DATE: Monday October 27, 2008.

TIME: 80 minutes; 1:00 p.m. - 2:20 p.m.

INSTRUCTIONS: The exam consists of FOUR (4) questions. Students are required to answer ALL FOUR (4) questions.

Answer all questions in the exam booklets provided. Be sure your student number is printed clearly on the front of all exam booklets used. Your name is optional.

Do not write answers to questions on the front page of the first exam booklet.

Please label clearly each of your answers in the exam booklets with the appropriate number and letter.

Please write legibly.

Tables of percentage points of the t-distribution and F-distribution are given on the last two pages of the exam.

This exam is CONFIDENTIAL. This question paper must be submitted in its entirety with your answer booklet(s); otherwise your exam will not be marked.

MARKING: The marks for each question are indicated in parentheses immediately above each question. Total marks for the exam equal 100.

GOOD LUCK!
QUESTIONS: Answer ALL FOUR questions.

All questions pertain to the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

\[ Y_i = \beta_0 + \beta_1 X_i + u_i \]  \hspace{1cm} (1)

where \( Y_i \) and \( X_i \) are observable variables, \( \beta_0 \) and \( \beta_1 \) are unknown (constant) regression coefficients, and \( u_i \) is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

\[ Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \hat{u}_i \]  \hspace{1cm} (i = 1, ..., N)  \hspace{1cm} (2)

where \( \hat{\beta}_0 \) is the OLS estimator of the intercept coefficient \( \beta_0 \), \( \hat{\beta}_1 \) is the OLS estimator of the slope coefficient \( \beta_1 \), \( \hat{u}_i \) is the OLS residual for the i-th sample observation, and \( N \) is sample size (the number of observations in the sample).

(15 marks)

1. Show that the OLS slope coefficient estimator \( \hat{\beta}_1 \) is a linear function of the \( Y_i \) sample values. Stating explicitly all required assumptions, prove that the OLS slope coefficient estimator \( \hat{\beta}_1 \) is an unbiased estimator of the slope coefficient \( \beta_1 \).

**Answer to Question 1:**

(5 marks)

- Show that the OLS slope coefficient estimator \( \hat{\beta}_1 \) is a *linear* function of the \( Y_i \) sample values.

\[
\hat{\beta}_1 = \frac{\sum_i x_i y_i}{\sum_i x_i^2} = \frac{\sum_i x_i (Y_i - \bar{Y})}{\sum_i x_i^2} = \frac{\sum_i x_i Y_i}{\sum_i x_i^2} - \frac{\bar{Y} \sum_i x_i}{\sum_i x_i^2} \\
= \frac{\sum_i x_i Y_i}{\sum_i x_i^2} \quad \text{because} \quad \sum_i x_i = 0 \quad \text{(5 marks)}
\]

\[
= \sum_i k_i y_i \quad \text{where} \quad k_i = \frac{x_i}{\sum_i x_i^2}.
\]
**ANSWER to Question 1 -- continued:**

(10 marks)
- Stating explicitly all required assumptions, prove that the OLS slope coefficient estimator \( \hat{\beta}_1 \) is an unbiased estimator of the slope coefficient \( \beta_1 \).

(1) **Substitute for** \( Y_i \) **the expression** \( Y_i = \beta_0 + \beta_1 X_i + u_i \) **from the population regression equation (or PRE).**

\[
\hat{\beta}_1 = \sum_i k_i Y_i
\]

\[
= \sum_i k_i (\beta_0 + \beta_1 X_i + u_i)
\]

\[
= \sum_i (\beta_0 k_i + \beta_1 k_i X_i + k_i u_i)
\]

\[
= \beta_0 \sum_i k_i + \beta_1 \sum_i k_i X_i + \sum_i k_i u_i
\]

\[
= \beta_1 + \sum_i k_i u_i, \quad \text{since } \sum_i k_i = 0 \text{ and } \sum_i k_i X_i = 1
\]

(2) **Now take expectations** of the above expression for \( \hat{\beta}_1 \) **conditional on the value** \( X_i \) **of** \( X \):

\[
E(\hat{\beta}_1) = E(\beta_1) + E[\sum_i k_i u_i]
\]

\[
= \beta_1 + \sum_i k_i E(u_i|X_i)
\]

\[
= \beta_1 + \sum_i k_i 0, \quad \text{since } E(u_i|X_i) = E(u_i) = 0 \text{ by assumption (A2)}
\]

\[
= \beta_1
\]
(15 marks)
2. Give a general definition of the F-distribution. Starting from this definition, derive the F-statistic for the OLS slope coefficient estimator \( \hat{\beta}_1 \). State all assumptions required for the derivation.

