utilities if policies are chosen ex ante. This implies, given the quasilinear form of utility with its constant marginal utility of income, that any redistribution of income has no effect on social welfare, meaning that our analysis can be interpreted solely as an efficiency analysis. The budget constraints of the federal government and the regions are, respectively:

$$S^k + \overline{S}^k = T^k \cdot (y^k + z^k + \overline{y}^k + \overline{z}^k) \qquad \forall\ k \qquad (3)$$

$$g = t^k \cdot (y^k + z^k) + S^k, \qquad \overline{g} = \overline{t}^k \cdot (\overline{y}^k + \overline{z}^k) + \overline{S}^k \qquad \forall\ k \qquad (4)$$

To complete our description of the basic one-period model, the timing of events is as follows:[5]

Timing of Events

Stage 1: The federal government announces state-contingent transfers $(S^k, \overline{S}^k)$ and tax rates T^k, anticipating the behavior of regional governments and households.

[4] The use of federal-regional transfers as instruments for insuring regions against adverse shocks has been analysed in Persson and Tabellini (1996), Lockwood (1999) and Bordignon, Manasse and Tabellini (2001).

[5] In fact, Stages 3 and 4 are interchangeable since the federal government could choose state-contingent policies either before or after shocks are revealed without affecting the results.

Stage 2: Regional governments simultaneously choose their public good provision $(g, \overline{g})$. Since these cannot be adjusted, they are the same for all states of nature.

Stage 3: Shocks $(z^k, \overline{z}^k)$ are revealed.

Stage 4: Depending on its ability to commit to the policy announced ex ante, the federal government may or may not change its transfers $(S^k, \overline{S}^k)$ and tax rates T^k.

Stage 5: Regions let $(t^k, \overline{t}^k)$ balance their budgets.

Stage 6: Households in each region make their production decisions $(y^k, \overline{y}^k)$.

In what follows, we consider the allocations achieved under alternative assumptions concerning the ability of the federal government to commit as well as alternative assumptions about the degree of cooperation among governments. In each case, allocations will be subgame perfect equilibria so our analysis proceeds by backward induction. Since the same characterization for household behavior in Stage 6 applies in all scenarios, it is useful to present that at the outset.

Household Behavior

By the time the household in the representative region chooses the level of output, all fiscal parameters (t^k, T^k, g) have been chosen and the shocks z^k have been revealed. Note that the federal and regional tax rates will depend on the state of nature k for the nation as a whole. Given the additivity of the utility function (1), we can suppress $b(g)$ from the household's problem.

Using (1) and (2), the household's problem in state of nature k is:

$$\max_{y^k} \quad (1 - t^k - T^k)(y^k + z^k) - h(y^k) \tag{5}$$

The first-order condition is $h'(y^k) = 1 - t^k - T^k$, with the second-order condition $h''(y^k) > 0$, which we assume to be satisfied everywhere. The solution to this problem is the output supply function $y^k(1 - t^k - T)$, with

$$y^{k\prime}(1 - t^k - T) = \frac{1}{h''(y^k)} > 0 \tag{6}$$

For future reference, note that:[6]

$$y^{k\prime\prime}(1-t^k-T) = -\frac{h'''(y^k)}{(h''(y^k))^2} \gtreqless 0 \tag{7}$$

Note also that y^k is not affected by the shock itself, a simplification that is due to our quasilinear utility function. This implies from the household budget constraint (2) that $\partial c^k/\partial z^k = (1-t^k-T^k)$.

The value function for the household problem is the indirect utility of consumption function $v(t^k+T^k, z^k)$. Define $v_t^k \equiv \partial v(\cdot)/\partial t^k$, and similarly for v_T^k and v_z^k. Then, by the envelope theorem, $v(t^k+T^k, z^k)$ has the following properties:

$$v_t^k = v_T^k = -(y^k+z^k), \qquad v_z^k = (1-t^k-T^k) \tag{8}$$

Analogous results apply for the other region.

3. The Second-Best Optimum in the One-Period Model

A useful benchmark case is the second-best optimum in which ressources are allocated efficiently, subject to the need to use distortionary taxation to finance regional public goods. A convenient way to characterize the second-best optimum is to imagine that there is a unitary national government that makes decisions on behalf of both regions subject to a single budget constraint. We begin with that case before considering how it might be decentralized in a federal setting.

The Unitary Nation Optimum

Let τ^k and $\overline{\tau}^k$ be the tax rates applied in the two regions in state of nature k. Then, the national budget constraint in state k may be written:

$$g+\overline{g} = \tau^k \cdot (y^k+z^k) + \overline{\tau}^k \cdot (\overline{y}^k+\overline{z}^k) \qquad \forall\ k \tag{9}$$

[6] In the constant elasticity case with $h(y) = y^{1+1/\sigma}/(1+1/\sigma)$, the elasticity of $y(1-t-T)$ is $(1-t-T)y'(\cdot)/y(\cdot) = h'/(h''y) = \sigma$. Then,

$$y''(1-t-T) = -\frac{h'''}{(h'')^2} = -\left(\frac{1}{\sigma}-1\right)\frac{y^{1/\sigma-2}}{(h'')^2\sigma} \gtreqless 0 \quad \text{as} \quad \sigma \gtreqless 1$$

The unitary national government maximizes the sum of utilities nationwide subject to budget constraint (9). The Lagrangian expression is:

$$\mathcal{L}(\tau^k, \overline{\tau}^k, g, \overline{g}, \Lambda^k) =$$

$$\sum_k p^k \left[v(\tau^k, z^k) + b(g) + v(\overline{\tau}^k, \overline{z}^k) + b(\overline{g}) + \Lambda^k (\tau^k \cdot (y^k + z^k) + \overline{\tau}^k \cdot (\overline{y}^k + \overline{z}^k) - g - \overline{g}) \right]$$

where outputs $(y^k, \overline{y}^k)$ are functions of the relevant tax rates. Using the envelope theorem (8), the first-order conditions simplify to:

$$b'(g) = b'(\overline{g}) = \sum_k p^k \Lambda^k \tag{10}$$

$$\Lambda^k = \frac{y^k + z^k}{y^k + z^k - \tau^k y^{k\prime}} = \frac{\overline{y}^k + \overline{z}^k}{\overline{y}^k + \overline{z}^k - \tau^k \overline{y}^{k\prime}} \; > \; 1 \qquad \forall\, k \tag{11}$$

These conditions have a straightforward interpretation. Equation (11) states that in each state of nature, the *marginal cost of public funds* (MCPF) is the same in both regions ($\text{MCPF}^k = \overline{\text{MCPF}}^k$), where MCPF^k is defined for the representative region as follows:

$$\text{MCPF}^k \equiv \frac{y^k + z^k}{y^k + z^k - \tau^k y^{k\prime}} = \left[1 - \frac{\tau^k y^{k\prime}}{y^k + z^k} \right]^{-1} \qquad \forall\, k \tag{12}$$

and similarly for the other region. This is analogous to a conventional MCPF expression, modified to take into account shocks to tax bases. Of course, although the MCPF is equalized across regions in each state, it is necessarily different in the four states.[7] Equation (10) states that the optimal level of public goods is identical in the two regions ($g = \overline{g}$), which is not surprising given that the regions are ex ante identical. Moreover, the level is such that the marginal benefit (which is identical in all states of nature) equals the expected MCPF.

Optimal national tax policy will depend on how Λ^k varies with the total tax rate τ^k. One can readily verify that whether Λ^k varies positively or negatively with τ^k depends on

[7] An analogous result has been suggested by Dahlby and Wilson (1994) in a deterministic setting, although Sato (2000) shows that when equity as well as efficiency are policy objectives, equality of MCPFs across regions no longer applies.

the sign of $y^{k\prime\prime}$, which is ambiguous as we have noted above. In what follows, we shall restrict attention to the case in which Λ^k varies positively with the tax rate, since that is the more likely case.

In the cases with symmetric shocks ($k = hh, \ell\ell$ and $z^k = \overline{z}^k$) both regions are identical ex ante and ex post, so a symmetric equilibrium will occur. In this case, $y^k = \overline{y}^k$ and $\tau^k = \overline{\tau}^k$, which immediately leads to $\text{MCPF}^k = \overline{\text{MCPF}}^k$. Given that the same revenue must be raised in both states of nature, the aggregate tax rate will be higher when the symmetric shock is negative, $k = \ell\ell$, than when it is positive, $k = hh$. And, the MCPF will be higher in the case where the shock is negative.

With asymmetric shocks, $k = h\ell, \ell h$ and $z^k = -\overline{z}^k$. Regional tax rates τ^k and $\overline{\tau}^k$ must be chosen such that the MCPF is equalized across regions. It is apparent that the tax rate must be higher in the region with the positive shock. To see this, imagine starting with equal tax rates. This will imply that the deterministic component of output will be the same in both regions. But, since z^k is higher in the region with the positive shock, its MCPF will be lower by the definition of MCPF in (12). Therefore, the tax must be raised in the region with the positive shock and lowered in the other region. This implies that more revenue will be raised in the region with the positive shock that in the region with the negative shock. Since the level of spending is the same in both regions, there is an implicit transfer from the former to the latter region.

Decentralizing the Second-Best Optimum: The Cooperative Outcome

As a convenient way of introducing decentralized decision-making by regional governments into the benchmark model, we begin with the case where there is full cooperation between federal and regional governments so that the second-best allocation can be achieved. This will enable us to define the optimal vertical fiscal gap in our model. In a decentralized setting, as we have mentioned, the federal government imposes a uniform state-contingent tax at the rate T^k in both regions and provides non-negative transfers $(S^k, \overline{S}^k)$ to the two regions. For their part, the regions impose state-contingent taxes $(t^k, \overline{t}^k)$ and supply public goods $(g, \overline{g})$ to their respective residents. Budget constraints (3) and (4) apply in each state of nature.

