QUEEN'S UNIVERSITY AT KINGSTON
Department of Economics

ECONOMICS 351* - Section A

Introductory Econometrics

Fall Term 2002 **MID-TERM EXAM ANSWERS** M.G. Abbott

DATE: **Monday October 28, 2002.**

TIME: **80 minutes; 2:30 p.m. - 3:50 p.m.**

INSTRUCTIONS: The exam consists of **SIX (6)** questions. Students are required to answer **ALL SIX (6)** questions.
Answer all questions in the exam booklets provided. Be sure your ***name*** and ***student number*** are printed clearly on the front of all exam booklets used.
Do not write answers to questions on the front page of the first exam booklet.
Please label clearly each of your answers in the exam booklets with the appropriate number and letter.
Please write legibly.
A table of percentage points of the t-distribution is given on the last page of the exam.

MARKING: The marks for each question are indicated in parentheses immediately above each question. **Total marks** for the exam **equal 100**.

QUESTIONS: **Answer ALL SIX questions.**

All questions pertain to the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \textbf{(1)}$$

where Y_i and X_i are observable variables, β_1 and β_2 are unknown (constant) regression coefficients, and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{u}_i$ is the OLS residual for the i-th sample observation, $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$ is the OLS estimated value of Y for the i-th sample observation, and N is sample size (the number of observations in the sample).

(15 marks)

1. State the Ordinary Least Squares (OLS) estimation criterion. State the OLS normal equations. Derive the OLS normal equations from the OLS estimation criterion.

ANSWER:

(3 marks)

- State the Ordinary Least Squares (OLS) estimation criterion. **(3 marks)**

The OLS coefficient estimators are **those formulas or expressions for $\hat{\beta}_1$ and $\hat{\beta}_2$ that <u>minimize</u> the sum of <u>squared</u> residuals RSS** for any given sample of size N.

The **OLS estimation criterion** is therefore:

$$\underset{\{\hat{\beta}_j\}}{\text{Minimize}}\ \text{RSS}\left(\hat{\beta}_1, \hat{\beta}_2\right) = \sum_{i=1}^{N} \hat{u}_i^2 = \sum_{i=1}^{N} \left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right)^2$$

(4 marks)

- State the OLS normal equations. **(4 marks)**

The ***first*** **OLS normal equation** can be written in *any one* of the following forms:

$$\Sigma_i\, Y_i - N\hat{\beta}_1 - \hat{\beta}_2\, \Sigma_i\, X_i = 0$$

$$- N\hat{\beta}_1 - \hat{\beta}_2\, \Sigma_i\, X_i = -\Sigma_i\, Y_i \qquad \textbf{(N1)}$$

$$N\hat{\beta}_1 + \hat{\beta}_2\, \Sigma_i\, X_i = \Sigma_i\, Y_i$$

The ***second*** **OLS normal equation** can be written in *any one* of the following forms:

$$\Sigma_i\, X_i Y_i - \hat{\beta}_1\, \Sigma_i\, X_i - \hat{\beta}_2\, \Sigma_i\, X_i^2 = 0$$

$$- \hat{\beta}_1\, \Sigma_i\, X_i - \hat{\beta}_2\, \Sigma_i\, X_i^2 = -\Sigma_i\, X_i Y_i \qquad \textbf{(N2)}$$

$$\hat{\beta}_1\, \Sigma_i\, X_i + \hat{\beta}_2\, \Sigma_i\, X_i^2 = \Sigma_i\, X_i Y_i$$

Question 1 (continued)

(8 marks)

- Show how the OLS normal equations are derived from the OLS estimation criterion.