**ANSWER to Question 2:**

- **General definition of the F-distribution:** Consider the two random variables \( V_1 \) and \( V_2 \) such that

  (1) \( V_1 \sim \chi^2[m_1] \)
  (2) \( V_2 \sim \chi^2[m_2] \)
  (3) \( V_1 \) and \( V_2 \) are independent.

Then the random variable

\[
F = \frac{V_1/m_1}{V_2/m_2} \sim F[m_1, m_2]
\]

where \( F[m_1, m_2] \) denotes the F-distribution with \( m_1 \) numerator degrees of freedom and \( m_2 \) denominator degrees of freedom.

- **Error Normality Assumption:** The random error term \( u_i \) is normally distributed with mean 0 and variance \( \sigma^2 \); that is,

\[
\begin{align*}
  &u_i|X_i \sim N\left[0, \sigma^2\right] \text{ for all } i \quad \text{OR} \quad u_i \text{ is iid as } N\left[0, \sigma^2\right]
\end{align*}
\]

- Use **three implications** of the error normality assumption to derive \( F(\hat{\beta}_1) \):

  1. \( \hat{\beta}_1 \sim N[\beta_1, \text{Var}(\hat{\beta}_1)] \) where
     \[
     \text{Var}(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^{N} x_i^2} = \frac{\sigma^2}{\sum_{i=1}^{N} (X_i - \bar{X})^2}
     \]

     Implications of normality of \( \hat{\beta}_1 \):

\[
Z(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{\text{se}(\hat{\beta}_1)} \sim N[0, 1]
\]
**ANSWER to Question 2 -- continued:**

\[
[Z(\hat{\beta}_1)]^2 = \frac{(\hat{\beta}_1 - \beta_1)^2}{\text{Var}(\hat{\beta}_1)} = \frac{(\hat{\beta}_1 - \beta_1)^2}{\sigma^2/\sum_i x_i^2} = \frac{(\hat{\beta}_1 - \beta_1)^2 \sum_i x_i^2}{\sigma^2} \sim \chi^2[1] \quad (1) \tag{1 mark}
\]

2. \[
\frac{(N - 2)\hat{\sigma}^2}{\sigma^2} \sim \chi^2[N - 2] \implies \frac{\hat{\sigma}^2}{\sigma^2} \sim \frac{\chi^2[N - 2]}{(N - 2)} \quad (2) \tag{1 mark}
\]

3. The estimators \(\hat{\beta}_1\) and \(\hat{\sigma}^2\) are statistically independent

**OR**

The statistics \(Z(\hat{\beta}_1)\) and \(\hat{\sigma}^2/\sigma^2\) are statistically independent.

- **The F-statistic for \(\hat{\beta}_1\).** The F-statistic for \(\hat{\beta}_1\) is therefore the ratio of (1) to (2): \(6\) marks

\[
F(\hat{\beta}_1) = \frac{(Z(\hat{\beta}_1))^2}{\sigma^2/\sigma^2} = \frac{(\hat{\beta}_1 - \beta_1)^2 \left(\sum_i x_i^2\right)/\sigma^2}{\sigma^2/\sigma^2} = \frac{(\hat{\beta}_1 - \beta_1)^2 \sum_i x_i^2}{\sigma^2} = \frac{(\hat{\beta}_1 - \beta_1)^2}{\text{Vâr}(\hat{\beta}_1)} \quad \text{since} \ \hat{\sigma}^2/\sum_i x_i^2 = \text{Vâr}(\hat{\beta}_1).
\]

- **Result:** The F-statistic for \(\hat{\beta}_1\) takes the form

\[
F(\hat{\beta}_1) = \frac{(\hat{\beta}_1 - \beta_1)^2}{\sigma^2/\left(\sum_i x_i^2\right)} = \frac{(\hat{\beta}_1 - \beta_1)^2}{\text{Vâr}(\hat{\beta}_1)} \sim F[1, N - 2].
\]
(32 marks)
3. A researcher is using data for a sample of 3240 female employees 25 years of age and over to investigate the relationship between employees’ hourly wage rates \( Y_i \) (measured in dollars per hour) and their age \( X_i \) (measured in years). The population regression equation takes the form of equation (1): \( Y_i = \beta_0 + \beta_1 X_i + u_i \). Preliminary analysis of the sample data produces the following sample information:

\[
\begin{align*}
N &= 3240 \\
\sum_{i=1}^{N} y_i^2 &= 78434.97 \\
\sum_{i=1}^{N} x_i^2 &= 25526.17 \\
\sum_{i=1}^{N} x_i y_i &= 3666.426 \\
\sum_{i=1}^{N} y_i &= 34379.16 \\
\sum_{i=1}^{N} x_i &= 96143.00 \\
\sum_{i=1}^{N} y_i^2 &= 443227.1 \\
\sum_{i=1}^{N} x_i^2 &= 2878451.0 \\
\sum_{i=1}^{N} x_i y_i &= 1023825.0 \\
\sum_{i=1}^{N} u_i^2 &= 77908.35
\end{align*}
\]

where \( x_i \equiv X_i - \bar{X} \) and \( y_i \equiv Y_i - \bar{Y} \) for \( i = 1, ..., N \). Use the above sample information to answer all the following questions. **Show explicitly all formulas and calculations.**

(10 marks)
(a) Use the above information to compute OLS estimates of the intercept coefficient \( \beta_0 \) and the slope coefficient \( \beta_1 \).

\[
\begin{align*}
\hat{\beta}_1 &= \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2} = \frac{3666.426}{25526.17} = 0.143634 = 0.1436 \\
\hat{\beta}_0 &= \bar{Y} - \hat{\beta}_1 \bar{X} \\
\bar{Y} &= \frac{\sum_{i=1}^{N} y_i}{N} = \frac{34379.16}{3240} = 10.61085 \\
\bar{X} &= \frac{\sum_{i=1}^{N} x_i}{N} = \frac{96143.00}{3240} = 29.67377
\end{align*}
\]

Therefore

\[
\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 10.610852 - (0.143634)(29.67377) = 10.610852 - 4.262162 = 6.34869
\]

\[= 6.349\]  
(5 marks)
**(4 marks)**

(b) Interpret the slope coefficient estimate you calculated in part (a) – i.e., explain in words what the numeric value you calculated for \( \hat{\beta}_1 \) means.

*Note:* \( \hat{\beta}_1 = 0.1436 \). \( Y_i \) is measured in *dollars per hour*, and \( X_i \) is measured in *years*.

The estimate 0.1436 of \( \beta_1 \) means that a 1-year *increase (decrease)* in age \( X_i \) is associated on average with an *increase (decrease)* in hourly wage rate equal to 0.1436 *dollars per hour*, or 14.36 *cents per hour*.

**(4 marks)**

(c) Calculate an estimate of \( \sigma^2 \), the error variance.

\[
\text{RSS} = \sum_{i=1}^{N} \hat{u}_i^2 = 77908.35; \quad N - 2 = 3240 - 2 = 3238
\]

\[
\hat{\sigma}^2 = \frac{\text{RSS}}{N - 2} = \frac{\sum_{i=1}^{N} \hat{u}_i^2}{N - 2} = \frac{77908.35}{3240 - 2} = \frac{77908.35}{3238} = 24.060639 = 24.0606 \quad (4 \text{ marks})
\]

**(6 marks)**

(d) Compute the value of \( R^2 \), the coefficient of determination for the estimated OLS sample regression equation. Briefly explain what the value you have calculated for \( R^2 \) means.

**4 marks**

\[
R^2 = \frac{\text{ESS}}{\text{TSS}} = \frac{\sum_{i=1}^{N} y_i^2 - \sum_{i=1}^{N} \hat{u}_i^2}{\sum_{i=1}^{N} y_i^2} = \frac{78434.97 - 77908.35}{78434.97} = \frac{526.62}{78434.97} = 0.006714
\]

*OR*

\[
R^2 = 1 - \frac{\text{RSS}}{\text{TSS}} = 1 - \frac{\sum_{i=1}^{N} \hat{u}_i^2}{\sum_{i=1}^{N} y_i^2} = 1 - \frac{77908.35}{78434.97} = 1 - 0.993286 = 0.006714
\]

**2 marks**

*Interpretation of \( R^2 = 0.006714 \): The value of 0.006714 indicates that 0.67 percent of the total sample (or observed) variation in \( Y_i \) (hourly wage rates) is attributable to, or explained by, the regressor \( X_i \) (age) or the sample regression function.*
(8 marks)

(e) Calculate the sample value of the t-statistic for testing the null hypothesis $H_0: \beta_1 = 0$ against the alternative hypothesis $H_1: \beta_1 \neq 0$. (Note: You are not required to obtain or state the inference of this test.)