The features of optimal policy can readily be outlined without resorting to formal analysis. The aggregate tax rates in the two regions will replicate the efficient tax rates derived above: $t^k + T^k = \tau^k$ and $\overline{t}^k + T^k = \overline{\tau}^k$. This will ensure that the MCPF is equalized between the two regions in each state. At the same time, transfers in each state $(S^k, \overline{S}^k)$ must be such that each region has sufficient funds to finance the optimal level of regional public goods $(g, \overline{g})$. It is apparent that in asymmetric-shock states, the transfer must be higher in the region facing the negative shock, since as discussed above there must be a transfer from the region with the positive shock to the one with the negative shock. In the case of symmetric shocks, transfers can be the same to the two regions.

In the optimum, the level of transfers $(S^k, \overline{S}^k)$ is indeterminate: an increase in the federal tax rate T^k accompanied by an increase in transfers to both regions and a reduction in both regions' tax rates will leave the allocation of resources unaffected. In other words, the vertical fiscal gap needed to support the second-best outcome will be indeterminate when policies are chosen cooperatively. To resolve this indeterminancy, and to make the notion of an optimal vertical fiscal gap well-defined, we assume that the federal government will always opt for the smallest non-negative transfers possible. Given that assumption, the cooperative second-best optimal policies will consist of the following. In states of nature with symmetric shocks, federal taxes and transfers are both zero: $S^k = \overline{S}^k = T^k = 0$ for $k = hh, \ell\ell$. In states of nature with asymmetric shocks, the federal tax rate is positive, the transfer to the region facing the negative shock is positive, and that to the other zero. The magnitude of the transfer is sufficient to equalize the MCPF across regions. So, for example, in state $k = h\ell$, $T^{h\ell} > 0, S^{h\ell} = 0$ and $\overline{S}^{h\ell} > 0$.

We can think of the optimal *vertical fiscal gap* (VFG) as being zero under symmetric shocks and positive for the region facing a negative shock when asymmetric shocks occur. This will serve as our benchmark in the decentralized non-cooperative cases to follow. More generally, if regions were ex ante heterogeneous, there would be a need for differential transfers even under symmetric shocks: the region with the lowest production opportunities to begin with would obtain a positive transfer under symmetric shocks. If there were more than two regions and many different sizes of possible shocks, there would be a positive VFG for a subset of regions in most states of nature. Thus, our finding that there is a VFG for only one region and only if there is an asymmetric shock is not as restrictive as it appears. However, for illustrative purpose we retain our simple model.

To summarize this section, let $(g^*, \overline{g}^*, T^{k^*}, t^{k^*}, \overline{t}^{k^*}, S^{k^*}, \overline{S}^{k^*})$ denote the second-best optimal policies in a decentralized setting, resulting in optimal marginal costs of public funds, $\text{MCPF}^{k^*}, \overline{\text{MCPF}}^{k^*}$. Then, the features of the second-best optimum are as follows:

Proposition 1: Assuming the smallest non-negative transfers are used, the decentralized second-best optimum has the following characteristics:

i. Regional public goods are chosen so that the marginal benefit equals the expected MCPF, and are identical in the two regions ($g^* = \overline{g}^*$).

ii. With symmetric shocks ($k = hh, \ell\ell$), regions are identical ex post. Federal taxes and transfers are zero ($T^{k^*} = S^{k^*} = \overline{S}^{k^*} = 0$). Regional tax rates ($t^{k^*}, \overline{t}^{k^*}$) and therefore MCPFs will be lower for $k = hh$ than for $k = \ell\ell$.

iii. With asymmetric shocks ($k = h\ell, \ell h$), the equilibrium is asymmetric. The optimal transfer, or VFG, will be positive for the region with the negative shock ($S^{\ell h^*}, \overline{S}^{h\ell^*} > 0$), and zero for the region with the positive shock ($S^{h\ell^*} = \overline{S}^{\ell h^*} = 0$), so the federal tax rate will be positive ($T^{k^*} > 0$). The MCPF will be equalized between regions ($\text{MCPF}^{k^*} = \overline{\text{MCPF}}^{k^*}$).

iv. The relation between aggregate tax rates and MCPFs for the representative region in different states satisfies:

$$\text{MCPF}^{hh^*} < \text{MCPF}^{h\ell^*} = \text{MCPF}^{\ell h^*} < \text{MCPF}^{\ell\ell^*}$$

$$(t^{hh^*} + T^{hh^*}) < (t^{h\ell^*} + T^{h\ell^*}) = (t^{\ell h^*} + T^{\ell h^*}) < (t^{\ell\ell^*} + T^{\ell\ell^*})$$

Analogous expressions apply for the other region.

4. Non-Cooperative Equilibrium under Full Commitment

In the cooperative outcome, regions endogenize any inter-jurisdictional externalities that arise from decentralized decision-making. When governments act non-cooperatively, that will no longer be the case. Given our assumption that tax bases are not mobile between regions, potential externalities are vertical ones between regional and federal governments. The nature of these externalities and their consequence for federal and regional policies will become clear by studying equilibrium outcomes with non-cooperative decision-making by governments. Federal and region choices can be made in different orders, depending on the ability of governments to commit to announced decisions. In our model, the only independent decision made by regions is the choice of their spending levels (regional taxes simply balance ex post budgets). Since regional spending choices must be made before the state of nature is revealed and cannot be revised, only the federal government's ability to commit is relevant. We begin with the case where the federal government can commit to policies announced in Stage 1 before regional spending decisions are made. Later, we consider the opposite case where commitment is not possible, that is, where federal decisions are made after regional ones.[8]

Under full commitment, the federal government announces its policies ex ante, anticipating the reaction of the regional governments, and does not adjust its announced policies once the shocks are revealed. In fact, there might be a limit to the policies to which the federal government can commit. Since regional tax rates—chosen after the federal government has announced its policies—will have an effect on the federal budget, this restricts the number of policies that the federal government will be able to commit to with credibility. Three options are possible in our simple model. If the regional governments

[8] An intermediate possibility is that the federal government and the regions make their decisions simultaneously, acting as Nash competitors. Although this is conceivable, it is typically assumed that because it is one big government acting against several smaller ones, the federal government has some first-mover advantage. In any case, the results for the Nash case would be between those obtained for the two cases we consider. Hayashi and Boadway (2001) estimated tax interaction effects for business income taxes in Canada. The presence of vertical fiscal interactions between federal and provincial tax rates was significant and robust to different specifications, but testing for Stackelberg versus Nash behavior was inconclusive.

recognize the effects their policies will have on the federal budget, the federal government can commit to either one of their two state-contingent policies, tax rates (T^k) or transfers ($S^k, \overline{S}^k$), the other being determined ex post by federal budget balance. On the other hand, if the regions are myopic and simply take announced federal policies as given, the federal government can commit to both tax rates and transfers, provided it selects them so that its budget is balanced ex post in every state of nature. It turns out that the qualitative results for each of these three cases are identical, and the same method of analysis can be used in this case. We therefore illustrate the results by studying one of the cases, that in which the federal government can commit to state-contingent tax rates.

The federal government announces its state-contingent tax rates T^k, anticipating the behavior of the regional governments and the households in the two regions and the consequences of these for the federal budget constraint. We proceed by analyzing the regions' behavior first, and then use that to consider the federal choice of T^k.

The Regional Governments' Ex Ante Spending Decisions

The two regions act simultaneously. Since they are ex ante identical, both will choose the same level of spending so we can concentrate on the problem of the representative region. The regional government chooses its level of provision of the public good g taking the federal tax rate in each state of nature T^k and the choice of policies by the other region as given.[9] It anticipates the behavior of households and the effect of its policies on federal transfers once the state of nature is revealed. In particular, given T^k and $\overline{t}^k$, the federal budget (3) can be used to determine how federal transfers S^k vary with the regional tax rate t^k.

To simplify the problem, recall that in the second-best optimum, transfers are only paid to the region suffering the negative shock in the asymmetric outcome. That turns out

[9] In fact, policies in the other region will be affected indirectly by the representative region's choice of g. A change in g may cause $\overline{S}^k$ to change, which would affect the other region's budget. If $\overline{g}$ is taken as given, $\overline{t}^k$ would adjust in response to changes in g, and this in turn will affect the federal budget. In our analysis, we ignore this complication by assuming that each region takes all policies of the other region—both spending and taxes—as given. This simplification does not affect the qualitative nature of our results.

also to be true in this non-cooperative case. The intuition for that will become clear, but basically the reason is that the federal government will always want to minimize its tax rate to reduce the size of the vertical fiscal externality that arises in the non-cooperative case. That being the case, it will want to set a zero tax rate in the symmetric-shock cases, and a tax rate just sufficient to transfer the desired amount of funds to the negative-shock region in the asymmetric-shock case, with zero transfers to the region receiving the positive shock. From the point of view of the representative region, the only relevant transfer is therefore $S^{\ell h}$, and that is determined by the following federal budget constraint, obtained from (3) with $\overline{S}^{\ell h} = 0$:

$$S^{\ell h} = T^{\ell h} \cdot (y^{\ell h} - \varepsilon + \overline{y}^{\ell h} + \varepsilon) = T^{\ell h} \cdot (y^{\ell h} + \overline{y}^{\ell h}) \tag{13}$$

Differentiating with respect to $t^{\ell h}$ and $T^{\ell h}$, we obtain:

$$\frac{\partial S^{\ell h}}{\partial t^{\ell h}} = -T^{\ell h} y^{\ell h\prime} < 0, \qquad \frac{\partial S^{\ell h}}{\partial T^{\ell h}} = y^{\ell h} + \overline{y}^{\ell h} - T^{\ell h}(y^{\ell h\prime} + \overline{y}^{\ell h\prime}) > 0 \tag{14}$$

where the latter inequality presumes that an increase in the federal tax rate increases federal tax revenues (i.e., that we are on the rising side of the Laffer curve), which is reasonable in the optimum. Thus, an increase in the representative region's tax rate in state ℓh will reduce federal tax revenues and therefore the transfer received by the region, while an increase in the federal tax rate will increase transfers received by the poor region, which is intuitive.