(4 marks)

Step 1: **Partially differentiate** the RSS $\left(\hat{\beta}_1, \hat{\beta}_2\right)$ function with respect to $\hat{\beta}_1$ and $\hat{\beta}_2$, using

$$\hat{u}_i = Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \quad \Rightarrow \quad \frac{\partial \hat{u}_i}{\partial \hat{\beta}_1} = -1 \quad \text{and} \quad \frac{\partial \hat{u}_i}{\partial \hat{\beta}_2} = -X_i .$$

$$\frac{\partial RSS}{\partial \hat{\beta}_1} = \sum_{i=1}^{N} 2\hat{u}_i \left(\frac{\partial \hat{u}_i}{\partial \hat{\beta}_1}\right) = \sum_{i=1}^{N} 2\,\hat{u}_i(-1) = -2\sum_{i=1}^{N} \hat{u}_i = -2\sum_{i=1}^{N}\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) \tag{1}$$

$$\frac{\partial RSS}{\partial \hat{\beta}_2} = \sum_{i=1}^{N} 2\hat{u}_i \left(\frac{\partial \hat{u}_i}{\partial \hat{\beta}_2}\right) = \sum_{i=1}^{N} 2\hat{u}_i(-X_i) = -2\sum_{i=1}^{N} X_i \hat{u}_i$$

$$= -2\sum_{i=1}^{N} X_i\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) \quad \text{since } \hat{u}_i = Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \tag{2}$$

$$= -2\sum_{i=1}^{N}\left(X_i Y_i - \hat{\beta}_1 X_i - \hat{\beta}_2 X_i^2\right).$$

(4 marks)

Step 2: **Obtain the first-order conditions (FOCs)** for a minimum of the RSS function by **setting the partial derivatives (1) and (2) *equal to zero*** and then dividing each equation by –2 and re-arranging:

$$\frac{\partial RSS}{\partial \hat{\beta}_1} = 0 \quad \Rightarrow \quad -2\sum_i \hat{u}_i = 0 \quad \Rightarrow \quad -2\sum_i\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) = 0$$

$$\Rightarrow \quad \sum_i\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) = 0$$

$$\Rightarrow \quad \sum_i Y_i - N\hat{\beta}_1 - \hat{\beta}_2 \sum_i X_i = 0$$

$$\Rightarrow \quad \sum_i Y_i = N\hat{\beta}_1 + \hat{\beta}_2 \sum_i X_i \tag{N1}$$

$$\frac{\partial RSS}{\partial \hat{\beta}_2} = 0 \quad \Rightarrow \quad -2\sum_i X_i \hat{u}_i = 0 \quad \Rightarrow \quad -2\sum_i X_i\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) = 0$$

$$\Rightarrow \quad \sum_i X_i\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) = 0$$

$$\Rightarrow \quad \sum_i\left(X_i Y_i - \hat{\beta}_1 X_i - \hat{\beta}_2 X_i^2\right) = 0$$

$$\Rightarrow \quad \sum_i X_i Y_i - \hat{\beta}_1 \sum_i X_i - \hat{\beta}_2 \sum_i X_i^2 = 0$$

$$\Rightarrow \quad \sum_i X_i Y_i = \hat{\beta}_1 \sum_i X_i + \hat{\beta}_2 \sum_i X_i^2 \tag{N2}$$

(15 marks)

2. Show that the OLS slope coefficient estimator $\hat{\beta}_2$ is a linear function of the Y_i sample values. Stating explicitly all required assumptions, prove that the OLS slope coefficient estimator $\hat{\beta}_2$ is an unbiased estimator of the slope coefficient β_2.

(5 marks)

- Show that the OLS slope coefficient estimator $\hat{\beta}_2$ is a *linear* function of the Y_i sample values.

$$\hat{\beta}_2 = \frac{\sum_i x_i y_i}{\sum_i x_i^2} = \frac{\sum_i x_i (Y_i - \overline{Y})}{\sum_i x_i^2} = \frac{\sum_i x_i Y_i}{\sum_i x_i^2} - \frac{\overline{Y}\sum_i x_i}{\sum_i x_i^2}$$

$$= \frac{\sum_i x_i Y_i}{\sum_i x_i^2} \qquad \text{because } \sum_i x_i = 0$$ **(5 marks)**

$$= \sum_i k_i Y_i \qquad \text{where } k_i \equiv \frac{x_i}{\sum_i x_i^2}.$$

(10 marks)

- Stating explicitly all required assumptions, prove that the OLS slope coefficient estimator $\hat{\beta}_2$ is an unbiased estimator of the slope coefficient β_2.