- The t-statistic for $\hat{\beta}_1$ is
  \[
  t(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{\text{s.e}(\hat{\beta}_1)}
  \]  
  (1)  
  (2 marks)

- From part (a), $\hat{\beta}_1 = 0.143634$; 
  \[\text{Vár}(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^{N} x_i^2} = \frac{24.060639}{25526.17} = 0.000942587.\]

- The estimated standard error of $\hat{\beta}_1$ is:
  \[\text{s.e}(\hat{\beta}_1) = \sqrt{\text{Vár}(\hat{\beta}_1)} = \sqrt{0.000942587} = 0.03070158 = 0.0307016\]

- Calculate the sample value of the t-statistic (1) under $H_0$: set $\beta_1 = 0$, $\hat{\beta}_1 = 0.143634$ and $\text{s.e}(\hat{\beta}_1) = 0.0307016$ in (1) above.
  \[t_0(\hat{\beta}_1) = \frac{(\hat{\beta}_1 - \beta_1)}{\text{s.e}(\hat{\beta}_1)} = \frac{0.143634 - 0}{0.0307016} = \frac{0.143634}{0.0307016} = 4.678388 = 4.68\]  
  (6 marks)
You have been commissioned to investigate the relationship between the birth weights of newborn females and the number of prenatal visits to a physician or midwife that their mothers made during pregnancy. The dependent variable is \(bwght_i\), the birth weight of the \(i\)-th newborn female, measured in grams. The explanatory variable is \(pnvisits_i\), the number of prenatal visits of the \(i\)-th newborn’s mother during pregnancy, measured in number of visits. The model you propose to estimate is given by the population regression equation

\[bwght_i = \beta_0 + \beta_1 pnvisits_i + u_i.\]

Your research assistant has used 857 sample observations on \(bwght_i\) and \(pnvisits_i\) to estimate the following OLS sample regression equation, where the figures in parentheses below the coefficient estimates are the estimated standard errors of the coefficient estimates:

\[bwght_i = 3199.02 + 14.1219 pnvisits_i + \hat{u}_i \quad (i = 1, ..., N) \quad N = 857 \quad (3)\]

\[(65.6909) \quad (5.36347) \quad \leftarrow \text{(standard errors)}\]

(a) Perform a test of the null hypothesis \(H_0: \beta_1 = 0\) against the alternative hypothesis \(H_1: \beta_1 \neq 0\) at the 1% significance level (i.e., for significance level \(\alpha = 0.01\)). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. What would you conclude from the results of the test?

\(H_0: \beta_1 = 0\)
\(H_1: \beta_1 \neq 0\)  a two-sided alternative hypothesis \(\Rightarrow\) a two-tailed test

- Test statistic is either \(t(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{s\hat{e}(\hat{\beta}_1)} \sim t[N - 2]\) or \(F(\hat{\beta}_1) = \frac{(\hat{\beta}_1 - \beta_1)^2}{\text{Var}(\hat{\beta}_1)} \sim F[1, N - 2].\)

- \(\hat{\beta}_1 = 14.1219;\quad s\hat{e}(\hat{\beta}_1) = 5.36347;\quad \text{Var}(\hat{\beta}_1) = \left(s\hat{e}(\hat{\beta}_1)\right)^2 = 28.76681\)

- Calculate the sample value of either the t-statistic or the F-statistic under \(H_0:\)
  set \(\beta_1 = 0, \hat{\beta}_1 = 14.1219, s\hat{e}(\hat{\beta}_1) = 5.36347,\) and \(\text{Var}(\hat{\beta}_1) = 28.76681.\)

\[t_0(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{s\hat{e}(\hat{\beta}_1)} = \frac{14.1219 - 0.0}{5.36347} = \frac{14.1219}{5.36347} = 2.63298 = 2.63\]

\(or\)
ANSWER to Question 4(a) -- continued:

\[
F_0(\hat{\beta}_1) = \frac{(\hat{\beta}_1 - \beta_1)^2}{\text{Var}(\hat{\beta}_1)} = \frac{(14.1219 - 0.0)^2}{28.76681} = \frac{199.42806}{28.76681} = 6.9326 = 6.93
\]

- Null distribution of \( t_0(\hat{\beta}_1) \) is \( t[N - 2] = t[857 - 2] = t[855] \)
- Null distribution of \( F_0(\hat{\beta}_1) \) is \( F[1, N - 2] = F[1, 857 - 2] = F[1, 855] \)