The ex ante problem of the representative region is to choose g to maximize the expected utility of its representative resident, $\sum_k p^k[v^k(\cdot) + b(g)]$, anticipating the effect its choice will have on the ex post values of $S^{\ell h}$ via (14) and on its own tax rates t^k in all states of nature k. A convenient way to take anticipations of the latter into account is to use t^k as artificial control variables by adding as constraints the region's budget constraints in each state k, given by (4). Regional tax rates t^k can be treated as control variables ex ante, since the federal government can commit to its announced tax rate. The Lagrangian expression for the representative region is:

$$\mathcal{L}(g, t^k, \lambda^k) = \sum_k p^k \left[v(t^k + T^k, z^k) + b(g) + \lambda^k \left(t^k \cdot (y^k + z^k) + S^k - g \right) \right] \tag{15}$$

where $S^k = 0$ for $k \neq \ell h$, $T^{hh} = T^{\ell\ell} = 0$ and $S^{\ell h}$ satisfies (14). From the first-order conditions on g and t^k and using (14), we obtain:

$$b'(g) = \sum_k p^k \lambda^k \tag{16}$$

$$\lambda^k = \frac{y^k + z^k}{y^k + z^k - t^k y^{k\prime}} \qquad k \neq \ell h \tag{17.1}$$

$$\lambda^{\ell h} = \frac{y^{\ell h} + z^{\ell h}}{y^{\ell h} + z^{\ell h} - (t^{\ell h} + T^{\ell h}) y^{\ell h\prime}} \tag{17.2}$$

Analogous results apply for the other region, with $\overline{\lambda}^k$ being the multiplier. Equation (17.1) reflects a vertical fiscal externality that affects the incentives the regional government faces. Comparing (17.1) with (11), we see that the regional government misperceives its MCPF whenever $T^k \neq 0$. Given our stochastic setup, the representative region underestimates its true MCPF in state $h\ell$, and as a result has an incentive to oversupply g. This is analogous to the well-known vertical fiscal externality discussed in Boadway and Keen (1996), Dahlby (1996) and Keen (1998). A regional government acting non-cooperatively neglects the fact that when it increases its own tax rate, it reduces the tax revenues raised by the federal government: part of the cost of regional tax rate increases are effectively borne by taxpayers in the other region.

The solution to regional problem (15) yields spending g and state-contingent tax rates t^k that depend on the tax rates T^k committed to by the federal government. The maximum value function for the region's problem will be denoted $w(T)$, where T denotes the vector of state-contingent federal tax rates and

$$w(T) = \max_{\{g, t^k\}} \left\{ \sum_k p^k \left(v(t^k + T^k, z^k) + b(g) \right) \text{ s.t. } t^k \cdot (y^k + z^k) + S^k = g, \ \forall\, k \right\}$$

The envelope theorem implies, using (14) and the first-order conditions from problem (15):

$$\frac{\partial w(T)}{\partial T^k} = -p^k \lambda^k (y^k + z^k) \qquad \forall\, k \neq \ell h \tag{18.1}$$

$$\frac{\partial w(T)}{\partial T^{\ell h}} = p^{\ell h} \lambda^{\ell h} \left(\overline{y}^{\ell h} + \overline{z}^{\ell h} - T^{\ell h} \overline{y}^{\ell h\prime} \right) \tag{18.2}$$

A similar problem applies for the other region. Since the regions are ex ante identical, it yields $g = \overline{g}$. The value function is $\overline{w}(T)$, and it has analogous properties for $w(T)$, though with $\overline{S}^{h\ell} > 0$ and transfers in all other states zero.

The Federal Government's Ex Ante Problem

If there were no restrictions on federal policies T^k, S^k and $\overline{S}^k$, it is straightforward to see that the federal government could induce the cooperative optimum. This requires that the levels of g and $\overline{g}$ satisfy (10), and that $\text{MCPF}^k = \overline{\text{MCPF}}^k$ for all k, as stated in (11). To achieve the latter in asymmetric-shock states, it is necessary that $T^{h\ell}, T^{\ell h} > 0$, as we have seen. This implies that there will be a vertical fiscal externality causing regional MCPFs to be lower than the social optimum in those states. Therefore, for g to be optimal, MCPF must be higher than socially optimal in symmetric-shock states to ensure that the expected MCPF over all four states equals the socially optimal expected MCPF. This, in turn, requires that $T^k < 0$ and $S^k < 0$ in those states. If it were permissible to impose negative transfers on the regions, the cooperative level of $g^* = \overline{g}^*$ could be replicated. Moreover, the federal government could set its transfers under asymmetric shocks such that $\text{MCPF}^{k^*} = \overline{\text{MCPF}}^{k^*}$, and we would get the full cooperative optimum.

However, the cooperative outcome can only be achieved if the federal government can impose negative transfers on the regions, an option that is difficult to enforce in a federation with autonomous regional governments. As mentioned, we rule this out by assuming that $S^k \geqslant 0$, for all k. The best the federal government can do is to announce zero taxes—and therefore zero transfers—under symmetric shocks: $T^{hh} = S^{hh} = T^{\ell\ell} = S^{\ell\ell} = 0$. We can therefore restrict attention to the choice of federal policies in the cases of asymmetric shocks: $k = \{h\ell, \ell h\}$.[10] Moreover, as discussed above, we know that the transfer to the region with the positive shock will be zero ($S^{h\ell} = \overline{S}^{\ell h} = 0$). The federal government would like to impose a negative transfer in these cases but is constrained from doing so.

The federal government's problem then consists simply of choosing $T^{h\ell}$ and $T^{\ell h}$ to

[10] Formally, we could impose the restriction $T^k \geqslant 0$ on the federal problem and let the federal government choose T^k. However, to avoid unnecessary complication, we simply take $T^k = 0$ for $k = hh, \ell\ell$ at the outset since we know that the constraint will be binding for those states.

maximize the sum of regional utilities as given by the maximum value functions from problem (15):

$$\max_{\{T^{h\ell},T^{\ell h}\}} \quad w(T) + \overline{w}(T)$$

Using the envelope theorem results (18.1) and (18.2) for the two regions, the first-order conditions for state ℓh are:

$$\frac{\partial w(T)}{\partial T^{\ell h}} + \frac{\partial \overline{w}(T)}{\partial T^{\ell h}} = p^{\ell h}\lambda^{\ell h}\left(\overline{y}^{\ell h} + \overline{z}^{\ell h} - T^{\ell h}\overline{y}^{\ell h\prime}\right) - p^{\ell h}\overline{\lambda}^{\ell h}\left(\overline{y}^{\ell h} + \overline{z}^{\ell h}\right) = 0$$

or,

$$\frac{\lambda^{\ell h}}{\overline{\lambda}^{\ell h}} = \frac{\overline{y}^{\ell h} + \overline{z}^{\ell h}}{\overline{y}^{\ell h} + \overline{z}^{\ell h} - T^{\ell h}\overline{y}^{\ell h\prime}} \ > \ 1 \tag{19}$$

An analogous expression applies for state $h\ell$ except that in that case $\lambda^{h\ell} < \overline{\lambda}^{h\ell}$. The implication is that the federal government chooses a transfer that results in a higher MCPF in the region hit by the negative shock. This implies that the transfer when shocks are asymmetric is smaller than in the social optimum: $S^{\ell h} < S^{\ell h^*}$, $\overline{S}^{h\ell} < \overline{S}^{h\ell^*}$. That is, the VFI is negative for the case of asymmetric shocks in the sense that the federal government is transfering less than the second-best optimal amount to the regions and forcing them to raise more revenues on their own. The intuition is that the federal government would like to reduce T^k in order the reduce the vertical fiscal externality that is causing the regions to overprovide g. But, the lower the federal tax rate, the smaller will the transfer S^k be, and the more will optimal policy diverge from equalizing λ^k between the two regions, as is required in the social optimum.

We can summarize the results for the non-cooperative case with full commitment as follows.

Proposition 2: Assuming the smallest non-negative transfers are used, the non-cooperative outcome with full commitment has the following characteristics:

i. Regional public goods are chosen so that the marginal benefit equals the expected MCPF, and are identical in the two regions, but larger than the second-best optimal amount ($g = \overline{g} > g^* = \overline{g}^*$).

ii. With symmetric shocks ($k = hh, \ell\ell$), federal taxes and transfers are zero ($T^k = S^k = \overline{S}^k = 0$). Regional tax rates ($t^k, \bar{t}^k$) and therefore MCPFs will be lower for $k = hh$ than for $k = \ell\ell$, but higher than in the second best.

iii. With asymmetric shocks ($k = h\ell, \ell h$), the optimal transfer will be positive for the region with the negative shock ($S^{\ell h}, \overline{S}^{h\ell} > 0$), and zero for the region with the positive shock ($S^{h\ell} = \overline{S}^{\ell h} = 0$), so the federal tax rate will be positive ($T^k > 0$). The MCPF will be higher in the region with the negative shock ($\text{MCPF}^{\ell h} > \overline{\text{MCPF}}^{\ell h}$, $\text{MCPF}^{h\ell} < \overline{\text{MCPF}}^{h\ell}$). There will be a negative VFI: the transfer to the region with the negative shock will be lower than in the second-best optimum ($S^{\ell h} < S^{\ell h*}, \overline{S}^{h\ell} < \overline{S}^{h\ell*}$).

iv. The relation between aggregate tax rates and MCPFs for the representative region in different states satisfies:

$$\text{MCPF}^{hh} < \text{MCPF}^{h\ell} < \text{MCPF}^{\ell h} < \text{MCPF}^{\ell\ell}$$

$$(t^{hh} + T^{hh}) < (t^{h\ell} + T^{h\ell}) = (t^{\ell h} + T^{\ell h}) < (t^{\ell\ell} + T^{\ell\ell})$$

Analogous expressions apply for the other region.