(1) Substitute for Y_i the expression $Y_i = \beta_1 + \beta_2 X_i + u_i$ from the population regression equation (or PRE). **(5 marks)**

$$\begin{aligned}
\hat{\beta}_2 &= \sum_i k_i Y_i \\
&= \sum_i k_i(\beta_1 + \beta_2 X_i + u_i) && \text{since } Y_i = \beta_1 + \beta_2 X_i + u_i \text{ by assumption (A1)} \\
&= \sum_i (\beta_1 k_i + \beta_2 k_i X_i + k_i u_i) \\
&= \beta_1 \sum_i k_i + \beta_2 \sum_i k_i X_i + \sum_i k_i u_i \\
&= \beta_2 + \sum_i k_i u_i, && \text{since } \sum_i k_i = 0 \text{ and } \sum_i k_i X_i = 1
\end{aligned}$$

(2) Now **take expectations** of the above expression for $\hat{\beta}_2$: **(5 marks)**

$$\begin{aligned}
E(\hat{\beta}_2) &= E(\beta_2) + E[\textstyle\sum_i k_i u_i] \\
&= \beta_2 + \sum_i k_i E(u_i) && \text{since } \beta_2 \text{ is a constant and the } k_i \text{ are nonstochastic} \\
&= \beta_2 + \sum_i k_i 0 && \text{since } E(u_i) = 0 \text{ by assumption (A2)} \\
&= \beta_2.
\end{aligned}$$

(10 marks)

3. Answer parts (a) and (b) below.

(a) Write the expression (or formula) for $\text{Var}(\hat{\beta}_2)$, the variance of $\hat{\beta}_2$. **(5 marks)**

ANSWER:

$$\text{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum_{i=1}^{N} x_i^2} = \frac{\sigma^2}{\sum_{i=1}^{N} (X_i - \bar{X})^2} \qquad \text{where} \quad x_i \equiv X_i - \bar{X}, \quad i = 1, \ldots, N$$

(b) Which of the following factors makes $\text{Var}(\hat{\beta}_2)$ *smaller*? **(5 marks)**

ANSWER: Correct answers are highlighted in bold.

(1) a smaller value of N, sample size

(2) smaller values of $x_i^2 = (X_i - \bar{X})^2$, i = 1, …, N

(3) a larger value of σ^2, the error variance

(4) a smaller value of σ^2, the error variance

(5) a larger value of N, sample size

(6) larger values of $x_i^2 = (X_i - \bar{X})^2$, i = 1, …, N

(10 marks)

4. Explain what is meant by each of the following statements about the estimator $\hat{\theta}$ of the population parameter θ.

(a) $\hat{\theta}$ is an unbiased estimator of θ.

(b) $\hat{\theta}$ is an efficient estimator of θ.

ANSWER:

(5 marks)

- **(a)** $\hat{\theta}$ is an ***unbiased*** **estimator** of θ.

  $\hat{\theta}$ is an unbiased estimator of θ if the mean, or expectation, of the estimator $\hat{\theta}$ equals the true parameter value θ for any finite sample size $n < \infty$.

  $$E(\hat{\theta}) = \theta \quad \Rightarrow \quad \text{Bias}(\hat{\theta}) \equiv E(\hat{\theta}) - \theta = 0.$$

  The condition $E(\hat{\theta}) = \theta$ says that the sampling distribution of the estimator $\hat{\theta}$ is centered on the true parameter value θ, that *on average* the estimator $\hat{\theta}$ is correct.

(5 marks)

- **(b)** $\hat{\theta}$ is an ***efficient*** **estimator** of θ.

  The estimator $\hat{\theta}$ is an efficient estimator if it is ***unbiased*** and has ***smaller variance*** than *any other* ***unbiased*** estimator of the parameter θ.