Decision Rule: At significance level \( \alpha \),

- \( \text{reject } H_0 \) if \( F_0(\hat{\beta}_1) > F_\alpha[1, 855] \) or \( \left| t_0(\hat{\beta}_1) \right| > t_{\alpha/2}[855] \),
  i.e., if either (1) \( t_0(\hat{\beta}_1) > t_{\alpha/2}[855] \) or (2) \( t_0(\hat{\beta}_1) < -t_{\alpha/2}[855] \);

- \( \text{retain } H_0 \) if \( F_0(\hat{\beta}_1) \leq F_\alpha[1, 855] \) or \( \left| t_0(\hat{\beta}_1) \right| \leq t_{\alpha/2}[855] \),
  i.e., if \(-t_{\alpha/2}[855] \leq t_0(\hat{\beta}_1) \leq t_{\alpha/2}[855] \).

Critical values of \( t[855] \)-distribution or \( F[1, 855] \)-distribution: in t-table, use \( df = \infty \).

- \( \text{two-tailed 1 percent critical value} = t_{\alpha/2}[855] = t_{0.005}[855] = 2.5816 = 2.58 \) (1 mark)
  \( = t_{0.005}[\infty] = 2.576 = 2.58 \)

Critical values of \( F[1, 855] \)-distribution: in F-table, use denominator \( df = \infty \).

- \( \text{1 percent critical value} = F_{\alpha}[1, 855] = F_{0.01}[1, 855] = 6.6646 = 6.66 \)
  \( = F_{0.01}[1, \infty] = 6.63 \)

Inference:

- At 1 percent significance level, i.e., for \( \alpha = 0.01 \),
  \[
  \left| t_0(\hat{\beta}_1) \right| = 2.63 > 2.58 = t_{0.005}[855] \Rightarrow \text{reject } H_0 \text{ vs. } H_1 \text{ at 1 percent level.}
  \]
  \[
  F_0(\hat{\beta}_1) = 6.93 > 6.63 = F_{0.01}[1, 855] \Rightarrow \text{reject } H_0 \text{ vs. } H_1 \text{ at 1 percent level.}
  \]
ANSWER to Question 4(a) -- continued:

- **Inference:** At the 1% significance level, the null hypothesis $H_0: \beta_1 = 0$ is rejected in favour of the alternative hypothesis $H_1: \beta_1 \neq 0$.

**Conclusion implied by test outcome:**

(1 mark)

Rejection of the null hypothesis $\beta_1 = 0$ against the alternative hypothesis $\beta_1 \neq 0$ means that the sample evidence favours the existence of a relationship between the number of prenatal visits of mothers and the birth weights of their female newborns.

**Question 4(a) -- Alternative Answer -- uses confidence interval approach**

- The two-sided $(1 - \alpha)$-level, or $100(1 - \alpha)$ percent, confidence interval for $\beta_1$ is:

  $$
  \hat{\beta}_1 - t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_1) \leq \beta_1 \leq \hat{\beta}_1 + t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_1)
  $$

  $$
  \hat{\beta}_{1L} \leq \beta_1 \leq \hat{\beta}_{1U}
  $$

- Required results and intermediate calculations:

  $N - K = 857 - 2 = 855; \quad \hat{\beta}_1 = 14.1219; \quad s\hat{e}(\hat{\beta}_1) = 5.36347$

  $1 - \alpha = 0.99 \Rightarrow \alpha = 0.01 \Rightarrow \alpha/2 = 0.005: \quad t_{\alpha/2}[N-2] = t_{0.005}[855] = 2.576 \quad (1 \text{ mark})$

  $$
  t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_1) = t_{0.005}[855]s\hat{e}(\hat{\beta}_1) = 2.576(5.36347) = 13.816299
  $$

  - **Lower 99% confidence limit for $\beta_1$ is:**

    (2 marks)

    $$
    \hat{\beta}_{1L} = \hat{\beta}_1 - t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_1) = 14.1219 - 2.576(5.36347) = 14.1219 - 13.816299 = 0.305601 = 0.3056
    $$

    $$
    = 14.1219 - 2.617(5.36347) = 14.1219 - 14.036201 = 0.08570
    $$

    $$
    = 14.1219 - 2.58(5.36347) = 14.1219 - 13.837753 = 0.28417 = 0.2841
    $$

  - **Upper 99% confidence limit for $\beta_1$ is:**

    (2 marks)

    $$
    \hat{\beta}_{1U} = \hat{\beta}_1 + t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_1) = 14.1219 + 2.576(5.36347) = 14.1219 + 13.816299 = 27.938199 = 27.9382
    $$

    $$
    $$

    $$
    = 14.1219 + 2.58(5.36347) = 14.1219 + 13.837753 = 27.959653 = 27.9597
    $$
Question 4(a) – Alternative Answer (continued)