5. Non-Cooperative Equilibrium without Commitment

Suppose now that the federal government cannot commit to any policies it announces before the regions choose their levels of public goods ($g, \overline{g}$).[11] At the same time, once chosen, g and $\overline{g}$ cannot be changed, even though the source of financing is not resolved until after the state of nature is revealed. The standard approach is to suppose that the outcome is a subgame perfect equilibrium in which the regions' choice of g is based on their correct anticipation of both the federal government's choice of T^k, S^k and $\overline{S}^k$ after state of nature k is revealed and the subsequent determination of t^k by regional budget balance. However, it is interesting for heuristic purposes to consider first the case where the regions are not so forward-looking, but assume incorrectly that federal ex ante policy

[11] The consequences of the federal government not being able to commit to its transfer policies has been studied in other contexts. See, for example, Mitsui and Sato (2001), Boadway et al (2002) Köthenbürger (2004) and Wildasin (2004).

announcements will in fact be carried out. Although the outcome will not be a subgame perfect equilibrium, it will be instructive nonetheless.

Myopic Regional Governments

If regional governments are myopic while the federal government is forward-looking, the latter will recognize that whatever it announces in the first period will be taken as given by the regions. It is clear that in this case, the federal government, by fooling the regions, can implement the cooperative outcome. The argument is outlined below for the case where the federal government announces state-contingent tax rates.

The federal government announces state-contingent tax rates T^k ex ante, anticipating the effect of its announcement on regional policies (but knowing that it is not committed to carry out its announced policy ex post). The myopic representative regional governments takes the tax rates as given, and selects g to maximize expected utility, $\sum_k p^k[v^k + b(g)]$, assuming that the federal government is committed to its announced policy. The other region does the same. Ex post, the federal government re-optimizes, given g and $\overline{g}$ selected by the regions. As above, it sets its tax rates such that $\lambda^k = \overline{\lambda}^k$, for all k.

The problem of regional governments is identical to that in Section 4 where the federal government can fully commit to its tax rates. The choice of g by the representative regional government satisfies (16) for the announced tax rates T^k. Ex ante, the federal government knows that it can renege on its announcement ex post, but that regional governments take the announcement as a commitment. The federal government will exploit that in order to induce the cooperative level of provision of g from regional governments. In particular, the federal government will announce T^k to maximize $w(T^k) + \overline{w}(T^k)$, anticipating regional governments' behavior.

The tax rates announced by the federal government under an asymmetric shock $(T^{h\ell}, T^{\ell h})$ will be strictly positive but smaller than the equilibrium tax rates under full commitment derived in Section 4. To see this, note that if the federal government were to announce $T^k = 0$ for all k, there would be no vertical fiscal externality distorting the decision of regional governments, but the expected MCPF perceived by regional governments would be larger than the MCPF in the cooperative optimum, MCPF^{k^*} and $\overline{\text{MCPF}}^{k^*}$. This is necessarily the case since the federal transfers that equalize the MCPF across regions in the optimum effectively minimize the expected MCPF in each region across states of nature k. Therefore, the level of provision of the public good selected by regional governments would be smaller than g^*. Furthermore, the federal government knows that it can renege on its announcement, and therefore does not have to take into account the trade-off between inducing regions to lower their g and minimizing the efficiency cost of taxation. Thus, the tax rates announced under an asymmetric shock will be smaller than those in the full commitment case.

It is clear then that, given the tax rates announced under symmetric shocks, which need not be zero in this case, there is an announcement of tax rates under an asymmetric shock, $\overline{T}^{k^*}$, that will ensure that (16) yields $g = \overline{g} = g^*$. Given that the federal government can induce $g = \overline{g} = g^*$ ex ante, it can then select its taxes and transfers ex post to yield the full cooperative outcome $(g^*, \overline{g}^*, T^{k^*}, t^{k^*}, \overline{t}^{k^*}, S^{k^*}, \overline{S}^{k^*}, \text{MCPF}_i^{k^*}\underline{\text{MCPF}}^{k^*})$, so Proposition 1 applies. Nonetheless, the announced tax rates under an asymmetric shock and the associated transfers are lower than in the cooperative optimum, but are always increased ex post following an asymmetric shock. Therefore, although regional governments will select g^*, they anticipate the existence of a negative VFI under an asymmetric shock.

The Subgame Perfect Equilibrium Outcome

Suppose now that regional governments correctly anticipate future outcomes when choosing their levels of public goods. The federal government cannot commit to announced state-contingent tax rates and transfers ex ante, so such announcements are pointless. Even in the case of symmetric shocks, in which the optimal fiscal gap is equal to zero, the federal government cannot commit to a zero tax rate because regional governments do not take the symmetric equilibrium as given when making their policy choice. If regional governments were to choose different levels of g, the federal government would find it optimal ex post to provide transfers under symmetric shocks. Therefore, the timing of Stages 1 and 2 is essentially reversed relative to the full commitment case: regional governments move first anticipating the effect of their spending decisions on the subsequent choice of federal transfers S^k and $\overline{S}^k$, or equivalently tax rates T^k. We proceed by considering first the federal government problem and then go back to the regional ones.

The Federal Government's Problem

The federal government chooses S^k and T^k to maximize the sum of utilities, given its budget constraint (3), taking g and $\overline{g}$ as given and anticipating the values of t^k and $\overline{t}^k$ that will balance regional budgets (4) ex post. Since at this stage regional governments have no more discretion, it is clear that the outcome of this problem will be that the federal government will choose T^k—and therefore implicitly S^k and $\overline{S}^k$—such that for all states k, $\text{MCPF}^k = \overline{\text{MCPF}}^k$, or:

$$\lambda^k(t^k, T^k) \equiv \frac{y^k + z^k}{y^k + z^k - (t^k + T^k)y^{k\prime}} = \frac{\overline{y}^k + \overline{z}^k}{\overline{y}^k + \overline{z}^k - (\overline{t}^k + T^k)\overline{y}^{k\prime}} \equiv \overline{\lambda}^k(\overline{t}^k, T^k) \quad \forall\ k \quad (20)$$

The Regional Governments' Problem

The representative region chooses g to maximize the expected utility of its resident household, anticipating federal government behavior as captured by (20) and ex post regional tax rates t^k determined by regional budget constraints (4). These anticipations can be captured by treating S^k, T^k and t^k as artificial control variables for the regional governments, and adding government budgets (3) and (4) and federal behavioral condition (20) as constraints in each state. The Lagrangian expression for the region is:

$$\mathcal{L}(g, t^k, T^k, S^k, \delta^k, \gamma^k, \eta^k) = \textstyle\sum_k p^k \Big[v(t^k + T^k, z^k) + b(g) + \delta^k \left[t^k \cdot \left(y^k + z^k\right) + S^k - g \right]$$

$$+\gamma^k \left[T^k \cdot \left(y^k + z^k + \overline{y}^k + \overline{z}^k\right) - S^k - \overline{S}^k \right] + \eta^k \left[\lambda^k(\cdot) - \overline{\lambda}^k(\cdot) \right] \Big] \quad (21)$$

The first-order conditions with respect to g, t^k and S^k become, using the envelope theorem:

$$b'(g) - \sum_k p^k \delta^k = 0 \quad (22)$$

$$-(y^k + z^k) + \delta^k \left[y^k + z^k - t^k y^{k\prime} \right] - \gamma^k T^k y^{k\prime} + \eta^k \frac{\partial \lambda^k}{\partial t^k} = 0 \qquad \forall\ k \quad (23)$$

$$\delta^k - \gamma^k = 0 \qquad \forall\ k \quad (24)$$

Combining these three conditions, we find

$$b'(g) = \sum_k p^k \delta^k = \sum_k p^k \frac{y^k + z^k - \eta^k \partial\lambda^k/\partial t^k}{y^k + z^k - (t^k + T^k)y^{k\prime}} \tag{25}$$

where $\delta^k = \text{MCPF}^k$. A similar expression applies for the other region with $\overline{\delta}^k = \overline{\text{MCPF}}^k$.

As (25) indicates, there is no longer a standard vertical fiscal externality in this case: the region takes full account of the federal tax rate T^k in the denominator of its MCPF expression. However, there is an incentive for the region to increase its spending in every state of nature because of the additional term in the numerator (given that $\partial\lambda^k/\partial t^k > 0$, as we are assuming). Since the region anticipates that the federal government will make transfers ex post to equalize MCPFs, each region has an incentive to increase its MCPF in order to receive transfers. This leads the regions to overprovide public goods relative to the second-best optimum. In equilibrium, the outcome will be symmetric under symmetric shocks, and tax rates will be higher than the second-best ones in order to finance the higher levels of g and $\overline{g}$. Under asymmetric shocks, aggregate tax rates will also be too high, and transfers to the poor region will therefore be too high as well since the MPCFs are being equalized across regions. Thus, contrary to the case where the federal government can commit, there will be a *positive* VFI in this case. This case is summarized as follows.