  If $\tilde{\theta}$ is *any other* ***unbiased*** *estimator* of θ, then $\hat{\theta}$ is an ***efficient*** **estimator** of θ if

  $$\text{Var}(\hat{\theta}) \le \text{Var}(\tilde{\theta}) \qquad \text{where } E(\hat{\theta}) = \theta \text{ and } E(\tilde{\theta}) = \theta.$$

  Note: Answer must recognize that **unbiasedness** is a ***necessary condition*** **for efficiency**.

(34 marks)

5. A researcher is using data for a sample of 25 business schools that offer MBA degrees to investigate the relationship between the annual salary gain of graduates Y_i (measured in *thousands* of dollars per year) and annual tuition fees X_i (measured in *thousands* of dollars per year). Preliminary analysis of the sample data produces the following sample information:

$$N = 25 \qquad \sum_{i=1}^{N} Y_i = 1{,}034.97 \qquad \sum_{i=1}^{N} X_i = 528.599 \qquad \sum_{i=1}^{N} Y_i^2 = 45{,}237.19$$

$$\sum_{i=1}^{N} X_i^2 = 11{,}432.92 \qquad \sum_{i=1}^{N} X_i Y_i = 22{,}250.54 \qquad \sum_{i=1}^{N} x_i y_i = 367.179$$

$$\sum_{i=1}^{N} y_i^2 = 2{,}390.67 \qquad \sum_{i=1}^{N} x_i^2 = 256.241 \qquad \sum_{i=1}^{N} \hat{y}_i^2 = 526.147$$

where $x_i \equiv X_i - \overline{X}$, $y_i \equiv Y_i - \overline{Y}$ and $\hat{y}_i \equiv \hat{Y}_i - \overline{Y}$ for i = 1, ..., N. Use the above sample information to answer all the following questions. **Show explicitly all formulas and calculations**.

(10 marks)

(a) Use the above information to compute OLS estimates of the intercept coefficient β_1 and the slope coefficient β_2.

- $\hat{\beta}_2 = \dfrac{\sum_i x_i y_i}{\sum_i x_i^2} = \dfrac{367.179}{256.241} = 1.4329 = \underline{\mathbf{1.433}}$ **(5 marks)**

- $\hat{\beta}_1 = \overline{Y} - \hat{\beta}_2 \overline{X}$

  $$\overline{Y} = \frac{\sum_{i=1}^{N} Y_i}{N} = \frac{1034.97}{25} = 41.3988 \quad \text{and} \quad \overline{X} = \frac{\sum_{i=1}^{N} X_i}{N} = \frac{528.599}{25} = 21.1440$$

  Therefore

  $$\hat{\beta}_1 = \overline{Y} - \beta_2 \overline{X} = 41.3988 - (1.4329)(21.1440) = 41.3988 - 30.2972 = \underline{\mathbf{11.101}}$$

  (5 marks)

(6 marks)

(b) Interpret the slope coefficient estimate you calculated in part (a) -- i.e., explain in words what the numeric value you calculated for $\hat{\beta}_2$ means.

Note: $\hat{\beta}_2$ = **1.433**. Y_i is measured in ***thousands of dollars***, and X_i is measured in ***thousands of dollars***.

The estimate **1.433** of β_2 means that an ***increase* (decrease) in tuition** X_i of ***1,000 dollars*** is associated on average with **an *increase* (decrease) in MBA graduates' annual salary gain equal to *1.433 thousand dollars***, or ***1,433 dollars***.

(6 marks)

(c) Calculate an estimate of σ^2, the error variance.

$$\text{RSS} = \sum_{i=1}^{N} \hat{u}_i^2 = \sum_{i=1}^{N} y_i^2 - \sum_{i=1}^{N} \hat{y}_i^2 = 2{,}390.67 - 526.147 = 1{,}864.523$$

$$\hat{\sigma}^2 = \frac{\text{RSS}}{N-2} = \frac{\sum_{i=1}^{N} \hat{u}_i^2}{N-2} = \frac{1{,}864.523}{25-2} = \frac{1{,}864.523}{23} = \underline{\mathbf{81.0662}}$$

(6 marks)

(d) Compute the value of R^2, the coefficient of determination for the estimated OLS sample regression equation. Briefly explain what the calculated value of R^2 means.