- Two-sided 99% confidence interval for \( \beta_1 \) is therefore:

\[
0.3056 \leq \beta_1 \leq 27.9382 \quad \text{or} \quad 0.08570 \leq \beta_1 \leq 28.1581 \quad \text{or} \quad 0.2841 \leq \beta_1 \leq 27.9597
\]

- **Decision Rule:** At significance level \( \alpha \),

\[
\text{reject } H_0 \quad \text{if the hypothesized value } b_1 \text{ of } \beta_1 \text{ specified by } H_0 \text{ lies outside the two-sided (1-}\alpha\text{-level confidence interval for } \beta_1, \text{i.e., if either}
\]

\[
(1) \ b_1 < \hat{\beta}_1 - t_{a/2}[855]\hat{s}(\hat{\beta}_1) \quad \text{or} \quad (2) \ b_1 > \hat{\beta}_1 + t_{a/2}[855]\hat{s}(\hat{\beta}_1).
\]

- **retain** \( H_0 \) if the hypothesized value \( b_1 \) of \( \beta_1 \) specified by \( H_0 \) lies inside the two-sided (1-\( \alpha \))-level confidence interval for \( \beta_1 \), i.e., if

\[
\hat{\beta}_1 - t_{a/2}[855]\hat{s}(\hat{\beta}_1) \leq b_1 \leq \hat{\beta}_1 + t_{a/2}[855]\hat{s}(\hat{\beta}_1).
\]

**Inference:**

- At 1 percent significance level, i.e., for \( \alpha = 0.01 \),

\[
b_1 = 0 < 0.3056 = \hat{\beta}_{1L} = \hat{\beta}_1 - t_{0.005}[855]\hat{s}(\hat{\beta}_1) \quad \Rightarrow \quad \text{reject } H_0 \text{ vs. } H_1 \text{ at 1 percent level.}
\]

- **Inference:** At the 1% significance level, the null hypothesis \( H_0 \): \( \beta_1 = 0 \) is rejected in favour of the alternative hypothesis \( H_1 \): \( \beta_1 \neq 0 \).

**Conclusion implied by test outcome:**

Rejection of the null hypothesis \( \beta_1 = 0 \) against the alternative hypothesis \( \beta_1 \neq 0 \) means that the sample evidence favours the existence of a relationship between the number of prenatal visits of mothers and the birth weights of their female newborns.
(8 marks)

(b) Perform a test of the proposition that newborn females’ birth weights are positively related to their mother’s number of prenatal visits during pregnancy, i.e., that an increase in the number of prenatal visits increases the birth weights of newborn females. Use the 1 percent significance level (i.e., $\alpha = 0.01$). State the null hypothesis $H_0$ and the alternative hypothesis $H_1$. Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test.

**ANSWER to Question 4(b):**

**Null and Alternative Hypotheses:**

$$H_0: \beta_1 = 0$$
$$H_1: \beta_1 > 0 \Rightarrow \text{a right-tailed } t\text{-test}$$

- Test statistic is $t(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{\text{s}\hat{e}(\hat{\beta}_1)} \sim t[N - 2]; \hat{\beta}_1 = 14.1219; \text{s}\hat{e}(\hat{\beta}_1) = 5.36347$.

- Calculate the *sample value* of the *t*-statistic under $H_0$:
  
  set $\beta_1 = 0$, $\hat{\beta}_1 = 14.1219$, and $\text{s}\hat{e}(\hat{\beta}_1) = 5.36347$.

  $t_0(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{\text{s}\hat{e}(\hat{\beta}_1)} = \frac{14.1219 - 0.0}{5.36347} = 2.63298 = 2.63$ (2 marks)

- **Null distribution** of $t_0(\hat{\beta}_1)$ is $t[N - 2] = t[857 - 2] = t[855]$

**Decision Rule:** At significance level $\alpha$,

- **reject** $H_0$ if $t_0(\hat{\beta}_1) > t_{\alpha}[855]$,
- **retain** $H_0$ if $t_0(\hat{\beta}_1) \leq t_{\alpha}[855]$.

**Critical value of $t[855]$-distribution:** from t-table, use $\text{df} = 120$ or $\text{df} = \infty$ or any number in between.