Proposition 3: Assuming the smallest non-negative transfers are used, the non-cooperative outcome without federal government commitment has the following characteristics:

i. Regional public goods are chosen so that the marginal benefit equals the expected MCPF. They are identical in the two regions, but larger than the second-best optimal amount ($g = \overline{g} > g^* = \overline{g}^*$).

ii. With symmetric shocks ($k = hh, \ell\ell$), federal taxes and transfers are zero ($T^k = S^k = \overline{S}^k = 0$). Regional tax rates ($t^k, \overline{t}^k$) and therefore MCPFs will be lower for $k = hh$ than for $k = \ell\ell$, but higher than in the second best.

iii. With asymmetric shocks ($k = h\ell, \ell h$), the optimal transfer will be positive for the region with the negative shock ($S^{\ell h}, \overline{S}^{h\ell} > 0$), and zero for the region with the positive shock ($S^{h\ell} = \overline{S}^{\ell h} = 0$), so the federal tax rate will be positive ($T^k > 0$). The

MCPFs will be equalized ($\text{MCPF}^{\ell h} = \overline{\text{MCPF}}^{\ell h}$, $\text{MCPF}^{h\ell} = \overline{\text{MCPF}}^{h\ell}$). There will be a positive VFI: the transfer to the region with the negative shock will be higher than in the second-best optimum ($S^{\ell h} > S^{\ell h*}, \overline{S}^{h\ell} > \overline{S}^{h\ell*}$).

iv. The relation between aggregate tax rates and MCPFs for the representative region in different states satisfies:

$$\text{MCPF}^{hh} < \text{MCPF}^{h\ell} = \text{MCPF}^{\ell h} < \text{MCPF}^{\ell\ell}$$

$$(t^{hh} + T^{hh}) < (t^{h\ell} + T^{h\ell}) = (t^{\ell h} + T^{\ell h}) < (t^{\ell\ell} + T^{\ell\ell})$$

Analogous expressions apply for the other region.

Note that while the federal government's inability to commit leads to overspending by the regions, the equilibrium level of spending is not necessarily higher than when the federal government can commit, at least in this one-period model. That is because when the regions are the first movers, there is no vertical fiscal externality. The fact that the regions anticipate the effect of their spending decision on federal transfers effectively internalizes the vertical fiscal externality since each region is induced to treat the federal budget as a common pool of funds.

In this one-period setting, the federal government has virtually no discretion if it cannot commit. That is, it cannot take any pre-committed actions to offset its inability to commit and that leads to overspending by the regional governments. To allow for this possibility, and more generally to enrich the setting to allow for federal and regional debt, we turn next to a two-period model that captures these additional features in the simplest way. Before so doing, it is useful to briefly consider a more extreme form of the inability to commit that corresponds with the well-known soft budget constraint.[12]

The Soft Budget Constraint Case

Roughly speaking, a soft budget constraint exists if decision-makers overspend relative to their revenues and have to be bailed out. In the context of our simple one-period model,

[12] For a survey of the soft budget constraint, see Kornai et al (2003).

this can be captured by assuming that regions choose their expenditures and taxes without feeling constrained by a budget, anticipating that the federal government will cover any budget shortfalls with increased transfers. Analytically, this is equivalent to assuming that Stages 4 and 5 are reversed: regions choose t^k and $\overline{t}^k$ before the federal government chooses its tax rates and transfers T^k, S^k and $\overline{S}^k$.

Without resorting to formal analysis, we can readily understand intuitively what the outcome will be, reasoning as usual by backward induction. In the last stage, the federal government will choose its taxes and transfers to maximize the sum of expected regional utilities and to ensure that regional budgets are satisfied. The regions will recognize this when they choose their tax and expenditure decisions. They will set their tax rates to zero and let the federal government transfer enough funds to cover the costs of their public goods in every state of nature. With zero regional tax rates, federal tax-transfer policy cannot equalize MCPFs at all: tax rates in both regions must be uniform. Transfers serve only the purpose of financing regional budgets so will be equal to g and $\overline{g}$ in all states of nature. The choice of public goods by the regions will again be determined by their expected MCPFs. In this two-region case, the cost of an incremental increase in g will come from federal tax revenues whose costs is spread between the two regions, equally on average. This implies a vertical fiscal externality that is even larger than the above no-commitment case so regional public goods are overprovided to an even greater extent. Moreover, the VFI is highly positive in this case, and is positive for both regions in all states of nature. Empirically, transfers would not change with shocks to the tax bases of the regions.

6. A Simple Two-Period Model

The two-period model replicates the one-period model in the sense that each region provides a regional public good for two periods, financing it by its own tax revenues and transfers from the federal government in both periods. The federal government imposes uniform taxes in both regions and uses the proceeds to make transfers to the regional governments. The ability of the federal and regional governments to raise revenues in each region again depends on shocks to the tax base that are identically and independently distributed across the ex ante identical regions. The shocks are assumed to last for both periods. To make matters as simple as possible, we assume there is no discounting: household utilities are additive over time, and the interest rate is zero. This does not affect the qualitative results derived.[13]

The extension to two periods allows for a richer sequence of policy choices as well as extending the policy space itself. With respect to the former, we introduce some elements of rigidity in the fiscal instruments. First-period tax and expenditure choices must be made before the state of nature is revealed. Once chosen, the level of public goods remains the same for both periods, but tax rates can be changed in the second period as required for balancing government budgets. Thus, tax rates are more fungible than public goods, as in the one-period model. The choice of regional government spending and the tax rates by the two levels of government in the first period determine the aggregate level of debt (positive or negative) that must be carried forward to the second period in each state of nature. Once the state is revealed in the first period, the federal government can choose its level of transfers to the two regions. Its choice of transfers will determine the allocation of debt between the federal and regional governments. In fact, the intertemporal pattern of transfers, in addition to redistributing between regions, effectively also determines how aggregate debt is shared by the federal and regional governments. Assuming both levels of governments are able to borrow and lend, a revenue-neutral reallocation of transfers from the first to the second period can be offset by an increase in federal debt and a reduction in regional debt. To make this determinate, we assume that transfers announced in the first period after the shock is revealed must be identical in each of the two periods. This is a reasonable restriction to impose given that there is typically some adjustment lag involved in changing transfers once they are in place. Finally, as in the one-period case, we assume that transfers to the regions must be non-negative, and we also assume that federal tax rates are non-negative.

[13] In fact, we do not model the capital market explicitly. This would be consistent with assuming that the country is a small open one, able to borrow and lend freely abroad.

To be more explicit, the timing of events in the two periods is as follows, where subscripts 1 and 2 refer to time periods:

Timing of Events

Period 1:

Stage 1: The federal government chooses its first-period tax rate T_1, and announces state-contingent transfers equal in both periods, S^k and $\overline{S}^k$, anticipating the subsequent behavior of regional governments and households.

Stage 2: Regional governments simultaneously choose the level of provision of their public goods g and $\overline{g}$, which are fixed for both periods, and their first-period tax rates t_1 and $\overline{t}_1$, again anticipating future decisions and outcomes.

Stage 3: Shocks z^k and $\overline{z}^k$ are revealed, and last for two periods.

Stage 4: Depending on its ability to commit to the policy announced ex ante, the federal government may or may not change its transfers, S^k, $\overline{S}^k$.

Stage 5: Households in each region make their first-period production decisions y_1 and $\overline{y}_1$, which depend only on taxes so are independent of state, and debts of federal and regional governments D^k, d^k and $\overline{d}^k$ are determined.

Period 2:

Stage 1: Second-period tax rates t_2^k, $\overline{t}_2^k$ and T_2^k are determined to balance the budget of each government, including repaying any debt from the first period.

Stage 2: Households in each region make their production decisions y_2^k and $\overline{y}_2^k$.

Household behavior is a straightforward extension of that in the one-period case. In each period, output supplies depend on the tax rate that applies in that period. For the representative region, the supply functions are $y_1(1 - t_1 - T_1)$ and $y_2^k(1 - t_2^k - T_2^k)$. With a zero interest rate and no discounting, our quasilinear utility function implies that households are indifferent about the allocation of their consumption between the two periods. For simplicity, we can assume no household borrowing or lending, so per period indirect utility functions are defined as before ($v(t_i^k + T_i^k, z^k)$ for $i = 1, 2$), and the envelope theorem conditions analogous to (8) apply in each period.

Government budget constraints are slightly more complicated since they apply for both periods. In what follows, it is useful to aggregate per period budget constraints for the federal and regional governments into the following state-contingent intertemporal budget constraints, given that the same state lasts for both periods:

$$2\left(S^k + \overline{S}^k\right) = T_1 \cdot (y_1 + z^k + \overline{y}_1 + \overline{z}^k) + T_2^k \cdot (y_2^k + z^k + \overline{y}_2^k + \overline{z}^k) \qquad \forall\ k \qquad (26)$$

$$t_1 \cdot (y_1 + z^k) + t_2^k \cdot (y_2^k + z^k) + 2S^k = 2g \qquad \forall\ k \qquad (27.1)$$

$$\overline{t}_1 \cdot (\overline{y}_1 + \overline{z}^k) + \overline{t}_2^k \cdot (\overline{y}_2^k + \overline{z}^k) + 2\overline{S}^k = 2\overline{g} \qquad \forall\ k \qquad (27.2)$$

These constraints reflect the facts that shocks last for two periods and first-period tax rates are not state-contingent.

