

Microeconomic
Theory
a Concise Course

James Bergin

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A Brief Outline of the Chapters

Chapter 1 considers decision theory. Starting with preference orderings, sufficient conditions are given for the existence of optimal choices and for the representation of preference orderings by utility functions. Following this, decision-making under risk is discussed. The starting point for the discussion is the von Neumann—Morgenstern model. This sets the stage for objections to the von Neumann—Morgenstern theory and consideration of possible alternatives that preserve some simplicity of structure. These separate into two groups—betweenness models in which indifference curves are linear but not parallel; and rank order models which permit nonlinear indifference curves. Finally, decisionmaking under uncertainty is discussed. This parallels the risk discussion, with the theory of Savage replacing that of von Neumann and Morgenstern. Objections to the theory of Savage are described and proposed alternatives are discussed.

Chapter 2 focuses on preferences and risk, beginning with a discussion of risk aversion in the context of von Neumann—Morgenstern preferences. Risk aversion and asset choice are considered briefly. It is shown how assumptions on risk aversion provide implications for portfolio choice. The state preference model is described, and assumptions on absolute and relative risk aversion are used to determine the shape of indifference curves in the state space. There are many measures of riskiness of a random return. Measuring riskiness in terms of dominance criteria is discussed at length. Definitions are given purely in terms of distributions, and these are connected to preference-based definitions. First-, second-, and higher-order stochastic dominance are explained along with mean preserving spreads, conditional stochastic dominance, monotone likelihood ratio dominance, and hazard rate dominance. The relations between these notions are examined. In addition, a semideviation model is presented which has some virtue as a risk criterion when decisions are based on risk–return pair comparisons.

In Chapter 3 the basic features of a game in strategic form are described. A variety of approaches to the selection of equilibrium outcomes are considered. These provide alternative perspectives on how a player might reasonably approach a strategic decision problem. The last part discusses Nash equilibrium, but avoids going into the knowledge considerations that lie behind its (modern) logical

A Brief Outline of the Chapters

foundations. The section ends with a short discussion of dynamic stability of Nash equilibrium in terms of a simple tatonnement model. Chapter 4 considers the existence of Nash equilibrium and describes some of the major equilibrium refinements (perfection, properness, and persistence.) The chapter also lists a few fixed point theorems that arise in traditional proofs of existence of equilibrium. The relation between properness and sequential equilibrium is explained, establishing an important connection between extensive and strategic form equilibria.

Mechanism design is introduced in Chapter 5. The key classifications of complete and incomplete information are given and the revelation principle is described. Direct mechanisms and dominant strategy implementation are discussed. The Gibbard–Satterthwaite theorem and some positive results are given for single-peaked and quasilinear preferences. Chapter 6 discusses complete and incomplete information implementation in both strategic and extensive form games. A variety of solution concepts are used—Nash, undominated Nash, virtual implementation, and subgame perfection. The key ideas on monotonicity and Bayesian monotonicity are explained and they highlight the essential role of preference reversals in designing implementing mechanisms. This also frames the discussion for mechanisms based on other solution concepts or game forms.

In Chapter 7 auctions with independent values are described. Detailed calculations of equilibrium strategies are given for the standard auctions (first price, second price, and so on.) The fundamental revenue equivalence theorem is illustrated by computing expected revenue for five different types of auction that all share key features sufficient for revenue equivalence (assignment to the buyer with highest valuation and lowest valuation types has an expected payment of 0). Then, reduced form auctions are discussed. These are the key to a full study of the structure of incentives in this environment. One important observation comes directly from incentive compatibility: apart from a common constant, the assignment rule fully determines the expected payment of every type of a bidder. A simple envelope theorem argument is used to give this result. The implications for revenue are immediate—maximizing revenue (the optimal auction) revolves around the optimal assignment rule. This is used to characterize the optimal auction. Finally, the chapter concludes with a section on risk aversion. The revenue equivalence link is broken; greater risk aversion produces more competitive bidding. This captures the intuition that greater risk aversion leads to greater loss from not winning the object, and hence more aggressive bidding. Chapter 8 considered auctions where valuations are not drawn independently. Equilibrium bidding behavior in the first price, second price, and English auctions is characterized. Revenue comparisons are given showing that the revenue equivalence theorem fails for the standard auctions when values are correlated. The expected revenue is at least as large in the second price auction as in the first price auction. The linkage principle (price is positively “linked” to information) is described. Finally, full surplus extraction is discussed at length. This is the analog of the optimal auction in the independent valuations environment.

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Chapter 9 introduces extensive form games. Information structures—perfect, imperfect, and incomplete information are explained. Pure behavioral and mixed strategies are defined. Finally, perfect recall and the equivalence of mixed and behavioral strategies (in terms of end point distributions) are considered. Next, equilibrium in extensive form games is considered in Chapter 10. This chapter covers Nash equilibrium, perfect equilibrium, sequential equilibrium, and perfect Bayesian equilibrium. The classic chain store paradox example is discussed to illustrate.