$$R^2 = \frac{\text{ESS}}{\text{TSS}} = \frac{\sum_{i=1}^{N} \hat{y}_i^2}{\sum_{i=1}^{N} y_i^2} = \frac{526.147}{2390.67} = \mathbf{0.220083} = \underline{\mathbf{0.2201}}$$ **(4 marks)**

Interpretation of R^2 = 0.2201: The value of 0.2201 indicates that **22.01 percent of the total sample (or observed) variation in Y_i** (annual salary gain of graduates) **is *attributable to*, or *explained by*, the regressor X_i** (annual tuition fees). **(2 marks)**

(6 marks)

(e) Compute the estimated variance of $\hat{\beta}_2$ and the estimated standard error of $\hat{\beta}_2$.

$$\hat{Var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum_{i=1}^{N} x_i^2} = \frac{81.0662}{256.241} = \mathbf{0.316367}$$ **(4 marks)**

$$\hat{se}(\hat{\beta}_2) = \sqrt{\hat{Var}(\hat{\beta}_2)} = \sqrt{0.316367} = \mathbf{0.562465}$$ **(2 marks)**

(16 marks)

6. You have been commissioned to investigate the relationship between annual R&D expenditures (Y) and total annual sales revenues (X) for chemical firms. You have assembled data for a sample of 32 chemical firms, where Y_i is annual R&D expenditures of the i-th firm (measured in *millions* of dollars per year) and X_i is total annual sales revenues of the i-th firm (measured in *millions* of dollars per year). Your research assistant has used the sample data to estimate the following OLS sample regression equation, where the figures in parentheses below the coefficient estimates are the *estimated standard errors* of the coefficient estimates:

$$\underset{}{Y_i} = \underset{(20.515)}{-0.5772} + \underset{(0.0024487)}{0.04063 X_i} + \hat{u}_i \qquad (i = 1, ..., N) \qquad N = 32 \qquad \textbf{(3)}$$

(8 marks)

(a) Compute the two-sided 95% confidence interval for the slope coefficient β_2.

The **two-sided (1 – α)-level,** or **100(1 – α) percent, confidence interval for β_2** is computed as

$$\hat{\beta}_2 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2) \le \beta_2 \le \hat{\beta}_2 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2) \qquad \textbf{(2 marks)}$$

where

- $\hat{\beta}_{2L} = \hat{\beta}_2 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2)$ = the ***lower*** **100(1 – α)% confidence limit for β_2**
- $\hat{\beta}_{2U} = \hat{\beta}_2 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2)$ = the ***upper*** **100(1 – α)% confidence limit for β_2**
- $t_{\alpha/2}[N-2]$ = the **α/2 critical value of the t-distribution with N–2 degrees of freedom**.

- Required results and intermediate calculations:

$N - k = 32 - 2 = 30$; $\hat{\beta}_2 = \mathbf{0.04063}$; $\hat{se}(\hat{\beta}_2) = \mathbf{0.0024487}$

$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow \boldsymbol{\alpha/2 = 0.025}$: $t_{\alpha/2}[N-2] = \mathbf{t_{0.025}[30] = 2.042}$

$t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2) = t_{0.025}[30]\,\hat{se}(\hat{\beta}_2) = 2.042(0.0024487) = \mathbf{0.005000245}$

- **Lower 95% confidence limit for β_2** is: **(3 marks)**

$$\hat{\beta}_{2L} = \hat{\beta}_2 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2) = \hat{\beta}_2 - t_{0.025}[30]\,\hat{se}(\hat{\beta}_2)$$
$$= 0.04063 - 2.042(0.002487) = 0.04063 - 0.005000245 = 0.035630 = \underline{\mathbf{0.03563}}$$

Question 6(a) -- continued

- **Upper 95% confidence limit for β_2** is: **(3 marks)**

$$\hat{\beta}_{2U} = \hat{\beta}_2 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2) = \hat{\beta}_2 + t_{0.025}[30]\,\hat{se}(\hat{\beta}_2)$$
$$= 0.04063 + 2.042(0.002487) = 0.04063 + 0.005000245 = 0.045630 = \underline{\mathbf{0.04563}}$$

- **Result:** The **two-sided 95% confidence interval for β_2** is:

[0.03563, 0.04563]

(8 marks)

(b) Perform a test of the null hypothesis H_0: $\beta_2 = 0$ against the alternative hypothesis H_1: $\beta_2 \neq 0$ at the 1% significance level (i.e., for significance level $\alpha = 0.01$). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.