7. The Two-Period Second-Best Optimum

The determination of the second-best efficient outcome is a straightforward extension of the one-period case. Suppose there is a unitary national government that chooses all policies for the regions and the nation as a whole subject to a consolidated national budget constraint. The problem of the unitary government will be to choose policies $(\tau_1, \overline{\tau}_1, \tau_2^k, \overline{\tau}_2^k, g, \overline{g})$ to solve the following problem:

$$\max \sum_{i=1,2} \sum_k p^k \left[v(\tau_i^k, z^k) + g + \overline{v}(\overline{\tau}_i^k, \overline{z}^k) + \overline{g}\right] \quad \text{subject to} \qquad (28)$$

$$2g + 2\overline{g} = \tau_1 \cdot (y_1 + z^k) + \overline{\tau}_1 \cdot (\overline{y}_1 + \overline{z}^k) + \tau_2^k \cdot (y_2^k + z^k) + \overline{\tau}_2^k \cdot (\overline{y}_2^k + \overline{z}^k) \qquad \forall\ k$$

Let Λ^k be the Lagrangian multiplier associated with the state-contingent budget constraints. The first-order conditions for this problem reduces to the following:

$$b'(g) = b'(\overline{g}) = \sum_k p^k \Lambda^k \qquad (29)$$

$$\Lambda^k = \frac{y_2^k + z^k}{y_2^k + z^k - \tau_2^k y_2^{k\prime}} = \frac{\overline{y}_2^k + \overline{z}^k}{\overline{y}_2^k + \overline{z}^k - \overline{\tau}_2^k \overline{y}^{k\prime}} \ > \ 1 \qquad \forall\ k \qquad (30)$$

$$\sum_k p^k \left[\Lambda^k \left(y_1 + z^k - \tau_1 y' \right) \right] = \sum_k p^k (y_1 + z^k) \tag{31.1}$$

$$\sum_k p^k \left[\Lambda^k \left(\overline{y}_1 + \overline{z}^k - \overline{\tau}_1 \overline{y}' \right) \right] = \sum_k p^k (\overline{y}_1 + \overline{z}^k) \tag{31.2}$$

The interpretations of these conditions are as follows. From (29) and (30), we can infer that the supply of public goods and first-period tax rates, neither of which are state-contingent, are identical between regions ($g = \overline{g}$, $\tau_1 = \overline{\tau}_1$), reflecting the fact that the regions are ex ante identical. The levels of public goods will be such that the marginal benefit per period is equal to the expected MCPF, again as expected. By (30), we see that tax rates are set in the second period so that MCPFs are equalized ($\text{MCPF}^k = \overline{\text{MCPF}}^k$ for all k). As in the one-period case, this means that in the asymmetric cases, the tax rate is higher in the region with the positive shock implying that there is implicit redistribution from that region to the one with the negative shock. Conditions (31.1) and (31.2) determine the relative sizes of the tax rates in the first and second periods. They can be interpreted in a sense as equating the expected MCPFs in the first period with those in the second. To see this, rewrite (31.1) as follows:

$$\frac{\sum_k p^k \Lambda^k \left(y_1 + z^k - \tau_1 y' \right)}{\sum_k p^k \left(y_1 + z^k - \tau_1 y' \right)} = \frac{\sum_k p^k (y_1 + z^k)}{\sum_k p^k \left(y_1 + z^k - \tau_1 y' \right)}$$

or,

$$\widetilde{\Lambda} = -\frac{E[\partial v(\tau_1, z^k)/\partial \tau_1]}{E[\partial r(\tau_1, z^k)/\partial \tau_1]}$$

where $\widetilde{\Lambda}$ is a weighted average of the state-contingent MCPFs and $r(\cdot)$ is the revenue obtained in the representative region. The righthand side can be interpreted as an expected first-period MCPF: it is the expected utility change resulting from the expected change in revenues when τ_1 is increased. In the second-best optimum, this is set equal to a weighted average of MCPFs.

This second-best optimum thus determines the supply of regional public goods that applies in both periods as well as the tax rates in each region in each period. Implicit in this fiscal outcome is a determinate level of aggregate debt in each state of nature. This debt will be positive for symmetric negative shocks and negative for symmetric positive shocks, but will be of indeterminate sign for the asymmetric cases (although equal in the two). These debts arise because the tax rates are not state-contingent in the first period. A given amount of spending g must be financed regardless of the state of nature revealed, but more tax revenue will be obtained with a positive shock than with a negative one.

As in the one-period case, we can imagine decentralizing this second-best outcome in a cooperative way to regional governments. There will again be an indeterminate vertical fiscal gap because of the fact that federal taxes and transfers to the regional are substitutable. We deal with this indeterminacy by assuming that the federal government makes the lowest non-negative aggregate transfer to the regions over the two periods.

Given this assumption, the following pattern of taxes and transfers will occur in the policy-constrained second-best optimum. First, the federal government will choose first-period tax rates to be zero: $T_1 = 0$ for all k. Clearly in the cases with symmetric shocks, this is optimal. There is no need for redistribution between regions in the second period and therefore no need for second-period federal taxes. Moreover, if the federal government does impose taxes in the first period, if a symmetric shock occurs, they will obtain revenues in the first period that must be disposed of through transfers to the regions in the second period. And with asymmetric shocks, any revenues that they require for redistributive purposes can be obtained using second-period taxes.

Given this, fiscal variables will be fully decentralized to the regions in the symmetric-shock cases.[14] This implies that $D^k = S_1^k = \overline{S}_1^k = S_2^k = \overline{S}_2^k = 0$ for $k = hh, \ell\ell$. The debt of regional governments will be negative under a positive symmetric shock and positive under a negative one.

With asymmetric shocks, the federal government must raise revenues in the second period to make region-specific transfers in order to equalize MCPFs across regions. Given the objective of minimizing transfers, transfers only go to the region with the negative shock. Under our institutional assumption that transfers should be the same in both

[14] This is a consequence of our assumption that there are no federal expenditures. If there were, the federal government would have a reason for raising taxes in all states of nature.

periods, these transfers to the unlucky region will be spread equally across the two time periods. In other words, the optimal VFG will be the same in both periods and will apply only to regions suffering negative shocks in the asymmetric-outcome cases. (Again, in more general cases with ex ante heterogeneous regions and more states of nature, asymmetric outcomes will almost always occur). Given that the federal government does not impose a tax in the first period, it must incur some debt in order to finance its transfers in the first period.

We can summarize as follows the decentralized second-best optimum that will be used as a benchmark, again using asterisks to denote optimal values:

Proposition 4: Assuming the smallest non-negative transfers are used and that they are the same in both periods ($S_1^k = S_2^k \equiv S^k,\ \overline{S}_1^k = \overline{S}_2^k \equiv \overline{S}^k$), the decentralized second-best optimum in the two-period model has the following characteristics:

i. Regional public goods are chosen so that the marginal benefit equals the expected MCPF and are identical in the two regions ($g^* = \overline{g}^*$).

ii. First-period regional tax rates are identically positive (${t_1}^* = {\overline{t}_1}^* > 0$), and federal tax rates are zero (${T_1}^* = 0$).

iii. With symmetric shocks ($k = hh, \ell\ell$), regions are identical ex post. Federal taxes and transfers are zero (${T_2^k}^* = {S^k}^* = {\overline{S}^k}^* = 0$). Regional second-period tax rates (${t_2^k}^* = {\overline{t}_2^k}^*$) and therefore MCPFs will be lower for $k = hh$ than for $k = \ell\ell$. Regions will incur positive debt under negative shocks, and vice versa, and there will be no federal debt.

iv. With asymmetric shocks ($k = h\ell, \ell h$), the equilibrium is asymmetric. The optimal transfer, or VFG, will be positive for the region with the negative shock (${S^{\ell h}}^* = {\overline{S}^{h\ell}}^* > 0$), and zero for the region with the positive shock (${S^{h\ell}}^* = {\overline{S}^{\ell h}}^* = 0$), so the federal second-period tax rate will be positive (${T_2^k}^* > 0$). The federal government will incur positive debt in the first period to pay for the first-period transfer.

v. The MCPF will be equalized between regions in the second period ($\text{MCPF}^{k^*} = \overline{\text{MCPF}}^{k^*}$). First-period regional tax rates will be set so that the expected MCPF in the first period is equal to a weighted average of the second period MCPF in both regions.

8. The Two-Period Non-Cooperative Outcome with Commitment

As in the one period model, there are various ways that commitment can be interpreted, depending on whether the regional governments anticipate the effects of their actions on the federal budget. Since the qualitative results do not depend on the precise commitment assumption that we make, we shall adopt the same assumption as in the one-period full-commitment case and assume that the federal government commits to future tax rates with transfers being determined by budget balance. The regions recognize this and take account of the effects their policy choices will have on future transfers to themselves, which are identical in the two periods for a given state. As before, we assume that each region takes as given the policy choices of the other region. This simplifies the exposition without affecting the qualitative results. We begin with the representative region's choice of public good g and first-period tax rate t_1 before turning to federal policies.

The Regional Governments' Ex Ante Spending Decisions

The representative regional government chooses g and t_1 to maximize the intertemporal sum of utilities of its representative household taking as given the taxes committed to by the federal government T_1 and T_2^k, and anticipating how its decision will affect its transfers in each period S^k and its own second-period tax rate t_2^k. The latter is determined by its budget constraint (27.1), while federal transfers are determined by the federal budget constraint (26) with $S_1^k = S_2^k \equiv S^k$.

We can simplify our analysis at the outset by suppressing some variables that we know to be zero. Given our assumption that the federal government chooses the minimum non-negative value for transfers, only the region hit by the negative shock in each of the asymmetric cases will receive a transfer, so $S^{h\ell} = \overline{S}^{\ell h} = 0$. In the symmetric-shock cases, there will be equal transfers to the two regions ($S^k = \overline{S}^k > 0$ for $k = hh, \ell\ell$) since the federal government will impose a positive tax in the first period. However, in the second period, federal taxes are zero in the symmetric case ($T_2^k = 0$ for $k = hh, \ell\ell$) to avoid an unnecessary vertical fiscal externality. These imply that transfers for the representative region by (26) are as follows:

$$S^k = \frac{T_1}{2}(y_1 + z^k + \overline{y}_1 + \overline{z}^k) + \frac{T_2^k}{2}(y_2^k + z^k + \overline{y}_2^k + \overline{z}^k) \qquad k = \ell\ell, hh \tag{32.1}$$

$$S^{\ell h} = T_1(y_1 + \overline{y}_1) + T_2^{\ell h}(y_2^{\ell h} + \overline{y}_2^{\ell h}) \tag{32.2}$$

These expressions can be used to determine how transfers respond to changes in the representative region's first-and second-period tax rates as well as federal first- and second-period tax rates. These changes are the straightforward analogs of (14) and we omit listing them for simplicity.