- right-tail 1 percent critical value $= t_{0.01}[855] = 2.331$ = 2.33 (1 mark)
- $= t_{0.01}[855] = 2.358 = 2.36$
- $= t_{0.01}[\infty] = 2.326 = 2.33$
ANSWER to Question 4(b) -- continued:

Inference: (1 mark)

♦ At 1 percent significance level, i.e., for $\alpha = 0.01$,

$$ t_0(\hat{\beta}_1) = 2.63 > 2.33 = t_{0.01}[855] \Rightarrow reject H_0 \text{ vs. } H_1 \text{ at 1 percent level.} $$

♦ Inference: At the 1% significance level, the null hypothesis $H_0: \beta_1 = 0$ is rejected in favour of the alternative hypothesis $H_1: \beta_1 > 0$. 
(14 marks)
(c) Compute the two-sided 95% confidence interval for the intercept coefficient $\beta_0$. Use this two-sided 95% confidence interval for $\beta_0$ to test the hypothesis that the mean birth weight of newborn females whose mothers made no prenatal visits to a physician or midwife equals 3,000 grams. State the null hypothesis $H_0$ and the alternative hypothesis $H_1$. State the decision rule you use, and the inference you would draw from the test.

**ANSWER to Question 4(c):**

- The **two-sided** $(1 - \alpha)$-level, or $100(1 - \alpha)$ percent, confidence interval for $\beta_1$ is computed as

\[
\hat{\beta}_0 - t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_0) \leq \beta_0 \leq \hat{\beta}_0 + t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_0)
\]

where

- $\hat{\beta}_{0L} = \hat{\beta}_0 - t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_0)$ = the **lower** $100(1 - \alpha)$% confidence limit for $\beta_0$
- $\hat{\beta}_{0U} = \hat{\beta}_0 + t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_0)$ = the **upper** $100(1 - \alpha)$% confidence limit for $\beta_0$
- $t_{\alpha/2}[N-2]$ = the $\alpha/2$ critical value of the t-distribution with $N-2$ degrees of freedom.

- Required results and intermediate calculations:

\[
N - K = 857 - 2 = 855; \quad \hat{\beta}_0 = 3199.02; \quad s\hat{e}(\hat{\beta}_0) = 65.6909
\]

\[
1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow \alpha/2 = 0.025: \quad t_{\alpha/2}[N-2] = t_{0.025}[855] = 1.963
\]

\[
= t_{0.025}[855] = 1.980
\]

\[
= t_{0.025}[\infty] = 1.960
\]

\[
t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_0) = t_{0.025}[855]s\hat{e}(\hat{\beta}_0) = 1.963(65.6909) = 128.95124
\]

- **Lower 95% confidence limit for $\beta_0$ is:**

\[
\hat{\beta}_{0L} = \hat{\beta}_0 - t_{\alpha/2}[N-2]s\hat{e}(\hat{\beta}_0) = \hat{\beta}_0 - t_{0.025}[855]s\hat{e}(\hat{\beta}_0)
\]

\[
= 3199.02 - 1.963(65.6909) = 3199.02 - 128.95124 = 3070.069 = 3070.07
\]

\[
= 3199.02 - 1.980(65.6909) = 3199.02 - 130.06798 = 3068.952 = 3068.95
\]

\[
= 3199.02 - 1.960(65.6909) = 3199.02 - 128.75416 = 3070.266 = 3070.27
\]
**ANSWER to Question 4(c) -- continued:**

- **Upper 95% confidence limit for \( \beta_0 \) is:**

  \[
  \hat{\beta}_{0U} = \hat{\beta}_0 + t_{\alpha/2}[N - 2]s\hat{e}(\hat{\beta}_0) = \hat{\beta}_0 + t_{0.025}[855]s\hat{e}(\hat{\beta}_0) \\
  = 3199.02 + 1.963(65.6909) = 3199.02 + 128.95124 = 3327.971 = 3327.97 \\
  = 3199.02 + 1.980(65.6909) = 3199.02 + 130.06798 = 3329.088 = 3329.09 \\
  = 3199.02 + 1.960(65.6909) = 3199.02 + 128.75416 = 3327.774 = 3327.77
  \]

- **Result:** The two-sided 95% confidence interval for \( \beta_0 \) is:

  \[3070.07 \leq \beta_0 \leq 3327.97 \text{ or } 3068.95 \leq \beta_0 \leq 3329.09 \text{ or } 3070.27 \leq \beta_0 \leq 3327.77\]

- **Null and Alternative Hypotheses:**

  \[H_0: \beta_0 = 3000 \]
  \[H_1: \beta_0 \neq 3000 \text{ a two-sided alternative hypothesis} \Rightarrow \text{a two-tailed test}\]

- **Decision Rule:** At significance level \( \alpha \),

  - **reject** \( H_0 \) if the *hypothesized value* \( b_0 \) of \( \beta_0 \) specified by \( H_0 \) lies *outside* the two-sided \((1 - \alpha)\)-level confidence interval for \( \beta_0 \), i.e., if either
  
  (1) \( b_0 < \hat{\beta}_0 - t_{\alpha/2}[855]s\hat{e}(\hat{\beta}_0) \) or
  (2) \( b_0 > \hat{\beta}_0 + t_{\alpha/2}[855]s\hat{e}(\hat{\beta}_0) \).

