**Formulation 2 of the Decision Rule: the p-value Rule**

**What is a Decision Rule?**

The decision rule for an hypothesis test is a rule that states when the null hypothesis $H_0$ is rejected or retained (not rejected) against the alternative hypothesis $H_1$ at some chosen significance level $\alpha$.

**Formulation 2:** Determine if the p-value for the calculated sample value of the test statistic $t_0$ or $F_0$ under the null hypothesis $H_0$ is smaller or larger than the chosen significance level $\alpha$.

**Definition:** The p-value (or probability value) associated with the calculated sample value of the test statistic is defined as the lowest significance level at which the null hypothesis $H_0$ can be rejected, given the calculated sample value of the test statistic.

**Interpretation**

- The p-value is the probability of obtaining a sample value of the test statistic as extreme as the one we computed if the null hypothesis $H_0$ is true.

- P-values serve as inverse measures of the strength of evidence against the null hypothesis $H_0$.

  - Small p-values – p-values close to zero – constitute strong evidence against the null hypothesis $H_0$.

  - Large p-values – p-values close to one – provide only weak evidence against the null hypothesis $H_0$.  


Examples of p-values for common types of hypothesis tests

♦ For a \textit{two-tail t-test}, let the calculated sample value of the t-statistic for a given null hypothesis be \( t_0 \). Then the p-value associated with the sample value \( t_0 \) is the probability of obtaining an \textbf{absolute value of the t-statistic greater than the absolute value of \( t_0 \) if the null hypothesis \( H_0 \) is true}, where the absolute value of \( t_0 \) is denoted as \(| t_0 |\). That is,

\[
two-tail \ p-value \ for \ t_0 = \Pr\left( |t| > |t_0| \mid H_0 \text{ is true} \right) \\
= \Pr\left( t > t_0 \mid H_0 \text{ is true} \right) + \Pr\left( t < -t_0 \mid H_0 \text{ is true} \right) \\
= 2 \cdot \Pr\left( t > t_0 \mid H_0 \text{ is true} \right) \quad \text{if } t_0 > 0 \\
= \Pr\left( t < t_0 \mid H_0 \text{ is true} \right) + \Pr\left( t > -t_0 \mid H_0 \text{ is true} \right) \\
= 2 \cdot \Pr\left( t < t_0 \mid H_0 \text{ is true} \right) \quad \text{if } t_0 < 0
\]

Remember: the t-distribution is symmetric about its mean of zero.

♦ For a \textit{one-tail t-test}, let the calculated sample value of the t-statistic for a given null hypothesis be \( t_0 \). Then the p-value associated with the sample value \( t_0 \) depends on whether the test is a \textit{right-tail} or \textit{left-tail} test.

(1) For a \textit{right-tail t-test}, the p-value associated with the sample value \( t_0 \) is the \textbf{probability of obtaining a t-statistic value greater than the calculated sample value \( t_0 \) if the null hypothesis \( H_0 \) is true} – i.e.,

\[
right-tail \ p-value \ for \ t_0 = \Pr\left( t > t_0 \mid H_0 \text{ is true} \right).
\]

(2) For a \textit{left-tail t-test}, the p-value associated with the sample value \( t_0 \) is the \textbf{probability of obtaining a t-statistic value less than the calculated sample value \( t_0 \) if the null hypothesis \( H_0 \) is true} – i.e.,

\[
left-tail \ p-value \ for \ t_0 = \Pr\left( t < t_0 \mid H_0 \text{ is true} \right).
\]
For an **F-test**, let the calculated sample value of the F-statistic for a given null hypothesis be $F_0$. Then the p-value associated with the sample value $F_0$ is the **probability of obtaining an F-statistic value greater than the calculated sample value** $F_0$ **if the null hypothesis** $H_0$ **is true** – i.e.,

$$p\text{-value for } F_0 = \Pr(F > F_0 | H_0 \text{ is true}).$$

Note that the F-distribution is defined only over non-negative values that are greater than or equal to zero.

**P-value Decision Rule -- Formulation 2**

1. If the **p-value** for the calculated sample value of the test statistic **is less than** the chosen **significance level** $\alpha$, **reject the null hypothesis** at significance level $\alpha$.

   $$p\text{-value} < \alpha \Rightarrow \text{reject } H_0 \text{ at significance level } \alpha.$$

2. If the **p-value** for the calculated sample value of the test statistic **is greater than or equal to** the chosen **significance level** $\alpha$, **retain (i.e., do not reject) the null hypothesis** at significance level $\alpha$.

   $$p\text{-value} \geq \alpha \Rightarrow \text{retain } H_0 \text{ at significance level } \alpha.$$