The Lagrangian expression for the representative region's problem can then be written as follows:

$$\mathcal{L}(g, t_1, t_2^k, \lambda^k) = \sum_k p^k \big[v(t_1 + T_1, z^k) + v(t_2^k + T_2^k, z^k) +$$

$$2b(g) + \lambda^k \left(t_1 \cdot (y_1 + z^k) + t_2^k \cdot (y_2^k + z^k) + 2S^k - 2g \right) \big] \tag{34}$$

where S^k satisfies (32.1) and (32.2). From the first-order conditions on g, t_1 and t_2^k, and using the derivatives of (32.1) and (32.2), we obtain (16) and:

$$\sum_k p^k \left[-(y_1 + z^k) + \lambda^k \left(y_1 + z^k - t_1 y_1' - \frac{T_1}{2} {y_1}' \right) \right] = 0 \tag{35}$$

$$\lambda^k = \frac{y_2^k + z^k}{y_2^k + z^k - t_2^k {y_2^k}'} \qquad k \neq \ell h \tag{36}$$

$$\lambda^{\ell h} = \frac{y_2^{\ell h} + z^{\ell h}}{y_2^{\ell h} + z^{\ell h} - (t_2^{\ell h} + T_2^{\ell h}) {y_2^{\ell h}}'} \tag{37}$$

These equations can be compared with those for the social optimum, (29)–(31.1) and have an analogous interpretation to the one-period case. Regional governments choose the level of g so that the per-period marginal benefit equals the perceived expected MCPF, but the latter differs from the expected social MCPF, $\sum_k p^k \Lambda^k$, determined by (30). There is a vertical fiscal externality for all states $k \neq h\ell$ due to the fact that the region neglects to take account of the effect of changes in its tax rate on the revenues of the federal government.

Thus, regions will have an incentive to overspend. As (35) indicates, that fiscal externality tends to spread to the first-period tax rate as well. The regional government will smooth its MCPF across periods, but in the first period it also neglects to take account of the full effect of its tax rate on national revenues. Similar conditions apply for the other region.

As in the one-period model the federal government chooses its policies to try and offset these vertical fiscal externalities while at the same time equalizing MCPFs between regions in the asymmetric case. And as in that case, it could do so fully if negative transfers were not precluded: it could choose zero taxes in all states and with asymmetric shocks make positive transfers to the region hit by the bad shock financed by negative transfers extracted from the other region. With a non-negative transfer constraint in effect, the qualitative features of the optimal federal policy are readily understood and have already been mentioned above. In the asymmetric-shock states, in order to reduce the disparity in the MCPFs between the two regions, the federal government makes positive transfers to the region hit by the negative shock and zero transfers to the other region. This entails a positive federal tax rate. Given that the distortion resulting from the vertical fiscal externality is convex in the federal tax rate, the federal government will want to spread the externality between the two periods by imposing a positive tax rate in the first period as well as in the asymmetric shock states in the second period. This implies that there will be federal transfers of equal size to the two regions in the symmetric shock cases.

Formally, the federal government's policy choice can be characterized as follows. First, from the above problem of the regional government, we obtain a maximum value function $w(T_1, T_2)$, where T_2 is a vector of state-contingent tax rates, indicating the expected intertemporal utility of the representative household when the regional government is optimizing. The following envelope theorem results analogous to (18.1) and (18.2) are obtained:

$$\frac{\partial w(\cdot)}{\partial T_2^{\ell h}} = p^{\ell h}\lambda^{\ell h}\left(\overline{y}_2^{\ell h} + \overline{z}^{\ell h} - T_2^{\ell h}\overline{y}_2^{\ell h\prime}\right) > 0, \; \frac{\partial w(\cdot)}{\partial T_2^{h\ell}} = -p^{h\ell}\lambda^{h\ell}\left(y_2^{h\ell} + z^{h\ell}\right) < 0$$

$$\frac{\partial w(\cdot)}{\partial T_1} = \sum_k p^k\left[-(y_1 + z^k) + \lambda^k\left(-t_1 y_1' + \frac{\partial S^k}{\partial T_1}\right)\right]$$

Similar results apply for the other region. The federal problem is then to choose its taxes T_1 and T_2^k to maximize the sum of expected utilities $w(T_1, T_2) + \overline{w}(T_1, T_2)$. Using these envelope results, the first-order condition for second-period taxes $T^{\ell h}$ can be written:

$$p^{\ell h}\lambda^{\ell h}\left(\overline{y}_2^{\ell h} + \overline{z}^{\ell h} - T_2^{\ell h}\overline{y}_2^{\ell h\prime}\right) - p^{\ell h}\overline{\lambda}^{\ell h}\left(\overline{y}_2^{\ell h} + \overline{z}^{\ell h}\right) = 0$$

or,

$$\frac{\lambda^{\ell h}}{\overline{\lambda}^{\ell h}} = \frac{\overline{y}_2^{\ell h} + \overline{z}^{\ell h}}{\overline{y}_2^{\ell h} + \overline{z}^{\ell h} - T_2^{\ell h}\overline{y}_2^{\ell h\prime}} \; > \; 1$$

This is the analog of (19) for the one-period case and along with the similar expression for $T^{h\ell}$ indicates that because of the vertical fiscal externality, the federal government does not equalize the MCPFs of the two regions in the asymmetric shock case. The implication is that the second-best outcome cannot be achieved under non-cooperative regional behavior. In the asymmetric-shock cases, the transfer is smaller than in the cooperative case so there is a negative VFI. However, in the symmetric cases, there is a positive transfer, or a positive VFI. These will be accompanied by a first-period federal surplus in the symmetric shock cases and a federal deficit in the asymmetric shock cases. This is all summarized in the following proposition.

Proposition 5: Assuming the smallest non-negative transfers are used and that they are the same in both periods, the non-cooperative full-commitment outcome in the two-period model has the following characteristics:

i. Regional public goods are chosen so that the marginal benefits equal the expected MCPFs, and are identical in the two regions but larger than the second-best amount ($g = \overline{g} > g^* = \overline{g}^*$).

ii. First-period federal and regional tax rates are both positive ($T_1 > 0,\ t_1 = \overline{t}_1 > 0$).

iii. With symmetric shocks ($k = hh, \ell\ell$), regions are identical ex post. Transfers are positive ($S^k = \overline{S}^k > S^{k*} = \overline{S}^{k*} = 0$) so there is a positive VFI, and second-period federal tax rates are zero ($T_2^k = 0$). Second-period regional tax rates ($t_2^k = \overline{t}_2^k$) and therefore MCPFs will be lower for $k = hh$ than for $k = \ell\ell$. Regions will incur positive debt under negative shocks, and vice versa, and the federal government will be in surplus.

iv. With asymmetric shocks ($k = h\ell, \ell h$), the equilibrium is asymmetric. The optimal transfer will be positive for the region with the negative shock but less than the second-best optimum ($S^{\ell h} = \overline{S}^{h\ell} < S^{\ell h*} = \overline{S}^{h\ell *} > 0$) (so there is a negative VFI), and zero for the region with the positive shock ($S^{h\ell} = \overline{S}^{\ell h} = 0$), so the federal second-period tax rate will be positive ($T_2^k > 0$). The federal government will incur positive debt in the first period to pay for the first-period transfer.

v. With asymmetric shocks, the MCPF will be not equalized between regions in the second period ($\text{MCPF}^{\ell h} > \overline{\text{MCPF}}^{\ell h}$, $\text{MCPF}^{h\ell} < \overline{\text{MCPF}}^{h\ell}$). First-period regional tax rates will be set so that the expected MCPF in the first period is equal to a weighted average of the second-period MCPF in both regions.

9. The Two-Period No-Commitment Outcome

The analysis here is parallel to the one-period case with the exception that the federal government can now influence the outcome by its initial choice of T_1. We begin as before with the federal government's ex post choices, which occur after the state of nature becomes known. Given k, it chooses S^k, $\overline{S}^k$ and T_2^k to maximize the sum of expected utilities, anticipating the tax rates required to balance regional government budgets. At this stage, the federal government takes as given first-period policies chosen by the regions $g = \overline{g}$ and $t_1 = \overline{t}_1$, as well as its own first-period tax rate T_1.

As in the one-period model, the federal will choose its policies such that $\text{MCPF}^k = \overline{\text{MCPF}}^k$, for all k, or:

$$\lambda^k(t_2^k, T_2^k) \equiv \frac{y_2^k + z^k}{y_2^k + z^k - (t_2^k + T_2^k)y_2^{k\prime}} = \frac{\overline{y}_2^k + \overline{z}^k}{\overline{y}_2^k + \overline{z}^k - (\overline{t}_2^k + T_2^k)\overline{y}_2^{k\prime}} \equiv \overline{\lambda}^k(\overline{t}_2^k, T_2^k) \quad \forall\ k \quad (38)$$

which is the analog of (20) in the one-period case.

The representative region, taking as given T_1 and anticipating the federal government's choice of S^k, $\overline{S}^k$ and T_2^k, as well as the value of t_2^k required to balance its budget, will choose g and t_1 to maximize intertemporal utility of its representative citizen. As in the one-period case, this problem can be solved by treating S^k, $\overline{S}^k$, T_2^k and t_2^k as artificial control variables and adding the federal and regional budgets as well as (38) as constraints.