  - **retain** \( H_0 \) if the *hypothesized value* \( b_0 \) of \( \beta_0 \) specified by \( H_0 \) lies *inside* the two-sided \((1 - \alpha)\)-level confidence interval for \( \beta_0 \), i.e., if

  \( \hat{\beta}_0 - t_{\alpha/2}[855]s\hat{e}(\hat{\beta}_0) \leq b_0 \leq \hat{\beta}_0 + t_{\alpha/2}[855]s\hat{e}(\hat{\beta}_0) \).

- **Inference:**

  - At 5 percent significance level, i.e., for \( \alpha = 0.05 \),

  \[
  b_0 = 3000 < 3070.07 = \hat{\beta}_{0L} = \hat{\beta}_0 - t_{0.025}[855]s\hat{e}(\hat{\beta}_0) \Rightarrow \text{reject } H_0 \text{ vs. } H_1 \text{ at 5 percent level.}
  \]

  - **Inference:** At the 5% significance level, the null hypothesis \( H_0: \beta_0 = 3000 \) is rejected in favour of the alternative hypothesis \( H_1: \beta_0 \neq 3000 \).
(8 marks)
(d) Perform a test of the proposition that each additional prenatal visit made by the mother is associated on average with an increase in their newborn females’ birth weight of less than 25 grams. Use the 5 percent significance level (i.e., $\alpha = 0.05$). State the null hypothesis $H_0$ and the alternative hypothesis $H_1$. Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test.

ANSWER to Question 4(d):

Null and Alternative Hypotheses:

$$H_0: \beta_1 = 25$$
$$H_1: \beta_1 < 25 \quad \Rightarrow \text{a left-tailed } t\text{-test}$$

- Test statistic is $t(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{\hat{s}(\beta_1)} \sim t[N - 2]; \hat{\beta}_1 = 14.1219; \hat{s}(\beta_1) = 5.36347$.

- Calculate the sample value of the $t$-statistic under $H_0$:
  set $\beta_1 = 25$, $\hat{\beta}_1 = 14.1219$, and $\hat{s}(\beta_1) = 5.36347$.

$$t_0(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{\hat{s}(\beta_1)} = \frac{14.1219 - 25.0}{5.36347} = -10.8781 = -2.02818 = -2.03$$

- Null distribution of $t_0(\hat{\beta}_1)$ is $t[N - 2] = t[857 - 2] = t[855]$.

Decision Rule: At significance level $\alpha$,

- reject $H_0$ if $t_0(\hat{\beta}_1) < -t_{\alpha}[855]$,
- retain $H_0$ if $t_0(\hat{\beta}_1) \geq -t_{\alpha}[855]$.

Critical value of $t[855]$-distribution: from t-table, use $df = 120$ or $df = \infty$ or any number in between.

- left-tail 1 percent critical value $= -t_{0.05}[855] = -1.6466 = -1.65$

$$= -t_{0.05}[855] = -1.658 = -1.66$$

$$= -t_{0.05}[\infty] = -1.645 = -1.65$$
ANSWER to Question 4(d) -- continued:

Inference: (1 mark)

♦ At 5 percent significance level, i.e., for $\alpha = 0.05$,

$$t_0(\hat{\beta}_1) = -2.03 < -1.65 = -t_{0.05}[855] \Rightarrow \text{reject } H_0 \text{ vs. } H_1 \text{ at 5 percent level.}$$

♦ Inference: At the 5% significance level, the null hypothesis $H_0: \beta_1 = 25$ is rejected in favour of the alternative hypothesis $H_1: \beta_1 < 25$. 
### Percentage Points of the t-Distribution

**Example**

\[ \Pr(t > 2.086) = 0.025 \]
\[ \Pr(t > 1.725) = 0.05 \quad \text{for } df = 20 \]
\[ \Pr(|t| > 1.725) = 0.10 \]

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*Note:* The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.


### Selected Upper Percentage Points of the F-Distribution

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