Repeated games are considered in Chapter 11. Apart from definitions and the familiar characterization results, the discussion explains issues surrounding randomization, observability, feasibility, and convexity. Games with payoff averaging and discounting are discussed along with finitely repeated games. The “no-gain-from-one-shot-deviation” property of games with continuous payoffs is discussed and the proof sketched. Finally, games of incomplete information are introduced.

Information models are considered in Chapter 12. The chapter begins with a discussion of utility maximization when the decisionmaker has some information. The main focus of the first part is to set the framework for a discussion and proof of Blackwell’s theorem on garbling and the value of information. Information and monotonic decisions are discussed: under what conditions do higher signal values lead to a higher optimal level for a choice variable? This issue is examined using simple arguments which then lead to a discussion of stochastic dominance of the distribution on states conditional on the signal, and subsequently to supermodularity of the utility function. This then leads to the introduction of monotone total positivity of densities—which is seen to correspond to the monotone likelihood condition. Some useful results relating to monotone total positivity are given. The material sets the stage for a brief review of supermodularity and submodularity and on how the concepts relate to optimization. Multiperson environments are then introduced. The first observation made is that more information is not necessarily valuable. Rational expectations concepts and potential nonexistence of equilibrium are explained. Here, a short proof is given of the fact that in a rational expectations equilibrium, no speculative gain is possible. Next, equilibrium in an abstract game of incomplete information is discussed. Finally, the discussion sets the stage for three classical information models by describing their distinct features; the information structures for principal–agent, screening, and signaling models are laid out.

In Chapter 13 the principal–agent problem is considered. The principal’s problem with full information is taken as the benchmark. Full insurance occurs with a risk neutral principal and risk averse agent. Then efficient risk allocation with a risk averse principal and risk averse agent is considered, again with full information. Turning to the incomplete information case, unobservable effort raises the key incentive problem. Optimizing subject to first-order conditions—the first-order approach—is considered at length, and sufficient conditions for validity of the first-order approach are given. The key conditions relate to the distribution

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function of output, conditional on effort. One of these conditions is the monotone likelihood ratio condition. Some distributions satisfying the sufficiency conditions are given. Finally the monotone likelihood ratio is used to interpret output as a signal of effort and the level of inefficiency is related to the informativeness of the likelihood ratio. In Chapter 14 the signaling model is considered. The model is used to highlight the difference between various equilibrium refinements. Chapter 15 considers the traditional screening model and covers the basic features of screening models, including pooling and separating equilibria.

In Chapter 16 common knowledge is discussed. The chapter describes information structures and sets up a framework for the discussion of common knowledge. The definition of common knowledge is given. Convergence of beliefs under iterative announcement is discussed and the implications of common knowledge of an aggregate statistic are described. In a game theoretic framework, it is shown how lack of common knowledge can lead to cooperative equilibrium in a finitely repeated prisoners' dilemma game. Finally, a no-trade theorem is given.

Chapter 17 deals with bargaining. The chapter begins with the axiomatic bargaining framework. Derivation of a bargaining set from an underlying environment is illustrated by example. Four axiomatic solutions are discussed: Egalitarian, Utilitarian, Nash, and Kalai–Smorodinsky. Proofs for the Nash and Kalai–Smorodinsky characterizations are given. Noncooperative bargaining is considered with emphasis on the alternating offers model and its recursive structure. The connection between Nash bargaining and the alternating offers model is described. This is the basic noncooperative foundations story: as the time between offers goes to 0, the alternating offers equilibrium division converges to the generalized Nash bargaining solution. Finally, difficulties that arise in the many multiperson case are discussed.

In Chapter 18 cooperative games are considered. The chapter discusses some of the key ideas in cooperative game theory. The core is introduced and the key idea behind nonemptiness (balancedness) is explained through the dual program. Since the coalitional function is commonly introduced without reference to underlying preferences and choice sets, it is discussed here through the notions of “alpha” and “beta” effectivity. Following this, von Neumann–Morgenstern solutions are discussed, and von Neumann–Morgenstern stability is defined. The chapter concludes with a description of the Shapley value.

Large games—games with a continuum of players are described in Chapter 19. Both one-shot and dynamic games are considered. The focus is on anonymous games where only the distribution of players and actions affects any given player. A section discusses the social planner formulation of equilibrium. No aggregate uncertainty is briefly described.

Chapter 20 studies evolution and learning. The chapter begins with a discussion of fictitious play and replicator dynamics—two early models of dynamic adjustment or learning. Following this, some detailed discussion of stochastic stability is given, including the computation of invariant distributions and minimum cost

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trees. An example illustrates how these computations connect directly to the relative sizes of the basins of attraction of absorbing states. A second computation illustrates how the minimum cost tree approach can be used to identify stochastically stable states. Blackwell approachability is used to define strategies that minimize regret across all actions. This is then connected to correlated equilibrium. Calibrated forecasts are defined and a connection to correlated equilibria is also noted. The chapter provides a brief discussion of Bayesian learning and the key role of the martingale convergence theorem. Finally, Blackwell approachability is discussed.