From the first-order conditions of the regional problem, we obtain the analog of (25):

$$b'(g) = \sum_k p^k \delta^k = \sum_k p^k \frac{y_2^k + z^k - \eta^k \partial\lambda^k / \partial t_2^k}{y_2^k + z^k - (t_2^k + T_2^k) y_2^{k\prime}} \tag{39}$$

Once again, the regional government chooses g such that the per period marginal benefit equals the expected MCPF in the second period. And, as in the one-period case, this implies an incentive to overspend: each region has an incentive to increase its MCPF given that ex post federal government policies will equalize MCPFs across regions.

However, now the federal government can take some action up front to influence this incentive. To analyze the federal choice of T_1, define the value function for the representative regional government's above problem as $w(T_1)$. Since both region's are identical ex ante, the federal government's ex ante problem can be stated as simply maximizing the utility of the resident in the representative region, $w(T_1)$. Doing so and using the envelope theorem and first-order conditions from the regional problem, we obtain:

$$\frac{\partial w(T_1)}{\partial T_1} = \sum_k p^k \Big[-(y_1 + z^k) + \gamma^k \left(y_1 + z^k - (t_1 + T_1) y_1{}' + \overline{y}_1 + \overline{z}^k - T_1 \overline{y}_1{}' \right) \Big] = 0 \tag{40}$$

where γ^k is the Lagrangian multiplier on the federal budget constraint in the region's problem and $\gamma^k = \delta^k$ from the first-order conditions on the representative regional government's problem. The term $\sum_k p^k (y_1 + z^k)$ is the expected marginal utility cost of raising the first period tax rate. The federal choice of T_1 equates this expected cost to the expected marginal benefit of raising T_1, i.e. $\sum_k p^k \gamma^k \left(y_1 + z^k - (t_1 + T_1) y_1{}' + \overline{y}_1 + \overline{z}^k - T_1 \overline{y}_1{}' \right)$, which is the expected change in the revenues of the federal government and of the representative regional government for a change in T_1 evaluated at the MCPF perceived by regions, γ^k. Comparing this condition with equation (39) indicates that, at the first-period tax rate chosen by the federal government T_1, the expected MCPF perceived by regional governments in each period are not equalized. The term $\eta^k \partial\lambda^k / \partial t_2^k$ in (39) tends to make the perceived MCPF in the second period smaller than that in the first period. On the other hand, the term $\gamma^k \left(\overline{y}_1 + \overline{z}^k - T_1 \overline{y}_1{}' \right)$ in equation (40) tends to work in the opposite direction. Therefore, whether the expected MCPF perceived by regional governments will be higher in the first or in the second period is ambiguous. Hence, it is also ambiguous whether the federal government will tend to have, in expected terms, a deficit or a surplus in the first period.

The Two-Period Soft Budget Case

Suppose now, as in the one-period case, that the regions can choose their fiscal variables before the federal government without regard to their budget constraints. They anticipate correctly that the federal government will transfer sufficient funds to cover their budget shortfalls. Obviously, regional governments will set their tax rate to zero in both periods, and will receive a transfer $S = g$, which will be independent of the state of nature.

The representative regional government chooses its expenditures g taking as given the first-period federal tax rate T_1 and anticipating the second-period federal tax rate that will be required for budget balance. The solution to this problem yields the following condition determining g:

$$b'(g) = \sum_k p^k \omega^k = \sum_k p^k \frac{y_2^k + z^k}{y_2^k + z^k + \overline{y}_2^k + \overline{z}^k - T_2^k \left(y_2^{k\prime} + \overline{y}_2^{k\prime} \right)} \tag{41}$$

The region's perceived MCPF takes account in the denominator of the fact that an increase in T_2^k required to finance an increase in g raises revenue not just from within the region but from the other region as well. This leads to a significant underestimate of the true MCPF (conceivably to less than unity) implying a strong incentive to overspend.

There is relatively little the federal government can do in the first period to counter this. The solution to the regional government's problem is the usual value function $w(T_1)$ (identical for both regions), which the federal government maximizes. The first-order condition reduces to:

$$\frac{\partial w(T_1)}{\partial T_1} = \sum_k p^k \left[-\left(y_1 + z^k\right) + \omega^k \left(y_1 + z^k + \overline{y}_1 + \overline{z}^k - T_1 \left(y_1{}' + \overline{y}_1{}' \right) \right) \right] = 0 \tag{42}$$

where ω^k is the MCPF from the regional problem. Again, the term $\sum_k p^k (y_1 + z^k)$ is the expected marginal utility cost of raising the first period tax rate, which is equated, at the federal choice of T_1, to the expected marginal benefit of raising T_1. However with $t_1 = 0$, the marginal benefit of raising T_1 is simply equal to the expected change in federal revenues evaluated at the MCPF perceived by regions, ω^k, i.e.

$\sum_k p^k \omega^k \left(y_1 + z^k + \overline{y}_1 + \overline{z}^k - T_1 \left({y_1}' + {\overline{y}_1}' \right) \right)$. Comparing (42) with the value of ω^k defined in (41) indicates that the first-period federal tax rate is set to equalize across periods the expected MCPF that is perceived by regional governments. Therefore, the first period federal tax rate is not directly used to induce regional governments to lower their provision of the public good. Hence, given the soft budget constraint, the federal government has limited power to pre-commit in the first period. Note, however, that since regional governments under-estimate the MCPF, and therefore ω^k is relatively low, the federal government will choose a relatively high T_1.

The upshot then is similar to the one-period case. Federal transfers to each region are set equal to $g = \overline{g}$ in each period. Regional taxes are zero in both periods, and there is no regional debt. The federal government runs a deficit under a negative symmetric shock and a surplus under a positive symmetric shock. The sign of the deficit is indeterminate under asymmetric shocks.

10. Conclusions

The objective of our analysis was to investigate how sequential decision-making by different levels of government and problems of commitment can give rise to vertical fiscal imbalances (VFIs) in decentralized federations. Our model relied on region-specific shocks to tax bases to provide an efficiency rationale for federal-regional transfers and to generate an optimal vertical fiscal gap. Transfers serve to equate the marginal costs of public funds (MCPFs) between regions that are hit by asymmetric shocks, but serve no purpose under symmetric shocks that leave both regions identical ex post. We argued that transfers in the decentralized equilibrium will tend to deviate from the optimal vertical fiscal gap and that such deviations can be seen as VFIs. Central to our analysis is the existence of vertical fiscal externalities, resulting from the co-occupancy of the same tax base by both levels of government, which distort the efficient provision of public goods by regional governments and induces the federal government to create a VFI.

More specifically, the analysis has shown that when the federal government can commit to policies announced before regional governments select their level of public spending, the federal government faces a trade-off between inducing efficiency in the provision of public goods and minimizing the MCPF in the federation as a whole under asymmetric shocks. The optimal resolution of this trade-off results in a negative VFI. In this context, the existence of a VFI can be seen as an optimal response of the federal government to a coordination failure between regional governments. In the absence of commitment ability, however, the time-consistent equilibrium involves a positive VFI. In this case, overprovision of the public good combined with the ex post desire of the federal government to equate the MCPF across regions necessarily results in a federal-regional transfer larger than the optimal vertical fiscal gap.

SUMMARIZE TWO-PERIOD RESULTS AND SUGGEST TESTABLE HYPOTHESES

Our model is an initial attempt to formalize the notion of a VFI. However, the analysis remains exploratory and could be extended in several directions. In particular, we could introduce adjustment costs in the case where the federal government cannot commit to its announced policy. Once the regional governments have made their policy choices and the shocks are revealed, the federal government would incur adjustment costs if it chose to alter its announced policy. These adjustment costs would, in effect, provide some commitment ability to the federal government and would likely affect the size of the VFI. Secondly, we could allow for some mobility of households across regions, which would introduce horizontal fiscal externalities into the model. Doing so would allow us to investigate how vertical and horizontal fiscal externalities jointly determine the optimal behavior of the federal government and the resulting VFI. Finally, we could examine the implications of ex ante heterogeneity across regions on the equilibrium VFI. In this case, region-specific transfers would assume a more important role as there would be a systematic difference in the ability of regions to raise revenues.

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Summary of Results in the Two-Period Case

Cooperative optimum

$g = \overline{g},\ t_1 = \overline{t}_1 > 0,\ T_1 = 0$

Symmetric States: $S^k = D^k = T_2^k = 0,\ t_2^k > 0,\ d^{\ell\ell} > 0,\ d^{hh} < 0$

Asymmetric States: $S^{\ell h} > 0,\ D^k > 0,\ T_2^k > 0$

Full Commitment

$g = \overline{g},\ t_1 = \overline{t}_1 > 0,\ T_1 > 0$

Symmetric States: $S^k > 0,\ D^k < 0,\ T_2^k = 0,\ t_2^k > 0,\ d^{\ell\ell} > 0,\ d^{hh} < 0$, positive VFI

Asymmetric States: $S^{\ell h} > 0,\ D^k > 0,\ T_2^k > 0$, negative VFI

No Commitment

$g = \overline{g},\ t_1 = \overline{t}_1 > 0,\ T_1 >> 0$

Symmetric States: $S^k > 0,\ D^k < 0,\ T_2^k = 0,\ t_2^k > 0,\ d^{\ell\ell} > 0, d^{hh} < 0$, positive VFI

Asymmetric States: $S^{\ell h} >> 0,\ D^k > 0,\ T_2^k > 0$, positive VFI

Soft Budget

$g = \overline{g},\ t_1 = \overline{t}_1 = 0,\ T_1 >> 0$

Symmetric States: $S^k = g,\ d^k = 0,\ T_2^k > 0,\ t_2^k = 0,\ D^{\ell\ell} > 0,\ D^{hh} < 0$, positive VFI

Asymmetric States: $S^k = g,\ d^k = 0,\ T_2^k > 0,\ D^k \gtrless 0$, positive VFI