H_0: $\beta_2 = 0$
H_1: $\beta_2 \neq 0$ a ***two-sided*** **alternative hypothesis** $\Rightarrow$ a ***two-tailed*** **test**

- Test statistic is $t(\hat{\beta}_2) = \dfrac{\hat{\beta}_2 - \beta_2}{\hat{se}(\hat{\beta}_2)} \sim t[N-2]$. (1)

- $\hat{\beta}_2 = 0.04063$ and $\hat{se}(\hat{\beta}_2) = 0.0024487$.

- Calculate the ***sample value*** **of the t-statistic** (1) under H_0: set $\beta_2 = 0$ in (1).

$$t_0(\hat{\beta}_2) = \frac{\hat{\beta}_2 - \beta_2}{\hat{se}(\hat{\beta}_2)} = \frac{0.04063 - 0.0}{0.0024487} = \frac{0.04063}{0.0024487} = 16.5925 = \underline{\mathbf{16.59}}$$ **(3 marks)**

- **Null distribution** of $t_0(\hat{\beta}_2)$ is **t[N – 2] = t[32 – 2] = t[30]**. **(1 mark)**

Decision Rule: At significance level α, **(1 mark)**

- ***reject*** **H_0** if $\left|t_0(\hat{\beta}_2)\right| > t_{\alpha/2}[30]$,
  i.e., if either (1) $t_0(\hat{\beta}_2) > t_{\alpha/2}[30]$ or (2) $t_0(\hat{\beta}_2) < -t_{\alpha/2}[30]$;

- ***retain*** **H_0** if $\left|t_0(\hat{\beta}_2)\right| \leq t_{\alpha/2}[30]$, i.e., if $-t_{\alpha/2}[30] \leq t_0(\hat{\beta}_2) \leq t_{\alpha/2}[30]$.

Question 6(b) -- continued

Critical value of t[30]-distribution: from t-table, use **df = 30**.

- ***two-tailed*** **<u>1 percent</u> critical value** = $t_{\alpha/2}[30] = t_{0.005}[30] = \underline{\mathbf{2.750}}$ **(1 mark)**

<u>Inference</u>: **(1 mark)**

♦ At **1 percent significance level**, i.e., for $\alpha = 0.01$,

$$\left|t_0(\hat{\beta}_2)\right| = 16.59 > 2.750 = t_{0.005}[30] \Rightarrow$$ ***reject*** $\mathbf{H_0}$ vs. H_1 **at 1 percent level**.

♦ **<u>Inference</u>: At the 1% significance level**, the null hypothesis $\beta_2 = 0$ is ***rejected*** in favour of the alternative hypothesis $\beta_2 \neq 0$.

Meaning of test outcome: **(1 mark)**
Rejection of the null hypothesis $\beta_2 = 0$ against the alternative hypothesis $\beta_2 \neq 0$ means that **the sample evidence favours the existence of a relationship between *annual R&D expenditures* and *total annual sales revenues***.

Percentage Points of the t-Distribution

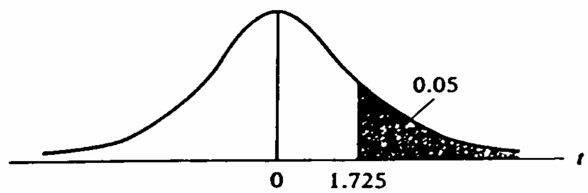
TABLE D.2
Percentage points of the *t* distribution

Example

Pr(t > 2.086) = 0.025

Pr(t > 1.725) = 0.05 for df = 20

Pr($|t|$ > 1.725) = 0.10



df \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.